

# Protecting the Vulnerable: the Tradeoff between Risk Reduction and Public Insurance

*Shantayanan Devarajan and William Jack*

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In a risky world should governments provide public goods that reduce risk or compensate the victims of bad outcomes through social insurance? This article examines a basic question in designing social protection policies: how should a government allocate a fixed budget between these two activities? In the presence of income and risk heterogeneities a simple public insurance scheme that pays a fixed benefit to all households that suffer a negative shock is an effective redistributive instrument of public policy. This is true even when a well functioning private insurance market exists, and so the role of public insurance is not to correct a market failure. In fact, the existence of a private insurance market means that the public system has desirable targeting properties—all but the poor and high-risk take up private insurance. The provision of public goods that reduce risk for all should therefore be complemented with public insurance that (automatically) benefits those who are especially vulnerable. JEL Codes: H41, H42, I38.

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When unanticipated disasters hit individuals, businesses, and communities, governments are often expected to respond. Governments provided much of the relief after the devastating Indian Ocean tsunami of 2004 and the 2005 earthquake in Pakistan, and they continue to do so. Similarly, the U.S. government was the main source of compensation for victims of the terrorist attacks of September 11, 2001. Governments are likewise called upon to distribute food aid in the event of drought, and they are expected to provide emergency medical care in response to disease outbreaks. All these post-event compensatory actions can be thought of as publicly provided insurance—public transfers to individuals in the event of bad luck—that spread risk across the population.

But governments can also affect the chances that individuals suffer direct negative shocks. For example, early warning systems for tsunamis and drought can reduce the negative shock associated with bad events. Similarly, dams prevent and control flooding, and mosquito spraying can lower the risk of malaria.

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How should spending on these two types of risk-management activities—public insurance and risk-reducing expenditures—be balanced? What is the tradeoff between a dam and public flood insurance<sup>1</sup> or between mosquito eradication and antimalarial drugs?<sup>2</sup>

These policy choices are important in the context of reducing poverty. There is increasing recognition that poverty is not only a situation of low income, but also one of vulnerability to severe income shocks, such as loss of work, ill-health, and the like. For example, in Indonesia (before the East Asian crisis) about twice as many people were vulnerable to being poor as were poor (defined as having an income in the 20th percentile).<sup>3, 4</sup> Although vulnerability has been widely studied and documented, much less has been written about what governments should do about it.<sup>5</sup> For this reason this article accounts for individual heterogeneity in both risk and income and for the distributive effects of government spending.

This article develops a model in which the government provides a public good that reduces the probability of a negative shock. There is no private provision of public goods, so no crowding-out of precautionary actions occurs. But publicly provided insurance that pays a fixed amount to all victims is allowed to affect the risk-sharing behavior of individuals and communities, as it is assumed that an efficient private insurance market exists. Individuals then have the option of availing themselves of the free but possibly incomplete coverage under the public scheme or opting out and purchasing private insurance. To be sure, governments sometimes provide insurance because private insurance markets are inefficient.<sup>6</sup>

1. Bangladesh has considered spending large amounts of money to build or reinforce dams along three rivers that frequently flood. An alternative use of these funds would be to provide flood insurance to compensate flood victims.

2. This kind of choice is also evident at the aggregate, or macroeconomic, level. Sound macroeconomic policies, with the associated political costs of fiscal restraint, can smooth fluctuations in incomes, while countercyclical transfers protect individuals who suffer during downturns.

3. In particular, 30–50 percent of the population had a 50 percent chance of having their income fall below that of the 20th percentile in the following year (Pritchett, Sumarto, and Suryahadi 2000).

4. The definition of vulnerability is itself the subject of debate. World Bank (2000) introduces dynamic and longer term concepts such as resilience, and Ligon and Schechter (2003) and Kamanou and Morduch (2003) develop more welfare economic concepts. This article does not contribute to this debate: vulnerability is measured here purely by a probability, that is, the chance of suffering a negative shock.

5. Albarran and Attanasio (2003), Dercon and Krishnan (2003), and Barrett, Holden, and Clay (2003) examine the effects of particular policy interventions on risk exposure. Morduch's (1999) review of safety net programs briefly compares public policies that reduce risk with means-tested transfers, unemployment benefits, health insurance, and social security. He argues, however, that risk-reducing expenditures are "policy areas that are on the table for other reasons, and are best judged by other criteria". This suggests that risk-reducing expenditures are best dealt with outside the design of public insurance schemes. By contrast, this article explicitly examines the tradeoff between these two policy instruments.

