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Equilibrium in competitive insurance markets with ex ante adverse selection and ex post moral hazard

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Abstract

Existence of pure strategy equilibria is studied in health insurance markets that exhibit both ex ante adverse selection of the Rothschild–Stiglitz–Wilson type, and ex post hidden information moral hazard. It is found that ex post moral hazard has two offsetting effects on the existence of equilibrium, and that in general it is difficult to say whether an equilibrium is more or less likely to exist. Numerical simulations, and an analytical example, confirm that moral hazard may increase the likelihood of equilibrium. These results are interpreted as evidence that Health Maintenance Organization (HMO) markets could be less likely to exhibit stable equilibria than are fee-for-service insurance markets. © 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

In important papers, Rothschild and Stiglitz (1976) and Wilson (1977) (henceforth RSW) identified a fundamental weakness of insurance markets. In the presence of asymmetric information about ex ante risk attributes, a competitive equilibrium may fail to exist. Some authors have responded to the problem of non-existence by modifying the structure of the original RSW model to establish existence of equilibria. For example, Dasgupta and Maskin (1986a,b) show that if mixed strategies are permitted an equilibrium always exists. Alternatively, Wilson (1977) extended his basic model to include firms' explicit dynamic reactions to

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new contract offers, and found that pure strategy equilibria generally exist. These papers increased the likelihood of an equilibrium existing (to unity) by, respectively, widening the strategy space and narrowing the set of feasible equilibrium-breaking contracts.

In this paper, I examine the existence of pure strategy competitive equilibrium in insurance markets that exhibit both adverse selection and, in addition, ex post moral hazard. In this set-up, insurers can observe neither the ex ante risk characteristics of purchasers of insurance (as in RSW), nor the state of the world that occurs ex post.¹ This information structure is meant to provide a more realistic description of orthodox fee-for-service medical insurance in which insurance companies are unable to monitor accurately the true health status (i.e., medical care needs) of claimants. I will argue below that the standard RSW model without ex post moral hazard is more descriptive of pre-paid insurance policies offered by US-style Health Maintenance Organizations (HMOs), and that a comparison of the two models allows an analysis of the relative market performance of the two organizational forms.

The intuition for the non-existence result in the standard model given by Rothschild and Stiglitz is that if an equilibrium exists, it must be a separating equilibrium in which individuals with different risks purchase different insurance contracts. (No pooling equilibrium exists.) This separation imposes a *cost* on low risks, who are only partially insured due to the self selection constraint. They are prepared to pay to reduce their risk exposure, and if there are enough of them an insurer could offer such a contract, and earn sufficient profits from the low risks to offset any loss incurred on high risks who also opt for the new contract. That is, a pooling contract might exist to break the potential separating equilibrium.

When the model is extended to include ex post moral hazard, both high and low risk individuals typically purchase less than full insurance, even in the absence of adverse selection. This is because the unobservability of the state of the world precludes optimal redistribution across such states, making full insurance too costly for both risk types. The costs to low risks of separation may thus be relatively less, and a pooling equilibrium that breaks the potential separating equilibrium less likely. Moral hazard could thus increase the range of parameters over which an equilibrium exists.

The main contribution of this paper is to confirm this intuition. Simple conditions under which fee-for-service markets are more likely to support an

¹What I refer to here as ex post moral hazard has been labelled *hidden information* moral hazard by, e.g., Hart and Holmstrom (1987). I do not consider what might be termed ex interim, or *hidden action* moral hazard, whereby individuals can take unobservable actions that affect the probability distribution over states of nature. Also, I do not consider the intermediate case of costly state verification (e.g., Townsend, 1979) in which a principle (the insurer) chooses an optimal auditing policy. Instead, the two cases considered here (with and without moral hazard) can be considered to represent infinitely costly and costless verification, respectively.

equilibrium than HMO markets are not available. However, with the help of an analytical example and numerical simulations I am able to show that for some parameterisations of preferences, an improved informational environment in which ex post moral hazard is absent (i.e., in which the state of the world is observable) is detrimental to the attainment of equilibrium.

To understand these results, it is useful to reinterpret the original RSW intuition regarding the conditions under which an equilibrium will not exist. It is not so much the *cost* (in terms of expected utility) of separation that makes a pooling contract both profitable to insurers and attractive to low risk individuals. Instead it is more useful to recognise that in a separating equilibrium, low risk individuals are *constrained* to purchase a sub-optimal quantity of insurance at *actuarially fair rates*. The relevant description of the impact of incomplete insurance is in terms of its effect on the willingness of low risk individuals to purchase *unconstrained* quantities of insurance at *actuarially unfair rates*. The less fair the rate — i.e., the higher the price — at which low risk individuals would be willing to purchase unconstrained quantities of insurance — the less likely is an equilibrium to exist.

Ex post moral hazard has two effects on the willingness of low risk individuals to purchase insurance at actuarially unfair rates. First, moral hazard reduces the demand for insurance at all prices, so the highest price at which otherwise constrained low risk individuals would be willing to purchase insurance *falls*. I refer to this as the ‘internal incentive effect’ as it is not a direct consequence of the presence of high risk individuals in the market.² The second effect is that, because high risk individuals also prefer to purchase incomplete insurance, the constrained level of insurance offered to low risk individuals at a separating equilibrium is reduced. This ‘external incentive effect’ *increases* the price at which low risk individuals are willing to purchase unconstrained quantities, essentially because the value of their outside option is reduced.³ These two offsetting effects make it difficult to analytically sign the change in the critical insurance price. Section 5.1 below presents an example in which there is no external incentive effect, so that the net impact of moral hazard is to unambiguously increase the likelihood of equilibrium. More generally, the numerical simulations provide examples of when each effect dominates.

Of course, when an equilibrium does exist in both types of market (i.e., with and without ex post moral hazard), the HMO allocations will Pareto dominate the fee-for-service ones. Thus, conditional on an equilibrium existing, more information (the removal of ex post moral hazard) is better, as would be expected. However, in those cases when improved information leads to non-existence of equilibrium, it is arguable that more information is actually worse, as it causes the market to break down relatively more often.

²It is, of course, just a consequence of moral hazard, but overuse of this term seems imprudent.

³This effect derives from the impact of moral hazard on the adverse selection incentive constraint. Again, solely for reasons of clarity, avoidance of these terms seems advisable.

There is a danger in comparing two institutional arrangements with different equilibrium existence properties. Ideally we should search for a more appropriate equilibrium concept, perhaps within the context of a dynamic model, that assures existence for all parameterizations. Although speculative at this stage, one might expect that in cases when a standard RSW equilibrium exists, the equilibrium in the more general model would coincide with it, but that when an RSW equilibrium does not exist the equilibrium of the more general model could exhibit cycling or some other form of non-stationarity. If there are sufficient welfare costs associated with the second type of equilibrium (for example, due to a lack of ‘life-time’ insurance), a more definitive comparison of institutional arrangements would be possible.

The only other paper to have examined the issue of equilibrium existence under different institutional structures that I am aware of is Chernew and Frick (1999). They allow insurance companies to offer contracts with both a cost-sharing (i.e., fee for service) component and a non-financial cost of obtaining care, identified as the degree of ‘managedness.’ Managedness reduces a consumer’s utility because it constrains his ex post choices. The existence of this second contractual dimension expands both the degree to which low risks can differentiate themselves from high risks, and the set of contracts that can break any potential separating equilibrium. In this sense, the authors’ finding that the effects of managed care on equilibrium existence are ambiguous are not too surprising. In the current paper however, the essential difference between the two institutions is an improvement in the information structure, which nonetheless can have detrimental effects on equilibrium existence.

Adverse selection in health insurance markets has recently received growing attention in the empirical literature. Cutler and Zeckhauser (1997) and Cutler and Reber (1998) both find evidence that when employers changed the implicit subsidies to, and hence relative prices of, alternative insurance packages, consumers responded in the way predicted by the standard theory. In a study of a natural experiment involving states in the US, Buchmueller and DiNardo (1999) found little evidence of an ‘adverse selection spiral,’ akin to Akerlof’s (1970) no-trade equilibrium, but did find that the data were consistent with the RSW model. None of these studies however have attempted to tackle the issue of existence.

The next section presents a formal model of health and insurance choice under full and symmetric information about ex ante risk types and ex post health status. Section 3 introduces adverse selection, while retaining the assumption of full and symmetric information ex post, and examines the equilibrium properties of the insurance market. The model is essentially the same as that of the original RSW framework, and represents the informational structure faced by HMOs. Section 4 models fee-for-service insurance by including ex post moral hazard through the assumption that health status is not observable by the insurer. It examines the effects of the additional information constraint on the nature and existence of equilibrium. Section 5 presents an analytical example of a situation in which moral

hazard must increase the chance of an equilibrium, and reports the results of numerical simulations. The final section concludes. All proofs are relegated to Appendix A.