6. It is well understood that if insurance markets are missing, public safety nets can enhance efficiency, not just because of the reduced uncertainty of income, but also because they can increase average incomes, due to changes in production techniques, for example, Jalan and Ravallion (2003). Banerjee (2003) argues that lack of insurance leads to similar dynamic inefficiencies and poverty traps.

Allowing for efficient private insurance, it can be demonstrated that there is a redistributive case for public insurance beyond correcting insurance market failure: to improve the welfare of individuals who are high risk, poor, or both.

Income and risk heterogeneity create the possibility of a redistributive role for government, although they themselves do not support public provision of insurance. Two superior instruments are a lump-sum transfer from low-risk households to high-risk households and a progressive income tax. It is assumed here that both instruments are unavailable. Taxes and transfers based on ex ante risk characteristics are very difficult to implement in even the most capacity-rich countries,<sup>7</sup> and sophisticated and well functioning income tax systems are particularly rare in developing countries (Thirsk 1998). In the absence of these instruments, publicly provided insurance can serve a redistributive role. Such a motivation underlies much of the increased interest in developing so-called “social protection” policies as part of broad poverty reduction strategies in developing countries.<sup>8</sup>

How can it be assumed that the government can operate a public insurance system but not a redistributive tax-and-transfer system? First, suppose there is an optimal nonlinear income tax, based on earned income. Because of standard asymmetric information problems (Mirrlees 1971), this tax system will not fully redistribute income across the population; there will be some (typically much) residual heterogeneity in after-tax incomes due to the distortionary effects of taxes on taxable income. The public insurance system described here is an additional redistributive instrument that can be grafted onto the tax system to improve welfare. The exogenous distribution of income assumed here can thus be interpreted as the distribution of after-tax income in a more complete model.

In this model public insurance pays a fixed amount to anyone in the system who suffers a negative shock. This amount complements transfers inherent in the income tax system because the flat insurance benefit is a function solely of the realized state of nature rather than of an individual’s realized income. This design feature is the second reason that governments can operate a public insurance system even if their tax systems are rudimentary: it is much easier to verify that an individual has suffered a shock than to estimate the value of the loss.<sup>9</sup> This feature in turn has two effects that are absent from an income tax: transfers are made to unlucky individuals (independent of their actual income), and this kind of insurance is more valuable to individuals who expect to have especially low incomes in the event of a negative shock or who expect to suffer such a shock more often. Thus the public insurance scheme represents a useful

7. See, for example, the literature on risk adjustment in health insurance: Glazer and McGuire 2000, 2002; Newhouse 2002.

8. See, for example, the chapter on social protection in World Bank (2002), available at [www.worldbank.org/poverty/strategies/chapters/socprot/socprot.htm](http://www.worldbank.org/poverty/strategies/chapters/socprot/socprot.htm).

9. After the Indian Ocean tsunami the Sri Lankan government compensated individuals who had lost their house, but the value of their house was not accurately assessed in calculating this payment.

additional targeting instrument: resources are directed to individuals with either low (expected) incomes, high risk, or both.

An important feature of this model is that individuals can choose to participate in the public insurance system or to purchase insurance on the private market, but not both. This rules out individuals using the public system but taking out complementary insurance to pay for uncovered losses. With this feature the public insurance scheme naturally targets the poor, who choose to opt into the system. Without it, everyone would use the (free) public system, and its targeting properties would be reduced. It would, however, still possess some targeting features, directing resources (on average) to individuals with a higher probability of suffering negative shocks, for example.<sup>10</sup>

This self-targeting property is why governments might find it desirable to prohibit complementary insurance. Private insurance companies may, conversely, want to insure only individuals who are not covered by the public system for moral hazard reasons. Although, for simplicity, it is assumed that the private market works under conditions of complete information, in practice most insurance policies provide incomplete coverage so as to maintain individuals' incentives to take precautions against negative shocks (engaging in active job search, exercising and eating well, and so on). If public insurance increases an individual's coverage, these incentives will be weakened. Similarly, moral hazard provides an additional reason that the government may prohibit complementary private coverage.<sup>11</sup>

It is necessary to be explicit regarding the government's institutional capacity to implement an insurance system with the targeting properties described here. First, the government must be able to determine whether an individual has experienced a negative shock; second, it must be able to exclude privately insured individuals from using the public system. Both of these may require a degree of administrative capacity beyond what already exists, especially in poor countries. In practice governments would be expected to base insurance benefits on more aggregate measures of shocks, including, say, local rainfall levels (as is currently under consideration in Ethiopia) and areas of disease outbreak, which are easily measured. Regarding the ability to exclude privately insured individuals, the incentives identified above for both public and private providers are strong enough for them to do so in practice, if only imperfectly. Finally, the government may have other instruments for redistributing income, including an income tax and public spending targeted to the poor. These policy instruments are not excluded, but the possibility is noted that public insurance could be an additional instrument to enhance social welfare.

10. It would not, by contrast, direct resources to individuals with low incomes in the good state and even lower incomes in the bad state.

11. In some developed countries complementary insurance (also referred to as "gap" insurance) is permitted—for example, *assurance complémentaire* in France—but in most it is prohibited.

The tradeoff between public risk reduction and public insurance depends of course on the effectiveness of the public good in reducing risk.<sup>12</sup> But the optimal allocation of public spending between the two depends on flows into and out of the public insurance scheme as budget allocations are altered. Increasing the generosity of public insurance directly benefits the poor and vulnerable, but also causes more individuals to opt into the scheme, thereby putting upward pressure on insurance expenditures. Investment in risk reduction has the opposite effect on the pool of publicly insured: it lowers the cost of private insurance and therefore induces relatively low-risk and relatively high-income individuals to leave the public system. Thus public good provision affects the targeting properties of the public insurance system. The analysis here focuses on the interaction of these effects, holding constant the technical efficiency of the public good in reducing risk.

This article illustrates the complexity of the public policy tradeoff between prevention and cure. As a result of this complexity, robust but nontrivial conclusions about optimal public spending allocations are not easily forthcoming. Simulation techniques are used to gain further insight into an issue that is relevant to the debate on the multidimensional nature of poverty. In particular, if being income poor is associated with exposure to greater risk, should public spending be focused more toward one instrument than the other? The simulation results show however that the optimal allocation of the budget between the public good and insurance is nearly independent of the correlation between income and risk. Risk makes the poor poorer, but it does not significantly affect the government's appropriate antipoverty program, at least in the framework considered here.<sup>13</sup>

## I. SETUP OF THE MODEL

There is a continuum of individuals in the economy, each of whom is endowed with certain income and risk characteristics. An individual's "income type,"  $y \in [y_{\min}, y_{\max}]$ , is exogenous, meaning that there is no labor supply decision. There are two states of the world: good and bad. In the good state an individual earns income  $y$ ; in the bad state he or she earns  $\alpha y$ , where  $\alpha \in (0, 1)$  is fixed and the same for all individuals, meaning that they cannot influence the size of a loss in the bad state (there is no hidden information moral hazard). In the absence of public good provision the good state occurs with an underlying probability  $p \in [0, 1]$ , which is exogenous to each individual, meaning that he or she cannot affect this probability (there is no hidden action moral hazard),

12. Earlier literature (for example, Schlesinger and Venezian 1986) examined the incentive for a profit-maximizing monopoly insurer to invest in risk reduction. The straightforward assumptions are that monopoly profit is a concave function of the probability that an insured individual suffers a loss and that a private insurer might want to alter the probability to maximize profits, net of the costs of manipulating individuals' exposure to risk. Even in a model with no public insurance the optimal level of public expenditure to reduce risks would likely be positive, for similar reasons.

13. It may of course mean that the overall budget should be increased.

but varies across individuals. The strong assumption is made that shocks are idiosyncratic (discussed in more detail in section III when specifying the government's budget constraint). Thus each individual in the economy is indexed by a pair  $(y, p)$ . Individuals are distributed over the set  $\Omega \subset [y_{\min}, y_{\max}] \times [0, 1]$  with suitably differentiable density function  $\phi(\cdot, \cdot)$ . All individuals have the same von Neumann–Morgenstern utility index  $u(\cdot)$ , defined over income.

The focus is the allocation of a fixed government budget  $R$ , which can be spent on a pure public good  $G$  and on state-contingent transfers (insurance).  $G$  decreases the probability of the bad state occurring (it increases  $p$ ). Thus let  $\pi(G, p)$  be the probability that a  $p$ -individual faces the good state given  $G$ , where  $\pi_G > 0$ ,  $\pi_p > 0$ , and  $\pi \in [0, 1]$ . The effect of  $G$  on  $\pi$  is assumed to be independent of an individual's income.<sup>14</sup>

An alternative public expenditure is state-contingent transfers or services at a uniform rate  $m$  per capita. For example, if the risk is health-related,  $m$  could be the level of medical care available to an individual contingent on the person being sick. Alternatively,  $m$  could be a flat dollar amount paid to workers who become unemployed or otherwise lose their livelihood.