2. Insurance choice with full and symmetric information

In this section I present a simple model of health insurance under full and symmetric information about ex ante risk types and ex post states of nature. Following the analysis of Jack and Sheiner (1997), an individual's utility $u(c, h)$ is assumed to be a well-behaved function of health h (not to be confused with health care services) and other consumption c . Exogenous income is denoted by w . The shock against which individuals wish to insure is the price of health, θ , and individuals purchase insurance to maximise their expected utility over states of the world.⁴ Consistent with the empirical literature, I will assume that the elasticity of demand for health is less than unity, so that sicker people spend more on health. Jack and Sheiner assumed a continuous distribution of θ in their model of insurance choice without ex ante adverse selection. Here, as I will later want to examine the interaction of ex ante adverse selection and ex post moral hazard, I simplify the ex post structure of the model and assume that θ can take on one of just two values, θ_g in the good state of the world, and $\theta_b > \theta_g$ in the bad state. As in RSW, individuals differ ex ante according to their risk of being in the good or bad state: high risk individuals have a probability p^h of being in the bad state (and $(1 - p^h)$ of being in the good state), while the corresponding parameter for low risk individuals is $p^l < p^h$. Low risk individuals make up a proportion λ of the population.

When individuals' risk types are observable by insurers, insurance contracts can differ according to risk. In this case, assuming a competitive insurance market and zero profits, insurance contracts are offered independently to each type. Also, as the ex post state of the world (i.e., health status) is observable, contracts can be made state-contingent, and optimally are characterised by lump-sum transfers between states of the world. Thus, for individuals of type i the contract is described by a pair of transfers $\{t_g^i, t_b^i\}$, with the components indexed to the states of the world. The transfers are interpreted as net transfers from individuals to the

⁴This interpretation can be derived from a simple health production function model. Assume that θ represents generic health status, and health h , is produced under constant returns to scale from inputs z , with

$$h = f(z, \theta) = z/\theta.$$

(Introducing decreasing returns does not alter the intuition of the paper.) Thus θ determines the productivity of health inputs. If input prices are p , then the minimum cost of attaining health h in state θ is $c(h, \theta) = p\theta h$. If p is normalized to unity, θ can be interpreted as the price of health.

insurance company, so it is natural to expect that $t_g^i > 0 > t_b^i$. The optimal contract for each risk type i solves the following optimisation problem:

$$\max_{\{t_b^i, t_g^i\}} E_i[v(\theta, w - t^i)] \text{ s.t. } E_i \pi_i = 0$$

where $v(\theta, w - t)$ is the indirect utility function and π_i is the profit an insurer earns from type i individuals. More explicitly, the optimal contract for type i individuals satisfies

$$\begin{aligned} \max_{\{t_b^i, t_g^i\}} & p^i v(\theta_b, w - t_b^i) + (1 - p^i) v(\theta_g, w - t_g^i) \\ \text{s.t.} & p^i t_b^i + (1 - p^i) t_g^i = 0 \end{aligned}$$

The solution to this problem is characterised by full insurance, in which the marginal utility of income is equated across states of the world. This can be seen by simply substituting the constraint equation into the objective function, which yields

$$v_2(\theta_b, w - t_b^i) = v_2(\theta_g, w - t_g^i)$$

where the subscript on the utility function denotes partial differentiation with respect to the second argument. In standard models of income (as opposed to price) risk (as in RSW) optimal insurance is characterised by equality of income, and hence of the marginal utility of income, across states of the world, and the optimal contract is located on the ‘certainty’ line. Here, the optimal contract is located on the ‘full insurance’ line, whose equation is denoted $m_b = \phi(m_g)$, where $m_s = w - t_s$ is net income in state of the world s , and which is shown in Fig. 1. The contracts for both high and low risk individuals are shown, denoted H_0 and L_0 , respectively. Note that the axes of the figure represent nominal income in each state of the world, and so the initial endowment is on the certainty line at point E . However, the equal marginal utilities or full insurance line along which the optimal contracts lie is to the north-west.

3. Adverse selection

In this section, I assume that insurers cannot distinguish between high and low risk individuals, but that if and when an illness or accident occurs, they can costlessly observe the nature and extent of medical need. In terms of the stylized model, insurers can observe the ex post price of health, θ . This assumption is meant to characterise in a simple way the informational structure faced by HMO insurance companies. Because HMO staff include physicians, who are assumed to be at least as good at diagnosing health conditions as individuals, it seems reasonable to ascribe to them the ability to observe θ . On the other hand, in

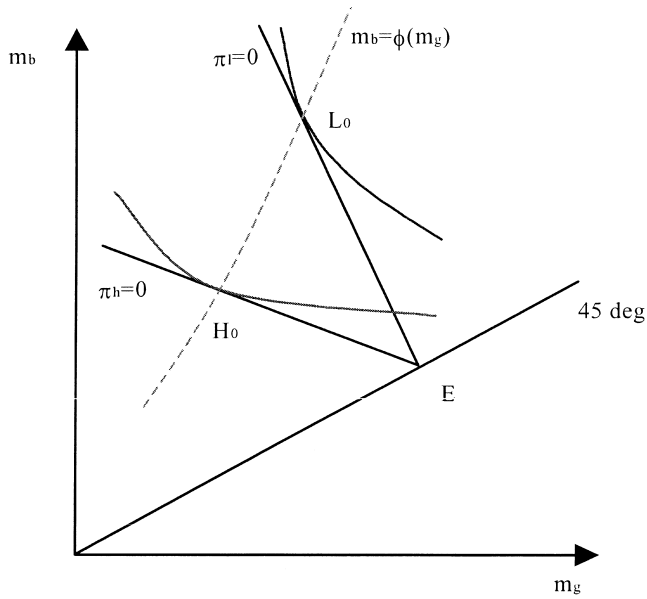


Fig. 1. Equilibrium contracts with full and complete information.

assuming that the HMO cannot observe risk characteristics, I endow individuals with more knowledge of likely health-related outcomes regarding their life-styles and predisposition towards disease contraction and accidents.⁵

If insurers cannot distinguish between different risk types ex ante, then the optimal contracts characterised above will not be sustainable in equilibrium. If the two contracts H_0 and L_0 are offered, both high and low risk individuals will choose the contract designed for low risks, L_0 , and profits will be negative. The equilibrium contracts now solve the following joint optimisation problem for $i = l, h$:

$$\begin{aligned}
 & \max_{\{t_g^i, t_b^i\}} E_i[v(\theta, w - t^i)] \\
 & \text{s.t.} \quad \pi_i = 0 \\
 & \text{and} \quad E_i[v(\theta, w - t^i)] \geq E_i[v(\theta, w - t^j)]
 \end{aligned}$$

Profits continue to be constrained to be zero for each risk type separately (i.e., there is no cross subsidisation). The second constraint requires that individuals

⁵Of course, it is reasonable to assume that HMOs can engage in some screening of risks at the time of enrollment. However, such screening is also carried out by fee-for-service insurers, and as I wish to differentiate between the types of insurance according to ex post observability of states of the world, the assumption of ex ante adverse selection appears reasonable.

self-select and choose contracts appropriate to their types. As RSW show, the impact of these constraints is that equilibrium contracts must differ between individuals (i.e., there is no pooling equilibrium), and that the equilibrium insurance coverage of low risk individuals is incomplete, at point L_1 in Fig. 2. While the contract they purchase transfers income between the two states at the actuarially fair price, they are constrained from purchasing their desired quantity of insurance at that price. High risk individuals continue to be fully insured at point H_0 .

The pair of contracts $\{H_0, L_1\}$ represents the only possible equilibrium. But it may not be sustainable. If there are sufficiently many low risk individuals so that the average market odds line EA is close enough to the zero profit line for low risks, $\pi_l = 0$, then the potential separating equilibrium will not be sustainable. The existence of a contract such as γ in Fig. 2 will break the equilibrium, since both high and low risks prefer it, and it earns positive total profits.

In general there exists some cut-off proportion of low risks in the population, λ^* , such that if the actual proportion of low risks λ is greater than λ^* , an equilibrium will fail to exist. λ^* is that proportion of low risks that results in the average market odds line EA being tangent to the low risk indifference curve through L_1 . The smaller is λ^* the less likely is an equilibrium to exist. Thus I identify λ^* with the chance or likelihood of a pure strategy equilibrium existing.

It is clear from Fig. 2 that, *ceteris paribus*, the further is L_1 from the full insurance point L_0 , the smaller is λ^* . In particular, for a given shape of the low

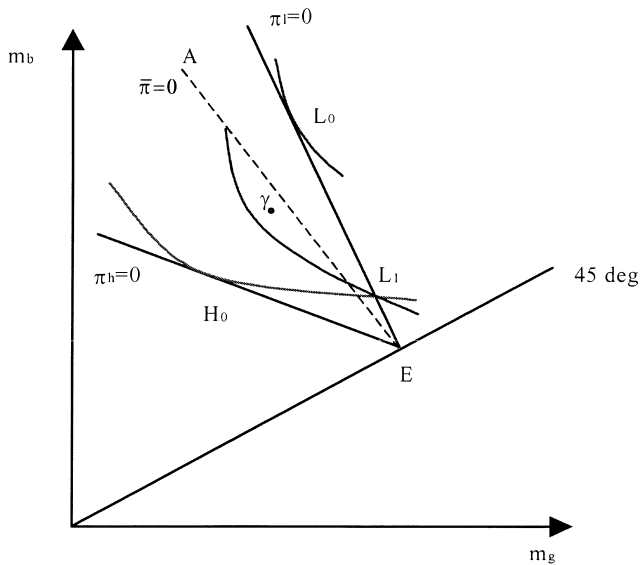


Fig. 2. Equilibrium in the RSW model of adverse selection.

risk indifference curves the greater the gap between full insurance and the actual insurance that low risks can obtain, the less likely is an equilibrium to occur. Low risk individuals would be willing to purchase insurance at a price higher than the actuarially fair rate of $p^l/1 - p^l$, as long as they faced an unconstrained quantity choice. The highest price at which they would be willing to purchase insurance is $p^*/1 - p^*$ where $p^* = \lambda^*p^l + (1 - \lambda^*)p^h$. Since high risk individuals will also wish to purchase insurance at this price, there needs to be sufficiently many low risk individuals in the population (i.e., λ must be greater than λ^*) to induce insurers to offer contracts at such prices.

Note however that λ^* also depends on the shape of the low risk indifference curve in Fig. 2. Heuristically, the higher the curvature in the relevant range, the lower will λ^* be, and the less likely will an equilibrium be to exist. Equivalently, the higher the curvature, say due to greater risk aversion, the higher will be the price above the actuarially fair rate at which a low risk individual will be willing to purchase an unconstrained quantity of insurance. Again, if there are sufficiently many low risk individuals to subsidise the high risks who also purchase insurance at the intermediate price, then insurers will be willing to supply insurance at these rates and make positive profits.

4. Adverse selection and moral hazard

In this section I introduce ex post moral hazard into the model. I retain the assumption of no ex interim or hidden action moral hazard, both because this may be of secondary importance in the health insurance context due to the often substantial non-pecuniary costs associated with ill health, and because some choices that affect health status risk are nowadays contractible (e.g., smoking). However, ex post moral hazard is thought to play an important role in determining behaviour in health care and health insurance markets (see, e.g., Feldstein and Friedman, 1977; Pauly, 1986; Manning et al., 1987). In particular, the information structure, in which insurers cannot observe the state of the world, is meant to characterise fee-for-service insurance. To analyse the equilibrium contracts that are offered in this setting, it is necessary first to describe the insurance choices of individuals in the absence of adverse selection, and then to analyse market equilibrium.

4.1. Incentive compatible contracts

The basic characteristic of feasible contracts under conditions of ex post moral hazard is that they must base net transfers not on the unobservable state of the world, but on health expenditures. This characterisation is consistent with the observation that most fee-for-service insurance plans base benefits on incurred costs (in a possibly highly non-linear fashion). When the possible states of the

world form a continuum, it is natural to write the net transfers defined by the insurance contract for individuals of risk type i as a function $t^i(q)$ where $q = \theta h$ is the expenditure on health by an individual in state of the world θ . For example, an insurance contract for risk type i that stipulates a premium P^i and a constant coinsurance rate κ^i (being the proportion of expenditures paid by the insured) would be represented by the net transfer function $t^i(q) = P^i - (1 - \kappa^i)q$.⁶

In the present case with just two possible states of the world, a generic contract stipulates a net transfer and level of health care expenditure in each state of the world. That is, the contract for risk type i is a pair of allocations $\{(q_g^i, t_g^i), (q_b^i, t_b^i)\}$, where $q_s^i = \theta_s h_s^i$ is the amount of health expenditure in state s , and t_s^i is the monetary transfer made to the insurance company by the individual in state s .⁷

For the purposes of describing feasible contracts under moral hazard, let us drop the risk superscript momentarily. For any pair of transfers $\{t_g, t_b\}$, the health expenditure allocations $\{q_g, q_b\}$ must satisfy the following three conditions.

1. q_s must be feasible, given t_s . That is, $w - t_s - q_s \geq 0$. When an individual in state s' chooses contract (q_s, t_s) her consumption of non-health goods is $c = w - t_s - q_s$ and her consumption of health is $h = q_s / \theta_{s'}$.
2. q_b and q_g must be incentive compatible. That is, writing $v(q_{s'}, t_{s'}, \theta_{s'}) = u(w - t_{s'} - q_{s'}, q_{s'} / \theta_{s'})$, it is necessary that

$$v(q_s, t_s, \theta_s) \geq v(q_{s'}, t_{s'}, \theta_{s'})$$

for $s' \neq s$.

3. q_s maximises $v(q_s, t_s, \theta_s)$ subject to the feasibility and incentive compatibility conditions.

The last condition reflects an assumption that due to competition, for a given pair of transfers and hence a given profit, insurers will offer individuals the most attractive consumption bundles subject to them being feasible and incentive compatible. Ignoring the incentive compatibility constraint, the maximal utility attainable in each state given the transfer t_s is $v(\theta_s, w - t_s) = u(w - t_s - q_s^*, q_s^* / \theta_s)$, where $v(.,.)$ is the indirect utility function, $q_s^*(t_s) = \theta_s h^*(\theta_s, w - t_s)$, and $h^*(.,.)$ is the Marshallian demand function for health. When there is no confusion, I shall just write q_s^* . Similarly, $c_s^* \equiv c^*(\theta_s, w - t_s) = w - t_s - q_s^*(t_s)$ is the efficient demand for the non-health consumption good. In fact, it turns out to be more

⁶This implies a net out-of-pocket expense of $P^i + \kappa^i q$.

⁷We are implicitly invoking the revelation principle (Myerson, 1979). A typical fee-for-service contract is an *indirect* mechanism, that stipulates a monetary transfer t as a function of endogenous expenditures q . Such a policy can always be implemented via a *direct* mechanism that assigns a transfer and required expenditure pair based on a report of the parameter θ , that is $(t(\theta), q(\theta))$. For an exposition of both the binary and continuum models in the isomorphic case of non-linear monopoly pricing, see Tirole (1988) Chapter 3.

convenient to describe the incentive compatibility constraint in terms of demand for the consumption good, c . For a given transfer pair (t_g, t_b) , consider the equation

$$v(\theta_g, w - t_g) = u(c, [w - t_b - c]/\theta_g).$$

For all transfer pairs (t_g, t_b) such that $t_g \geq t_b$, this equation has two solutions $\tilde{c}_1(t_g, t_b)$ and $\tilde{c}_2(t_g, t_b)$, with $\tilde{c}_1 \leq \tilde{c}_2$. Considering only contracts that increase nominal income in the bad state relative to the good state, the following partial characterisation lemma obtains:

Lemma 1. Assume c is normal and that the price elasticity of demand for health is less than one. When $t_g \geq t_b$, the contract pair $\{(q_g^*, t_g), (q_b^*, t_b)\}$ is incentive compatible if and only if $c^*(\theta_b, w - t_b) \leq \tilde{c}_1(t_g, t_b)$.

This lemma is illustrated in Fig. 3. When the transfer pair satisfies the conditions

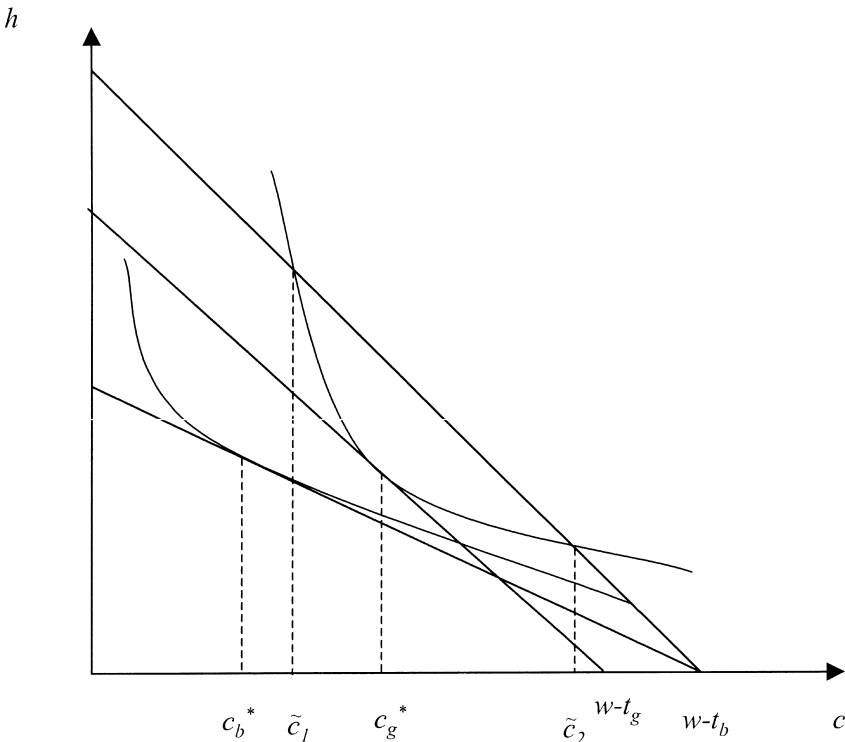


Fig. 3. For income transfer (t_g, t_b) , the efficient consumption bundles are incentive compatible.

of the lemma, the individual’s expected utility across states of the world is

$$V(t_g, t_b) = pv(\theta_g, w - t_g) + (1 - p)v(\theta_b, w - t_b).$$

When the transfer pair is such that $c^*(\theta_b, w - t_b) > \tilde{c}_1(t_g, t_b)$ and the efficient health expenditure allocations are offered, individuals in the good state of the world will prefer to choose the (q_b^*, t_b) contract (as will those in the bad state). To ensure that those in the good state choose the contract designed for them, switching to the bad state allocation must be made less desirable. This is achieved by distorting the consumption bundle in the bad state away from $(c^*(\theta_b, w - t_b), h^*(\theta_b, w - t_b))$, as illustrated in Fig. 4.