Individuals can purchase insurance at actuarially fair prices in a private market. It is assumed that there are no administrative costs associated with insurance. Thus, individuals with higher incomes (and hence higher losses in the bad state) tend to purchase more insurance in the private market than those with lower incomes, and individuals with less risk pay lower premiums. For reasons outlined in the introduction, an individual with private insurance is not permitted to use the public system—that is, by purchasing private coverage individuals effectively opt out of the public system.<sup>15</sup> The decision to do so is of course endogenous and depends on the government's choice of policy instruments  $m$  and  $G$ .

## II. PARTICIPATION IN THE PUBLIC SYSTEM

Let  $S(y, \pi; m)$  be defined as the net surplus an individual earns from purchasing private insurance instead of enrolling in the public insurance system when income in the good state is  $y$  and the probability of being therein is  $\pi$ . Clearly  $S_m(y, \pi; m) < 0$ : the more generous the public scheme, the greater an individual's expected utility of enrolling. An individual whose income in the good

14. The sign of the cross derivative,  $\pi_{Gp}$ , is not specified at this stage, but as  $p \rightarrow 1$ , it is necessary that  $\pi_G \rightarrow 0$ . There is little effect of the public good on individuals who already have virtually no chance of being in the bad state. Thus the public good naturally favors those who are more vulnerable (have lower  $p$ ) but public insurance does as well.

15. In addition to the reasons mentioned in the introduction, some kinds of public insurance are likely to be provided in kind rather than in cash, making individuals not want to double dip. For example, it may be difficult to use both public hospital services and private medical care for the treatment of a given condition.

state is less than  $m/(1 - \alpha)$  will definitely choose the public system. In this case his or her income in the bad state is  $\alpha y + m$ , which is higher than in the good state, providing expected utility greater (albeit with some risk) than could be obtained with full private insurance, that is,  $S < 0$ . Public insurance overinsures the very poor. Conversely, the very rich will definitely purchase private insurance. To see this, consider a very large  $y$  (and hence  $\alpha y$ ): public insurance yields a pair of incomes,  $(y, \alpha y + m)$ , which is close to  $(y, \alpha y)$  and so delivers virtually no improvement in expected utility. Private insurance by contrast yields a first-order increase in expected utility, so  $S > 0$ . On the basis of these limiting properties, the net surplus earned from private insurance is assumed to be increasing in  $y$  over the whole range of incomes. That is,  $S_y(y, \pi; m) > 0$ .<sup>16</sup>

When  $\pi = 1$ , the net surplus from private insurance is of course zero; neither public nor private insurance increases the individual's expected utility. But when  $\pi = 0$ , the net surplus is unambiguously negative; private insurance does nothing for the individual, but the public system guarantees a transfer of  $m$ .  $S$  is either always negative (except at  $\pi = 1$ , when  $S = 0$ ), in which case all individuals choose the public system, or first negative, then positive, and then zero at  $\pi = 1$ .<sup>17</sup> In this case, there is a value  $\hat{\pi}(y; m)$  such that  $S \geq 0$  as  $\pi \geq \hat{\pi}$ . Because it is assumed that  $S_y > 0$ ,  $\hat{\pi}$  is decreasing in  $y$ , whereas it is increasing in  $m$  (figure 1).

This behavior of  $\hat{\pi}$  with respect to  $y$  and  $m$  allows the decomposition of the population into those who opt into the public system and those who opt out (figure 2).<sup>18</sup> Assuming a given level of  $G$ , and hence a given mapping from  $p$  into  $\pi$ , the set of individuals who join the public system is denoted by  $P$ , its complement by  $P'$ , and the set of those who are indifferent between the public and private systems by  $\partial P$ . It is convenient in the next section to describe the boundary of the participation set,  $\partial P$ , as a function  $\hat{y}(p; m, G)$ .

An increase in the generosity of the public system (an increase in  $m$ ) shifts the boundary  $\partial P$  to the right, increasing the share of the population publicly insured. But an increase in public good spending  $G$ , holding  $m$  fixed, shifts  $\partial P$

16. A sufficient, but not necessary, condition for this is that  $u'''(y) < 0$ .

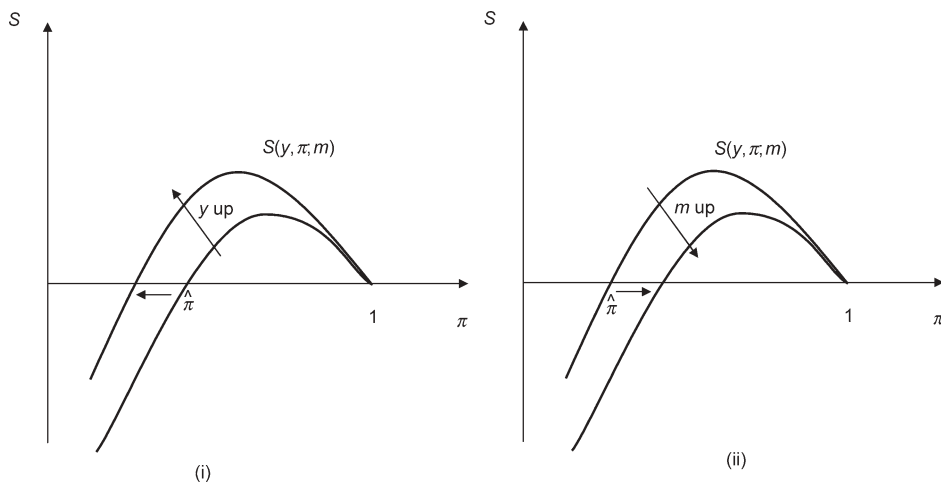
17. The net surplus function can be written

$$\begin{aligned} S(y, \pi, m) &= u([\pi + \alpha(1 - \pi)]y) \\ &\quad - [\pi u(y) + (1 - \pi)u(\alpha y + m)] \\ &\equiv v(y, \pi) - \omega(y, \pi, m) \end{aligned}$$

Note that  $\omega(\cdot)$  is linear in  $\pi$ , whereas  $v(\cdot)$  is concave. Also,  $v(y, 0) < \omega(y, 0; m)$ , and  $v(y, 1) = \omega(y, 1; m)$ , so that either  $S(y, \pi; m) < 0$  for all  $\pi$  and all individuals opt into the public system or only those for whom  $\pi$  is high enough do.

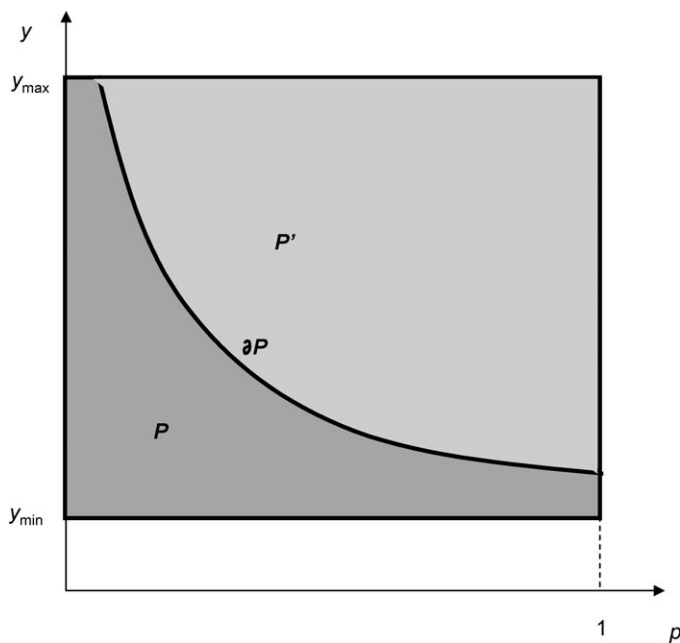
18. Figure 2 is drawn assuming that individuals with very low risk ( $p$  near 1) but low incomes ( $y$  near  $y_{\min}$ ) opt into the public system and that those with high incomes (up to  $y_{\max}$ ) but high risk ( $p$  near 0) also opt in. This need not be the case—that is, the line  $\partial P$  may intersect one or both of the axes—but it has no substantive impact on the analysis.

FIGURE 1. Net Surplus Earned from Purchasing Private Insurance Instead of Participating in the Public System



Note:  $y$  is an individual's income in the good state,  $\pi$  is the probability of an individual being in the good state, and  $m$  is the benefit paid by the government insurance scheme in the bad state.

FIGURE 2. Individuals' Decision Whether to Opt in to the Public Insurance System



Note:  $P$  is the set of individuals who join the public insurance system,  $P'$  is the set of individuals who join the private insurance system, and  $\partial P$  are those individuals who are indifferent between the public and private systems.

to the left, reducing participation in the public system. Public good spending reduces the value of public insurance relative to private insurance, even holding  $m$  constant, and hence allows the public system to be better targeted to individuals with low incomes and high risk.