In particular, by setting c_b equal to either $\tilde{c}_1(t_g, t_b)$ or $\tilde{c}_2(t_g, t_b)$, and $q_b = w - t_b - c_b$, the incentive for individuals in the good state to switch can be removed. The particular choice of c_b is determined by which of the two allocations $(\tilde{c}_1, \tilde{h}_1) \equiv (\tilde{c}_1, [w - t_b - \tilde{c}_1]/\theta_b)$ and $(\tilde{c}_2, \tilde{h}_2) \equiv (\tilde{c}_2, [w - t_b - \tilde{c}_2]/\theta_b)$ is both feasible and yields

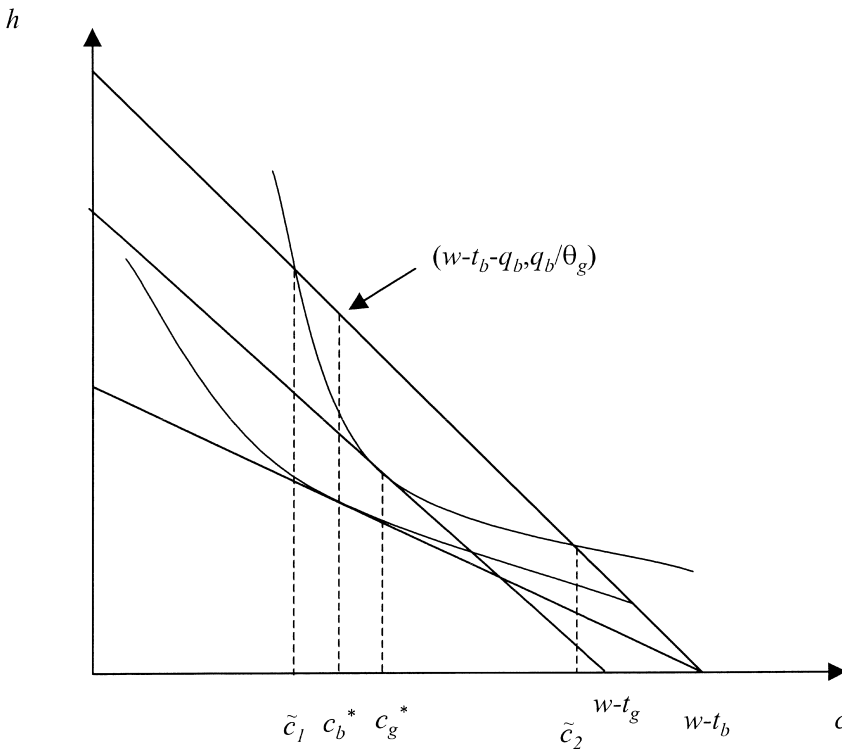


Fig. 4. With larger income transfers (t_g, t_b) , the efficient consumption bundles become incentive incompatible.

the consumer higher utility in the bad state.⁸ It is expected that in most cases of interest, $c_b = \tilde{c}_1(t_g, t_b)$ will be optimal. Sufficient conditions for this are given in the following lemma:

Lemma 2. *Suppose that for all $m > 0$, for all $\theta \in (\theta_g, \theta_b)$, and for $c \in (0, m)$,*

$$\frac{d}{dc} \left[\left(\frac{m - c}{\theta} \right) \frac{\partial u}{\partial h} \left(c, \frac{m - c}{\theta} \right) \right] > 0.$$

Then for all transfer pairs (t_g, t_b) such that $t_g > t_b$ and (q_g^, q_b^*) is not incentive compatible, $u(\tilde{c}_1, \tilde{h}_1) > u(\tilde{c}_2, \tilde{h}_2)$, and the health expenditure allocation that provides maximal utility in each state and is incentive compatible for individuals in the good state is $q_g = q_g^*(t_g)$ and $q_b = [w - t_b - \tilde{c}_1(t_g, t_b)]$.*

Thus under the conditions of Lemma 2, an incentive compatible insurance contract leads to excessive health expenditures by those in the bad state.⁹ Note that to show that the allocation (q_g, q_b) is incentive compatible for individuals in the bad state, it is necessary to show that those in the bad state will not wish to choose the contract (t_g, q_b) . That is, it is still required that $u(\tilde{c}_1, \tilde{h}_1) > u(c_g^*, [w - t_g - c_g^*] / \theta_b)$.

Lemma 3. *Suppose preferences over consumption and health are quasi-linear, $u(c, h) = c + \zeta(h)$, and suppose the coefficient of relative risk aversion associated with the gross surplus function $\zeta(\cdot)$ is always greater than unity, that is, $-h\zeta''(h) / \zeta'(h) > 1$ for all h . Then given a transfer pair (t_g, t_b) such that $t_g > t_b$, the expenditure allocation $q_g = q_g^*(t_g)$ and $q_b = [w - t_b - \tilde{c}_1]$ is incentive compatible for individuals in both states.*

As an example, suppose the gross surplus function is given by $\zeta(h) = -a/h^n$ for some positive constants a, n . Then

$$u(c, h) = c - a/h^n$$

and $-h\zeta'' / \zeta' = (n + 1) > 1$, so the conditions of Lemma 2 are satisfied.

⁸Note that distorting the consumption bundle of individuals in the good state, given their transfers, will only increase their incentive to switch, because such distortion makes them worse off. Thus such a distortion is not warranted.

⁹In a continuous state space model, a linear contract (premium plus coinsurance rate) will lead to excessive health expenditures in all states. But under a fully optimal non-linear contract, the health expenditure of the individual in the best health state will be undistorted. This is just the familiar no distortion at the top result from optimal tax theory (Mirrlees, 1971).

In general, for any transfer pair (t_g, t_b) with associated $(q_g(t_g, t_b), q_b(t_g, t_b))$ satisfying conditions 1–3, expected utility is

$$V(t_g, t_b) = pu([w - t_g - q_g], q_g / \theta_g) + (1 - p)u([w - t_b - q_b], q_b / \theta_b)$$

Now suppose there is a contract consisting of equal transfers, $t_0 < w$, in each state of the world (t_0 could be less than zero).¹⁰ In a trivial sense the pair $\{(q_g^*(t_0), t_0), (q_b^*(t_0), t_0)\}$ is efficient and incentive compatible. The individual's welfare with this allocation is $V(t_0, t_0) \equiv V_0$, and as the elasticity of demand for health is less than one, $c^*(\theta_b, w - t_0) < c^*(\theta_g, w - t_0) = \tilde{c}_1(t_0, t_0)$. Now consider reducing the size of the transfer to the individual in the good state of the world (i.e., increasing t_g), and increasing the transfer to her in the bad state (i.e., reducing t_b), so as to leave expected utility unchanged at V_0 . At each successive transfer pair, the health expenditure allocations in each state must adjust so as to continue to satisfy conditions 1–3 above. That is, they must remain both feasible and incentive compatible, while affording the consumer maximal utility in each state subject to these constraints. For small changes in (t_g, t_b) in the direction indicated, the efficient health expenditure allocations $(q_g^*(t_g), q_b^*(t_b))$ satisfy the required conditions. However, as the process continues, at some point the incentive constraint binds, and the efficient allocation will cease to be incentive compatible. More formally, let $t_b = t_b(t_g; V_0)$ solve the equation $V(t_g, t_b) = V_0$. Then the following lemma obtains.

Lemma 4. *Assume c is normal and $t_0 < w$. There exists $\hat{t}_g(t_0) > t_0$ such that for all $t_g \in (t_0, \hat{t}_g(t_0))$ the efficient allocations are incentive compatible, i.e., $q_g(t_g, t_b(t_g; V_0)) = q_g^*(t_g)$ and $q_b(t_g, t_b(t_g; V_0)) = q_b^*(t_b)$, while for all $t_g > \hat{t}_g(t_0)$, the efficient allocations are not incentive compatible.*

Now consider the mapping

$$\begin{aligned} \gamma: (-\infty, w) &\rightarrow \mathbb{R}_+^2 \\ t_0 &\mapsto (w - \hat{t}_g(t_0), w - \hat{t}_b(t_0)) \end{aligned}$$

The image of this map in \mathbb{R}_+^2 defines the boundary of two regions, and the equation of this boundary is denoted by $m_b = \tau(m_g)$. It is straightforward to confirm that $\tau'(m_g) > 0$. τ is called the transition line: to the right of it the efficient allocations are incentive compatible, and to the left they are not.

As well as characterizing transfers under which efficient allocations are implementable, the transition line tells us how moral hazard effectively alters individuals' preferences over implementable insurance policies. To each value of t_0 there corresponds a curve of constant expected utility that crosses the transition

¹⁰Clearly these transfers yield positive (negative) profits whenever $t > (<) 0$. At this stage, I am only interested in describing individuals' preferences over net income in the two states.

line. To the right of the transition line the indifference curves coincide with those that arise when there is no moral hazard. However, they are not differentiable along the transition line, and have discontinuously greater slopes (in absolute value) than the smooth indifference curves that exist in the absence of moral hazard. In particular, in the absence of moral hazard, the indifference curve is smooth at all points, including $\{\hat{t}_g(t_0), \hat{t}_b(t_0)\}$, and has slope there given by $dm_b/dm_g|_{V=V_0}$. This is also the value of the right-hand derivative of the indifference curve with moral hazard at $\{\hat{t}_g(t_0), \hat{t}_b(t_0)\}$, since for $m_g > w - \hat{t}_g(t_0)$ the indifference curves with moral hazard are identical to those without. However, the left-hand derivative of the indifference curve with moral hazard is (algebraically) less than $dm_b/dm_g|_{V=V_0}$, since a marginal increase in t_g (which lowers m_g) must be offset by a larger reduction in t_b (increase in m_b) than in the absence of moral hazard.¹¹

4.2. Equilibrium

The position of the transition line relative to the full insurance line, $\phi(m_g)$, determines whether ex post moral hazard has any effect on the potential equilibrium contracts. If the transition line is to the north-west of the full insurance line, then the ex post incentive constraints only bind at very high transfer levels. These transfers are greater than those that would be chosen without moral hazard, so the potential separating equilibrium is not altered, as shown in Fig. 5. Similarly, the likelihood of an equilibrium existing is unchanged, as the critical value λ^* is unaffected.¹²

However, when the transition line lies to the south-east of the full insurance line, then both the potential equilibrium contracts and the likelihood of an equilibrium existing in general change. In this case, at a potential separating equilibrium both high and low risk individuals obtain incomplete insurance, at points H_2 and L_2 , respectively. The contract for high risks, H_2 , is characterised by the fact that it provides maximal expected utility subject to earning zero profits. As drawn in Fig. 6, this occurs at the intersection of the transition line and the $\pi_h = 0$ locus, due to the non-differentiable kink in indifference curves at that point. It is plausible that H_2 actually lies further to the left of the transition curve (but still to

¹¹In models of hidden action (or ex interim) moral hazard (e.g., Arnott and Stiglitz, 1988), non-convexities can easily arise. However in the current model of ex post moral hazard the indifference curves remain convex.

¹²Recall that in the absence of ex post moral hazard, all low-risk indifference curves cross the full insurance line with a slope of $-(1-p_l)/p_l$. Thus, in the absence of moral hazard, the point of tangency between the market odds line defined in terms of the critical proportion of low risks, λ^* , and the relevant low risk indifference curve, is below the full insurance line. As the potential equilibrium contracts are unaffected by the existence of moral hazard when the transition line is above the full insurance line, neither is this tangency point, and the critical proportion λ^* of low risks is unchanged.

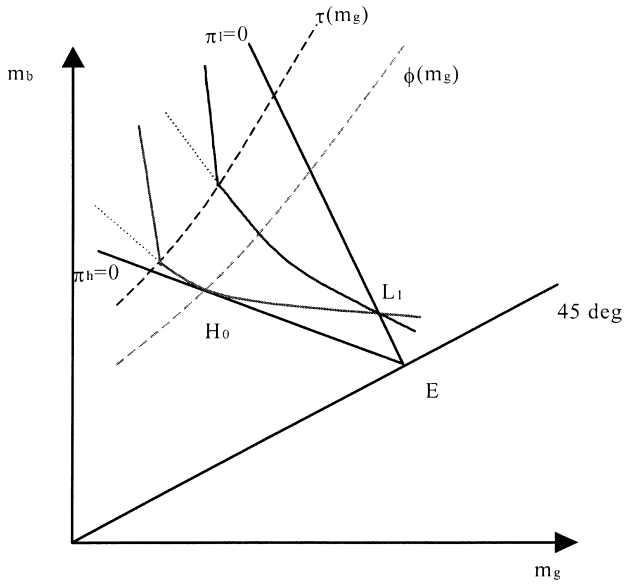


Fig. 5. The position of the transition line relative to the full insurance line determines whether moral hazard will affect the existence of equilibrium.

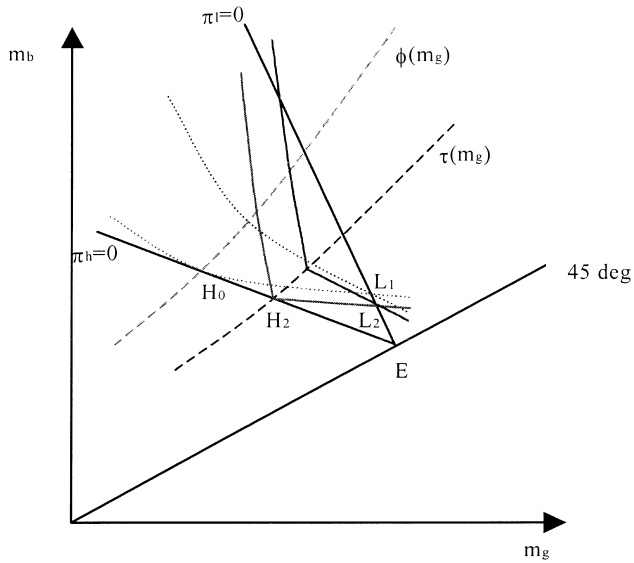


Fig. 6. Impact of ex post moral hazard on potential equilibrium when the transition line lies below the full insurance line.

the right of the full insurance line), but for illustrative purposes it is useful to assume that the kink in the indifference curves is large enough to avoid this possibility.¹³

As high risk individuals have less than complete coverage, the incompleteness of the coverage of low risk individuals is further increased relative to the case without moral hazard. The intuition for this is that, because the coverage of low risks must be sufficiently unattractive to high risks to stop the latter from choosing it, the level of that coverage must fall. However, the ‘cost’ of adverse selection to low risk individuals, in terms of their reduction in welfare, may well be substantially reduced when there is moral hazard, since the choice of insurance without adverse selection is itself incomplete. For example, the insurance level chosen by low risks in the presence of ex post moral hazard, but without adverse selection, is at point L' in Fig. 6.

It is clear from the diagram that the change due to moral hazard, if any, in the critical value λ^* above which an equilibrium will fail to exist, cannot be unambiguously signed. On the one hand, because of the lower level of insurance at the potential separating equilibrium, low risk individuals will be willing to purchase an unconstrained quantity of insurance at higher prices than previously, which reduces λ^* and the likelihood of an equilibrium. This is the ‘external incentive effect’ mentioned in the introduction. But on the other hand, because of the kink in their indifference curves, they will be unwilling to purchase insurance at such a high price as in the case of no moral hazard. That is, transferring income between states of the world at actuarially unfair rates is less attractive to low risk individuals than before, since they will be constrained to consume income in the bad state inefficiently. This ‘internal incentive effect’ increases λ^* and the likelihood of an equilibrium.

5. Examples

In this section, we first define a set of preferences under which moral hazard must increase the likelihood of equilibrium existence. We then report results from numerical simulations for a broader range of preferences.

5.1. An analytical example

I construct an analytical example in which the introduction of moral hazard must increase the chance of an equilibrium existing. Only the intuition behind this

¹³Indeed, as one objective is to find situations in which an equilibrium with moral hazard is more likely to exist, it is sufficient to have an upper bound on the extent to which the low risk contract, L_2 , moves down the $\pi_l = 0$ line. H_2 provides such an upper bound.

example is provided here, all details being relegated to Appendix B. Suppose at low income levels preferences are Leontief, but that at higher incomes they are quasi-linear. Such ordinal preferences imply that at low incomes the Engel curve for health is linear, and that for higher incomes it is flat, giving health the properties of a necessity good. This general relationship is not inconsistent with much empirical evidence on the demand for health (e.g., Manning et al. (1987)).

In this case, for low incomes there are no distortionary effects of price subsidies (i.e., expenditure-based insurance), so transfers in the bad state must be very large relative to those in the good state in order to be incentive incompatible. The transition line thus lies above the full insurance line. On the other hand, in the quasi-linear range at higher incomes, the distortionary costs are positive, and net transfers from the good to the bad state become incentive incompatible earlier. The transition line therefore lies below the full insurance line in this region. At an intermediate point, the two lines intersect.

Now assume that endowments are such that a fully insured high risk individual consumes at the intersection. This situation is shown in Fig. 7. In this case, the introduction of moral hazard has no effect on the potential equilibrium contract offered to high risks, as optimal insurance for them continues to be complete. Thus

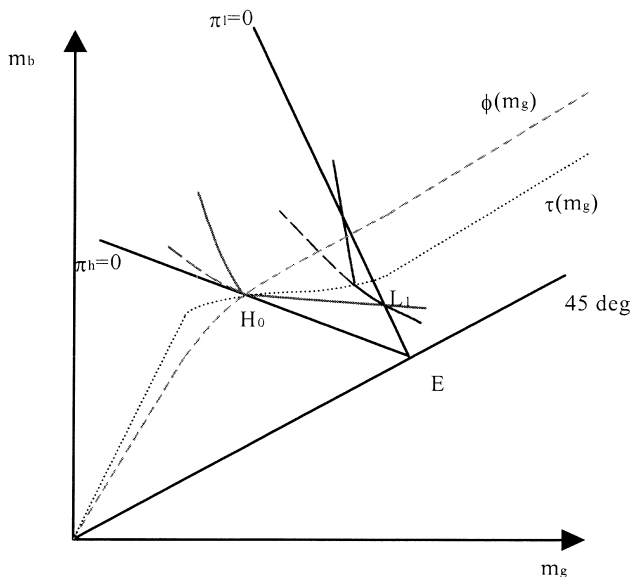


Fig. 7. An example in which there is no external incentive effect, and moral hazard can only improve the chances of equilibrium existence.

there is no ‘external incentive effect’ on low risks, and the only impact of moral hazard is the ‘internal incentive effect’ which implies that the highest price at which otherwise constrained low risk individuals are willing to purchase an unconstrained quantity of insurance falls, and the likelihood of an equilibrium existing increases.