The reason low-income and high-risk individuals opt into the public system while others opt out is that the value of public insurance differs correspondingly, with the payment in the bad state independent of income. Such a payment is worth more to individuals with lower incomes (due to declining marginal utility of income) and to individuals with higher risk, that is, those who expect to receive the payment more often. This kind of insurance is worth less to individuals with a low marginal utility of income and to individuals who do not expect to need it very often. Thus, undifferentiated public insurance is necessarily valued differently by different individuals and is thereby self-targeted to the poor and vulnerable.<sup>19</sup>

Of course, more closely targeted transfer systems are usually considered better because they allow more to be spent on each recipient. However, the improvement in targeting associated with an increase in  $G$  does not obviously increase welfare because  $m$  is unlikely to increase or remain constant. Expenditure on the public good must be financed by reductions in public insurance payments, and although there are fewer recipients, those who quit the system have low risk and receive the transfer infrequently (compared with those who remain). By contrast, unlike administrative expenditures (for example, on outreach and monitoring) designed to improve the targeting of certain transfer programs, the public good embodies direct benefits of its own, both to individuals who opt out of the public insurance system and to those who stay in (unless  $m$  is so large that the transfer-inclusive income of publicly insured individuals is higher in the bad state than in the good state). The optimal balance between public insurance and risk reduction accounts for both these targeting and risk reduction effects of the public good.

### III. OPTIMAL PUBLIC EXPENDITURE

Since shocks are independently (but not identically) distributed across the population, there is no aggregate uncertainty about public (or private) insurance payments. Given the policy variables  $m$  and  $G$ , the cost of running the public system is

$$(1) \quad M(m, G) = m \int_p (1 - \pi(G, p)) \phi(y, p) dy dp.$$

19. A similar targeting mechanism is used in Besley and Coate's (1991) article on in-kind transfers.

The costs of public insurance plus the public good must be no greater than the revenue available, that is<sup>20</sup>

$$(2) \quad R \geq M(m, G) + G.$$

Since increasing  $G$  induces more individuals to opt out of public insurance, the impact on the level of per capita spending,  $m$ , is ambiguous. On the one hand, in the case of medical care, for example, fewer public patients means higher quality ( $m$ ) for those remaining. On the other hand, the smaller insurance budget (equal to  $R - G$ ) means lower quality per capita.

Using the definitions of  $\omega(y, \pi; m)$  and  $v(y, \pi)$  in footnote 17 and assuming a utilitarian welfare function,<sup>21</sup> the government's optimization problem is

$$(3) \quad \begin{aligned} \max_{G, m} W(G, m) &= \int_P \omega(\pi, y, m) \phi(y, p) dp dy \\ &+ \int_{P'} v(\pi, y) \phi(y, p) dp dy \\ \text{subject to } \pi &= \pi(G, p) \\ \text{and } R &\geq M(m, G) + G. \end{aligned}$$

This optimization problem is potentially nonconvex. To begin however, the first-order conditions are assumed to be sufficient for a maximum. A heuristic illustration of a possible nonconvexity and its implication is then provided.

#### *First-Order Approach*

To derive the first-order conditions for the government's problem, the Lagrangian is defined as

$$(4) \quad \mathcal{L}(G, m; \lambda) = W(G, m) + \lambda[R - (G + M(m, G))]$$

20. In a world with covariate risk, aggregate public (and indeed private) insurance spending would be uncertain, and how unusually high (or low) expenditures would be financed would need to be carefully specified. If governments and insurers have access to reinsurance markets, equation (2) holds in expected value and individuals can be shielded from the aggregate fluctuations. They can be partially shielded if governments and insurers can borrow on capital markets to cover unusually large costs. Without access to such markets, governments will have to save and dissave as events necessitate, but the mix of public spending is unlikely to be significantly affected.

21. Some readers might prefer to adopt a welfare function that exhibits a degree of inequality aversion, such as the type suggested by Atkinson (1970). Within the standard expected utility framework, risk aversion induces declining marginal utility of income, so that even a utilitarian welfare function would lead to redistributive policies.