5.2. Numerical simulations

The analytical example of the previous sub-section used a very specific utility function. In this section, I present the results of numerical simulations using a simpler representation of preferences, in part to show that the special analytical case is not exceptional. In particular, I now assume that preferences are quasi-linear over all income ranges. Thus for all c and h , the utility index is of the form $c - a/h^n$, where a and n are positive constants, so that the elasticity of demand for health is equal to $1/(n + 1) \in (0,1)$. Individuals are assumed to have identical attitudes towards risk, represented by a constant coefficient of risk aversion ξ , yielding a complete description of preferences over health and consumption across states of the world of

$$u(c,h) = \frac{1}{1-\xi}(c - a/h^n)^{(1-\xi)}.$$

Using this representation of preferences, I examine the conditions under which the existence of moral hazard has an effect on equilibrium insurance contracts, the extent of the changes in coverage that it induces, and the effect of moral hazard on the likelihood of an equilibrium existing.

As shown in Appendix B for quasi-linear preferences, the full insurance line is given by

$$m_b = \phi(m_g) = m_g + \phi_0$$

where $\phi_0 = [\zeta(h(\theta_g)) - e_g] - [\zeta(h(\theta_b)) - e_b] = \sigma_g - \sigma_b$, e_s is unconstrained health expenditure in state s , and σ_s is the surplus from health consumption in state s . Similarly, the transition line is defined simply as

$$m_b = \tau(m_g) = m_g + \tau_0$$

where

$$\tau_0 = [\zeta(h(\theta_g)) - e_g] - \left[\zeta\left(\frac{\theta_b}{\theta_g}h(\theta_b)\right) - e_b \right] < \phi_0.$$

Thus, over all income ranges, the full insurance line and the transition line are parallel, and the latter lies always to the south of the former. In this case then, moral hazard always affects potential equilibrium contracts. Similarly, it is easy to check that $\tau_0 > 0$, so the transition line lies above the equal incomes line.¹⁴

Even with these relatively simple functional forms, it is not possible to arrive at simple conditions under which the external or internal incentive effect dominates, and thus under which the likelihood of an equilibrium will decrease or increase. Table 1 thus reports some illustrative results from numerical simulations using the CRRA/quasi-linear specification of utility above. Entries in the table represent the percentage increase in the critical value λ^* under conditions of ex post moral hazard.¹⁵ For these simulations, it was assumed that health expenditures in the good state of the world amounted to about 2% of income ($\theta_g = 0.1, m = 10$), and that the low risk individuals' probability of being in the bad state is 0.1.

Table 1
Percentage increase in the likelihood of equilibrium in the presence of ex post moral hazard

ξ	θ_b	$\varepsilon_h = 0.25$			$\varepsilon_h = 0.1$		
		P_h			P_h		
		0.2	0.5	0.8	0.2	0.5	0.8
2	0.2	+0.1	+0.1	+0.1	+0.6	+0.2	+0.1
	1.0	-0.2	+0.5	+0.7	+2.1	+1.1	+1.0
	1.8	-1.3	+0.5	+1.2	+2.2	+1.5	+1.7
4	0.2	+0.1	+0.1	+0.2	+1.0	+0.4	+0.3
	1.0	-2.2	+0.3	+1.2	+1.9	+1.5	+1.8
	1.8	-4.5	-0.3	+1.9	+1.3	+1.9	+3.2
6	0.2		-2.9	-1.6		-1.9	-1.0
	1.0	-5.0	-0.3	+1.5	+0.9	+1.7	+2.7
	1.8	-8.9	-1.6	+2.6	-0.9	+2.0	+4.8
8	0.2						
	1.0		-11.8	-2.4		-1.6	-0.7
	1.8		-16.2	+1.1		-7.3	+8.2

¹⁴When $n \in (-1,0)$ and the elasticity of demand is greater than unity, $\tau_0 < 0$.

¹⁵Blank cells represent parameter values for which the simulations did not converge.

Simulations are performed for two values of the elasticity of demand for health, $\varepsilon_h (= 1/(n + 1))$, consistent with empirical estimates of about 0.2. The price of health in the bad state varies between 2, 10, and 18 times the price in the good state, and health expenditures are correspondingly 4, 13, and 20% of income in the bad state when $n = 3$.

The results from Table 1 confirm that the impact of moral hazard on the likelihood of a pure strategy equilibrium existing can be positive or negative. In general it is not to be expected that monotonic relationships should necessarily exist between the various parameters of the model and the size of the impact of moral hazard on the likelihood of existence of a pure strategy equilibrium. However, some general properties of the model can be established from the numerical simulations. First, for many specifications, the impact of moral hazard seems to be rather small, altering the chance of an equilibrium existing by only a few percentage points. On the other hand, there are some parameterisations that yield large percentage changes in the critical value of λ when moral hazard is introduced, in both the positive and negative directions.

The results suggest that the impact of moral hazard on equilibrium existence is likely to be positive when the bad state is not too bad (i.e., when θ_b is relatively low), at least for those cases in which individuals are not too risk averse. Table 1 also suggests that ex post moral hazard appears more likely to increase the chance of equilibrium existence when the elasticity of demand for the good whose price is uncertain (i.e., health) is lower. Thus, not only are the distortionary costs due to incomplete insurance smaller when demand elasticities are smaller, it seems that the market is also relatively less likely to fail, in the RSW sense, in such situations.

In addition, the impact is likely to be positive when the two risk types have significantly different probability distributions over the two states (i.e., when p_h is sufficiently greater than p_l). This second property is illustrated further in Table 2, which presents percentage changes in the critical value of λ as the ratio of the probability of being in the bad state for good and bad risks varies. In particular, p_l

Table 2
Effect of the divergence of risks on the impact of moral hazard on the likelihood of equilibrium

p_l	p_h	% Change in λ^*
0.02	0.98	75.0
0.05	0.95	9.0
0.10	0.90	16.5
0.15	0.85	7.3
0.20	0.80	5.9
0.25	0.75	0.3
0.30	0.70	-8.0
0.35	0.65	-22.8
0.40	0.60	-48.4

varies from 0.02 to 0.4, while p_h is set equal to $(1 - p_l)$.¹⁶ The simulations are reported for $\varepsilon_h = 0.1$, $\theta_g = 0.1$, $\theta_b = 1.8$, and $\xi = 8$.

6. Concluding remarks

This paper has begun an investigation of the existence of pure strategy equilibria in insurance markets under conditions of ex ante adverse selection and ex post hidden information moral hazard. Since the work of Rothschild and Stiglitz, and Wilson, it has been well known that the first type of information asymmetry can lead to badly functioning markets, in the sense that a pure strategy equilibrium may fail to exist. The inclusion of the second information asymmetry is particularly motivated by the common concern amongst health economists and policy makers that the existence of fee-for-service health insurance leads to overspending on health care. The model shows that under plausible assumptions, the introduction of this further information asymmetry can have the surprising effect of increasing the likelihood that the insurance market will work well, in the sense that an equilibrium will exist.

This result is both theoretically interesting and of relevance to the policy debate. The idea that more information is not always better is not new, particularly when the additional information is held asymmetrically amongst agents. In fact, the original RSW model is a case in point: when neither individuals nor insurance companies know risk attributes, an equilibrium will always exist, but when individuals know their own risk types, the market can break down. However, this paper has shown that, for some parameterisations at least, making some asymmetrically held information public (i.e., removing ex post moral hazard by making the state of the world observable) may well make matters worse.

On the policy side, the result suggests that US-style Health Maintenance Organizations could have ambiguous effects on the health insurance market. If an HMO market exhibited an equilibrium, the absence of (or at least reduction in) ex post moral hazard due to the amalgamation of insurance and medical services may well increase aggregate welfare (assuming that there are no other negative effects of HMOs, e.g., related to the quality of services). However, if the growth of HMOs led to poorly functioning insurance markets with many people uninsured, then the welfare implications could be less benign.

As mentioned in Section 1, comparison of two institutional arrangements with different equilibrium existence properties is subject to questions of interpretation. A more complete model would ideally admit the possibility of a mixed equilibrium, in which both HMO and fee-for-service insurance contracts coexist. While there is a widespread belief that risk selection in the US may well be affected

¹⁶Of course, there is no reason that $p_l + p_h$ should be constrained to equal unity. This is just a convenient parameterisation.

through choice of organizational form (i.e., with lower risk individuals opting for HMO coverage), such an interpretation relies on quality differences between the two forms. In this paper I have assumed the only difference between them to be one of information, so a more elaborate extension to a model admitting mixed equilibria awaits future research.

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Appendix A

Proof of Lemma 1. (\Leftarrow) Suppose $c^*(\theta_b, w - t_b) \leq \tilde{c}_1(t_g, t_b)$. It is first necessary to check that it is unprofitable for individuals in the good state to choose the expenditure-transfer pair designed for those in the bad state. But, if $c_b^* \equiv c^*(\theta_b, w - t_b) \leq \tilde{c}_1(t_g, t_b)$, then

$$\begin{aligned} \nu(q_b^*, t_b, \theta_g) &= u(c_b^*, [w - t_b - c_b^*] / \theta_g) \\ &\leq u(\tilde{c}_1, [w - t_b - \tilde{c}_1] / \theta_g) \\ &= \nu(q_g^*, t_g, \theta_g). \end{aligned}$$

To check that individuals in the bad state do not choose the (q_g^*, t_g) contract, let $B(s, s')$ be the budget set of an individual in state s who chooses a contract with transfer $t_{s'}$. I show that $\nu(q_b^*, t_b, \theta_b) \geq \nu(q_g^*, t_g, \theta_b)$. Since $t_g \geq t_b$, $B(b, g) \subseteq B(b, b)$, and since q_b^* maximises utility subject to the constraint $B(b, b)$, the inequality must hold.

(\Rightarrow) Proceed in a contrapositive fashion. Suppose that $c_b^* \equiv c^*(\theta_b, w - t_b) > \tilde{c}_1(t_g, t_b)$. Then by the elasticity assumption, $c_b^* < c^*(\theta_g, w - t_b) \leq \tilde{c}_2$, so $c_b^* \in (\tilde{c}_1, \tilde{c}_2)$. But then

$$\begin{aligned} \nu(q_b^*, t_b, \theta_g) &= u(c_b^*, [w - t_b - c_b^*] / \theta_g) \\ &> u(\tilde{c}_1, [w - t_b - \tilde{c}_1] / \theta_g) \\ &= \nu(q_g^*, t_g, \theta_g) \end{aligned}$$

so the efficient allocation is not incentive compatible for individuals in the good state. \square

Proof of Lemma 2. Suppose (t_g, t_b) is such that (q_g^*, q_b^*) is not incentive compatible. Then, writing $h'_i = [w - t_b - \tilde{c}_i] / \theta_g$,

$$u(\tilde{c}_1, h'_1) = u(\tilde{c}_2, h'_2) \tag{1}$$

with $\tilde{c}_1 < \tilde{c}_2$. The condition of the lemma says that as one moves down the budget line in the good state (i.e., as c increases), the change in utility from a proportional increase in h , holding c fixed, increases. Since for each $i = 1, 2$, $\tilde{h}_i / h'_i = \theta_g / \theta_b$, given (1), this implies that

$$u(\tilde{c}_1, \tilde{h}_1) > u(\tilde{c}_2, \tilde{h}_2)$$

as required. Thus $(q_g, q_b) = (q_g^*(t_g), [w - t_b - \tilde{c}_1])$ is the allocation that is incentive compatible for individuals in the good state and that provides maximal utility for individuals in both states. \square

Proof of Lemma 3. First show that the condition of Lemma 2 is satisfied. Indeed,

$$\begin{aligned} \frac{d}{dc} \left[\left(\frac{m-c}{\theta} \right) \frac{\partial u}{\partial h} \left(c, \frac{m-c}{\theta} \right) \right] &= \frac{d}{dc} \left[\left(\frac{m-c}{\theta} \right) \zeta' \left(\frac{m-c}{\theta} \right) \right] \\ &= - \frac{\zeta'((m-c)/\theta)}{\theta} - \left(\frac{m-c}{\theta} \right) \frac{\zeta''((m-c)/\theta)}{\theta} \\ &= - \frac{1}{\theta} [h\zeta''(h) + \zeta'(h)] \\ &> 0. \end{aligned}$$

Thus, the allocation is incentive compatible for individuals in the good state.

Next, the allocation is incentive compatible for individuals in the bad state if and only if

$$v(q_b, t_b, \theta_b) = \tilde{c}_1 + \zeta(\tilde{h}_1) > c_g^* + \zeta((m_g - c_g^*) / \theta_b) = v(q_g, t_g, \theta_b).$$

Recalling the fact that $c_g^* + \zeta((m_g - c_g^*) / \theta_g) = \tilde{c}_1 + \zeta((m_b - \tilde{c}_1) / \theta_g)$, so that $c_g^* - \tilde{c}_1 = \zeta((m_b - \tilde{c}_1) / \theta_g) - \zeta((m_g - c_g^*) / \theta_g)$, incentive compatibility for those in the bad state is equivalent to

$$\zeta((m_b - \tilde{c}_1) / \theta_b) - \zeta((m_g - c_g^*) / \theta_b) > \zeta((m_b - \tilde{c}_1) / \theta_g) - \zeta((m_g - c_g^*) / \theta_g)$$

or that $\zeta(x) - \zeta(y) > \zeta(\alpha x) - \zeta(\alpha y)$ for $x > y$ and $\alpha > 1$. But this condition is satisfied if and only if the function $h\zeta(h)$ is decreasing in h , which is equivalent to the condition of the corollary. \square

Proof of Lemma 4. By the elasticity assumption, at the initial transfer pair $(t_g, t_b) = (t_0, t_0)$, $c^*(\theta_b, w - t_0) < c^*(\theta_g, w - t_0) = \tilde{c}_1(t_0, t_0)$, so $q_g(t_g, t_b) = q_g^*(t_0)$ and $q_b(t_g, t_b) = q_b^*(t_0)$. Normality of consumption implies that $\partial c^*(\theta_b, w - t_b) / \partial t_b < 0$, and $c^*(\theta_b, w - t_b) \rightarrow \infty$ as $t_b \rightarrow -\infty$. Similarly, $\partial \tilde{c}_1 / \partial t_g < 0$ and $\partial \tilde{c}_1 / \partial t_b > 0$ jointly imply that $\tilde{c}_1 \rightarrow 0$ as $t_b \rightarrow -\infty$. By continuity, there exists a $\hat{t}_g(t_0) > t_0$, and an

implied critical transfer pair $\{\hat{t}_g(t_0), \hat{t}_b(t_0)\} = \{\hat{t}_g(t_0), t_b(\hat{t}_g(t_0); V_0)\}$ such that $c^*(\theta_b, w - \hat{t}_b(t_0)) = \tilde{c}_1(\hat{t}_g(t_0), \hat{t}_b(t_0))$ and $V(\hat{t}_g(t_0), \hat{t}_b(t_0)) = V_0$. \square

Appendix B. An analytical example

For $h > 0$, let $\omega(c, h) = c + \zeta(h) = c - a/h^n$, for some positive constant a . Given θ_b and θ_g , let

$$\underline{h} = \left(\frac{an}{\theta_b}\right)^{1/(n+1)} \text{ and } \underline{c} = \underline{h}.$$

(\underline{h} is the amount of health that would be chosen by an individual with quasi-linear utility function $\omega(\cdot)$ facing a price of health θ_b .) Also, define $\underline{\omega} = \omega(\underline{c}, \underline{h})$, $\bar{\omega} = \underline{c}$, and $\tilde{c}(h) = \underline{c} + a/h^n$. Finally, for (c, h) such that $\omega(c, h) \in (\underline{\omega}, \bar{\omega})$, define

$$\alpha(c, h) = \frac{c - \underline{c}}{\tilde{c}(h) - \underline{c}} = \frac{c - \underline{c}}{a/h^n}.$$

Define five regions of the (c, h) -plane as follows:

- A = $\{(c, h): \omega(c, h) < \underline{\omega} \text{ and } h < \underline{c}\}$
- B = $\{(c, h): \omega(c, h) > \bar{\omega}\}$
- C = $\{(c, h): \underline{\omega} < \omega(c, h) < \bar{\omega} \text{ and } h < \underline{h}\}$
- D = $\{(c, h): c < \underline{c} \text{ and } h > \underline{c}\}$ and
- E = $\{(c, h): \underline{\omega} < \omega(c, h) < \bar{\omega} \text{ and } h > \underline{h}\}$

These are shown in Fig. 8. Then consider the indifference curves described by the utility index

$$\Omega(c, h) = \begin{cases} \omega(c, h) & \text{for } (c, h) \in A \cup B \cup C \\ c & \text{for } (c, h) \in D \\ \alpha(c, h)\underline{\omega} + (1 - \alpha(c, h))\bar{\omega} & \text{for } (c, h) \in E \end{cases}.$$

Thus, in regions A, B, and C, indifference curves are quasi-linear with respect to consumption of the non-health good. In region D additional health does not increase utility unless it is accompanied by additional non-health consumption. The utility index increases linearly in region E as non-health consumption increases, holding the level of h fixed. It is straightforward to show that the implied preference relation is continuous.