where  $\lambda$  is the multiplier on the constraint. The first-order condition for  $G$  is

$$(5) \quad \int_P \frac{\partial \omega}{\partial G} d\Phi + \int_{P'} \frac{\partial v}{\partial G} d\Phi - \lambda \left[ 1 - m \left( \int_P \frac{\partial \pi}{\partial G} d\Phi - \int_{\partial P} (1 - \pi) \hat{y}_G d\Phi \right) \right] = 0$$

where  $\hat{y}_G(p; m, G)$  is the increase in the income of individuals with probability  $p$  who are indifferent between public and private insurance, given the policy variables  $m$  and  $G$ , and  $d\Phi$  is shorthand for  $\phi(y, p)dy dp$ . The following expressions for the partial derivatives can then be substituted into this first-order condition:

$$(6) \quad \frac{\partial \omega}{\partial G} = [u(y) - u(\alpha y + m)] \pi_G$$

$$(7) \quad \frac{\partial v}{\partial G} = u'(\bar{y})(1 - \alpha)y \pi_G.$$

Condition (4) can be usefully interpreted as balancing the marginal benefits and costs of expanding the public good.

$$(8) \quad \underbrace{\int_P \frac{\partial \omega}{\partial G} d\Phi}_{\text{Marginal benefit to insiders}} + \underbrace{\int_{P'} \frac{\partial v}{\partial G} d\Phi}_{\text{Marginal benefit to outsiders}} = \lambda \left[ \underbrace{1}_{\text{Marginal cost of public good}} - m \left( \underbrace{\int_P \frac{\partial \pi}{\partial G} d\Phi}_{\text{Savings on insiders}} - \underbrace{\int_{\partial P} (1 - \pi) \hat{y}_G d\Phi}_{\text{Fall in public enrollment}} \right) \right]$$

The first term represents the benefits to users of the public system (insiders) and the second term the benefits to users of the private system (outsiders). The marginal cost of expanding the public good, priced at the shadow cost of public funds ( $\lambda$ ), has three elements. The financial cost of an extra unit of  $G$  is simply one dollar; as a result of the expansion, the probability of the bad state falls, so expenditures on users of the public insurance system fall; finally, the reduction in risk for all individuals induces some of them (those on the boundary  $\partial P$ ) to opt out of the public system (note  $\hat{y}_G < 0$ ), yielding a per capita cost saving of  $m$  with probability  $(1 - \pi)$  to the public budget.

The first-order condition for  $m$  is

$$(9) \quad \int_P \frac{\partial \omega}{\partial m} d\Phi - \lambda \left[ \int_P (1 - \pi(G, p)) d\Phi + m \int_{\partial P} (1 - \pi(G, p)) \hat{y}_m d\Phi \right] = 0$$

where

$$(10) \quad \frac{\partial \omega}{\partial m} = (1 - \pi(G, p)) u'(\alpha y + m).$$

The first-order condition thus simplifies to

$$(11) \quad \underbrace{\int_P (1 - \pi) u'(\alpha y + m) d\Phi}_{\text{Marginal benefit}} = \lambda \left[ \underbrace{\int_P (1 - \pi) d\Phi}_{\text{Intensive marginal cost}} + m \underbrace{\int_{\partial P} (1 - \pi) \hat{y}_m d\Phi}_{\text{Extensive marginal cost}} \right]$$

The term on the left side is the marginal social benefit of expanded quality of public insurance, comprising the expected marginal utility of additional income in the bad state for users of the public system. The term on the right side is the marginal social cost, again valued in terms of public revenue, comprising the cost of paying an extra dollar to public insurance beneficiaries in the bad state and the cost of paying the full benefit  $m$  in the bad state to individuals who join the public system as a result of the increased benefits.

### *Heuristic Approach*

Equations (8) and (11) show the policy tradeoffs at the optimum, assuming that the second-order conditions are satisfied. However, even if simple functional forms are assumed for utility and the effect of the public good, they prove too complex to solve analytically. This section presents a more heuristic analysis of the tradeoff between public insurance and risk reduction.

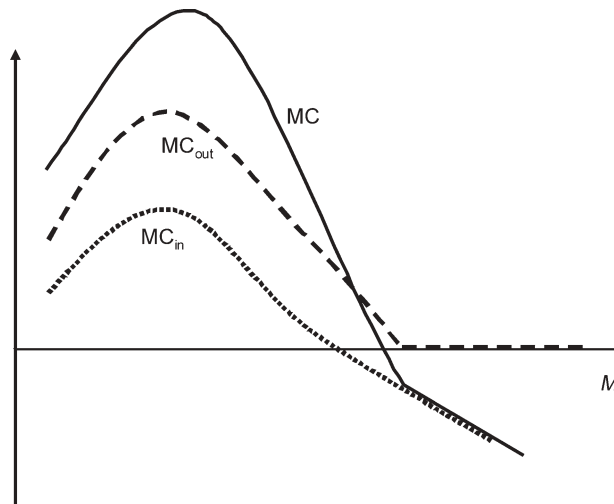
An increase in the public insurance budget,  $M$ , would be effected through an increase in  $m$  and would be matched by a reduction in public good spending,  $G$ . For individuals who participate in the public system there is a direct benefit: payments in the bad state increase, even with the increased participation. The social marginal benefit (the sum of the marginal benefit across participants) may initially increase with  $M$ , as participation increases dominate. At some point it is assumed that the marginal benefit to insiders begins to fall as  $M$  increases, while remaining positive. The marginal benefit per dollar of extra spending is the ratio of the left side of equation (11) to the square-bracketed term on the right side.