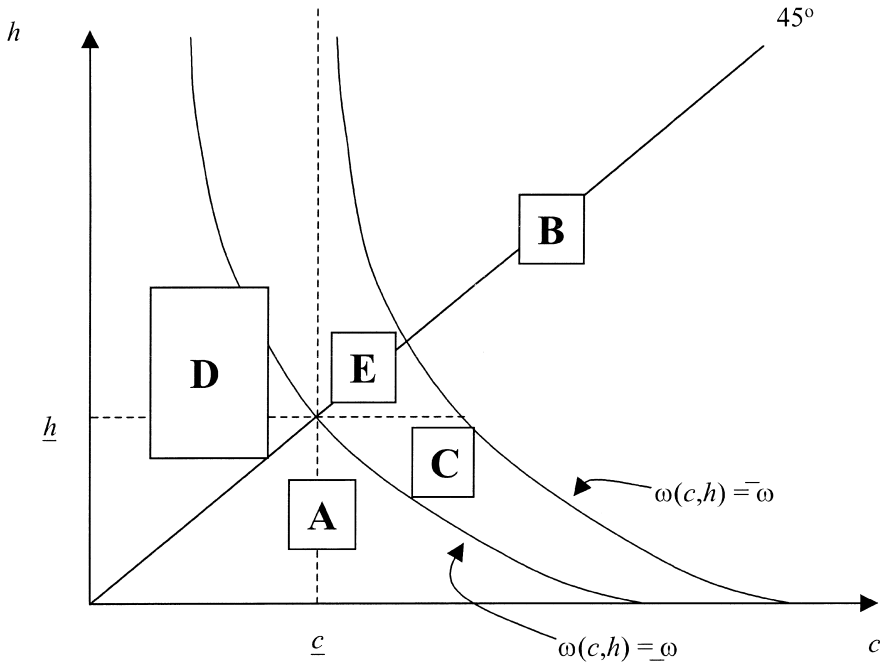


Fig. 8. Preferences are quasi-linear in regions A, B, and C; they are independent of h in region D; in region E they are linearly interpolated.

I assume that the cardinal utility function that incorporates the individual’s attitudes towards risk is a monotonic increasing and strictly concave transformation, $f(\cdot)$, of the utility index $\Omega(c,h)$. Thus

$$u(c,h) = f(\Omega(c,h)).$$

Also define the indirect utility function

$$v(\theta,m) = \max_{c,h} u(c,h) \text{ s.t. } c + \theta h \leq m.$$

I now characterise the full insurance and transition lines associated with these preferences. The essential feature of this example is that the two lines intersect. I examine two income ranges separately. First, for $j = g$ or b , let $\underline{m}_j = \underline{h}(1 + \theta_j)$. Then for all $m_j < \underline{m}_j$, demands in state j are Leontief. In fact, for all $m_g \leq \underline{m}_g$, the equation of the transition line is $m_b = \tau(m_g) = (1 + \theta_b / 1 + \theta_g)m_g$ — that is, given income in the bad state, to be incentive compatible income in the good state must be low enough that demands are identical across states.

When income in the good state satisfies $m_g \leq \underline{m}_g$, the full insurance line $m_b = \phi(m_g)$ satisfies $m_g < \phi(m_g) < \tau(m_g)$. That is, the full insurance line lies

between the 45° line and the transition line. To see this, let $\delta > 0$, and for all $m_g < \underline{m}_g$, let $m_b = \tau(m_g)$. Then $v(\theta_g, m_g + \delta) = f[(m_g + \delta)/(1 + \theta_g)] = f[m_g/(1 + \theta_g) + \delta/(1 + \theta_g)]$ and $v(\theta_b, m_b + \delta) = f[m_g/(1 + \theta_g) + \delta/(1 + \theta_b)]$. Letting $\delta \rightarrow 0$, this confirms that $v_m(\theta_g, m_g) > v_m(\theta_b, m_b)$ along the transition line. As long as $v_{mm} < 0$ (as assumed), this implies that $\phi(m_g) < \tau(m_g)$ as required.

In region *B* of the plane, indifference curves are quasi-linear. Define $\bar{m}_b = \bar{\omega} + \theta_b \underline{h} + a/\underline{h}^n$, so that for all $m_b > \bar{m}_b$ demand for health by an individual in the bad state is equal to \underline{h} , in which case the transition line is given by $m_b = \tau(m_g) = m_g + \tau_0$, where

$$\tau_0 = [\zeta(h(\theta_g)) - e_g] - \left[\zeta\left(\frac{\theta_b}{\theta_g} h(\theta_b)\right) - e_b \right],$$

where e_s is unconstrained health expenditure in state *s*. Thus, whenever $m_g > \bar{m}_b - \tau_0$, this is the equation of the transition line. Similarly, when indifference curves are quasi-linear, the full insurance line is $m_b = \phi(m_g) = m_g + \phi_0$ where $\phi_0 = [\zeta(h(\theta_g)) - e_g] - [\zeta(h(\theta_b)) - e_b] \equiv \sigma_g - \sigma_b > \tau_0$, as long as $m_b > \bar{m}_b$. Combining these conditions, we observe that for $m_g > \bar{m}_b - \phi_0$, we have

$$\phi(m_g) = m_g + \phi_0 > m_g + \tau_0 = \tau(m_g).$$

It just remains to confirm that $\underline{m}_g < \bar{m}_b - \phi_0$. But this is equivalent to $\underline{m}_g + \sigma_g < \bar{m}_b + \sigma_b$, and the left hand side is $\underline{\omega}$ and the right hand side is $\bar{\omega}$. Thus, $\tau(m_g) > \phi(m_g)$ for $m_g < \underline{m}_g$ and $\tau(m_g) < \phi(m_g)$ for $m_g > \bar{m}_g - \phi_0$. Both $\tau(\cdot)$ and $\phi(\cdot)$ are continuous functions, so $\Delta(m_g) = \tau(m_g) - \phi(m_g)$ is also continuous, with $\Delta(\underline{m}_g) > 0$ and $\Delta(\bar{m}_g - \phi_0) < 0$. Therefore, there exists $\hat{m}_g \in (\underline{m}_g, \bar{m}_g - \phi_0)$ such that $\Delta(\hat{m}_g) = 0$, or $\tau(\hat{m}_g) = \phi(\hat{m}_g)$.

Now, suppose that a high risk individual with preferences as above has money income m_0 such that she is fully insured at actuarially fair rates at $(m_g, m_b) = (\hat{m}_g, \hat{m}_b)$ where $\hat{m}_b = \tau(\hat{m}_g)$. That is,

$$\arg \max_{m_b, m_g} p_h v(\theta_b, m_b) + (1 - p_h) v(\theta_g, m_g) = (\hat{m}_g, \hat{m}_b).$$

Fig. 7 in the text plots the transition and full insurance lines for this case.

References

- Akerlof, G., 1970. The market for 'lemons': quality uncertainty and the market mechanism. *Quarterly Journal of Economics* 84, 488–500.
- Arnott, R., Stiglitz, J., 1988. The basic economics of moral hazard, NBER Working Paper No. 2484.
- Buchmueller, T., DiNardo, J., 1999. Did community rating induce an adverse selection death spiral?: evidence from New York, Pennsylvania, and Connecticut, NBER Working Paper No. 6872.
- Chernew, M.E., Frick, K.D., 1999. The impact of managed care on the existence of equilibrium in health insurance markets. *Journal of Health Economics* 18, 573–592.

- Cutler, D., Zeckhauser, R., 1997. Adverse selection in health insurance, NBER Working Paper No. 6107.
- Cutler, D., Reber, S., 1998. Paying for health insurance: the tradeoff between competition and adverse selection. *Quarterly Journal of Economics* 113, 433–666.
- Dasgupta, P., Maskin, E., 1986a. The existence of equilibrium in discontinuous economic games, I: Theory. *Review of Economic Studies* LIII, 1–26.
- Dasgupta, P., Maskin, E., 1986b. The existence of equilibrium in discontinuous economic games, II: Applications. *Review of Economic Studies* LIII, 27.
- Feldstein, M., Friedman, B., 1977. Tax subsidies, the rational demand for insurance, and the health care crisis. *Journal of Public Economics* 7, 155–178.
- Hart, O., Holmstrom, B., 1987. The theory of contracts. In: Bewley, Truman, (Eds.), *Fifth World Congress, Advances in Economic Theory*. Cambridge University Press, Cambridge, UK, pp. 71–155.
- Jack, W., Sheiner, L., 1997. Welfare improving health expenditure subsidies. *American Economic Review*, 87 pp.
- Manning, W., Newhouse, J., Duan, N., Keeler, E., Leibowitz, A., Marquis, S., 1987. Health insurance and the demand for medical care: evidence from a randomized experiment. *American Economic Review* 77, 251–277.
- Mirrlees, 1971. An exploration in the theory of optimum income taxation. *Review of Economic Studies* 38, 175–208.
- Myerson, R., 1979. Incentive compatibility and the bargaining problem. *Econometrica* 47, 61–73.
- Pauly, M., 1986. Taxation, health insurance, and market failure in the medical economy. *Journal of Economic Literature* XXIV (2), 629–675.
- Rothschild, M., Stiglitz, J., 1976. Equilibrium in competitive insurance markets: an essay on the economics of imperfect information, *Quarterly Journal of Economics*, pp. 629–649.
- Tirole, J.-J., 1988. In: *The Theory of Industrial Organization*. MIT Press, Cambridge, MA.
- Townsend, R.M., 1979. Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory* 21, 265–293.
- Wilson, C., 1977. A model of insurance markets with incomplete information. *Journal of Economic Theory* 16, 167–207.