An increase in  $M$  is costly to the extent that it must be matched by a reduction in  $G$ . This cutback in public good spending has direct implications

for individuals in the public insurance system and individuals who opt out. For insiders, the marginal cost is assumed to be initially positive and increasing, but as long as the total budget,  $R$ , is large enough, it must become negative (and hence decrease) at some (possibly large) value of  $M$ . This is because when  $M$ , and hence  $m$ , is large, at least some publicly insured individuals receive higher income in the bad state than in the good, and a fall in  $G$  (which increases the likelihood of the bad state) increases their expected utility. The marginal cost imposed on insiders is shown in figure 3 as the dotted line  $MC_{in}$ . This corresponds precisely to the ratio of the first term on the left side of equation (8) to the square-bracketed term on the right (with suitable change of sign).

For individuals who do not participate in the public scheme, the increase in insurance budget reduces welfare, and further increases in  $M$  initially prove costly to those outsiders. Of course, as  $M$  increases, participation in the public system becomes more attractive (both because  $m$  is higher and because  $G$  is lower), so the increase in total costs imposed by the shift in spending on outsiders as a group is less than if participation was fixed. Indeed, if  $M$  increases enough, the whole population might join the public system, and the marginal cost imposed on outsiders (of whom there are now none) would be zero. This is shown as the dashed curve  $MC_{out}$  in figure 3 and corresponds to the ratio of the second term on the left side of equation (8) to the square-bracketed term

FIGURE 3. Marginal Costs Associated with Decreased Public Good Spending and an Increase in the Public Insurance Budget



*Note:*  $MC$  is the total marginal cost,  $MC_{out}$  is the marginal cost to outsiders (users of the private insurance system), and  $MC_{in}$  is the marginal cost to insiders (users of the public insurance system).

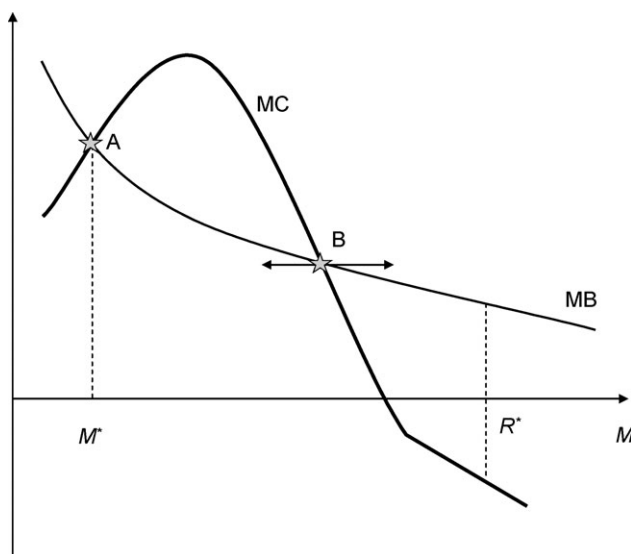
on the right (again, with suitable change of sign). Total marginal costs of increasing  $M$  are denoted  $MC$ .

Figure 4 combines the marginal cost and marginal benefit curves. Point A is a local maximum, at which the budget devoted to the public insurance system is  $M^*$ . Point B is a local minimum, and welfare is increased by either spending more or less on the system (as indicated by the arrows). Clearly, the primary determinants of the optimal level of spending on public insurance (and hence also on the public good) are the levels of the two curves  $MB$  and  $MC$ . In particular, because  $MC$  reflects the marginal benefits of public good spending, the position of this curve will depend crucially on how effective such spending is at reducing risk. If it is ineffective, the  $MC$  curve will be lower and more spending should be allocated to public insurance (point A shifts right). It is even possible that the public good is so ineffective that  $MC$  lies below  $MB$  everywhere, in which case the whole budget should be spent on the insurance scheme.

Increasing the available overall budget,  $R$ , means that for a given insurance budget,  $M$ , there is more spending on the public good, making the bad state less likely. This shifts the  $MB$  and  $MC$  curves down in figure 4, with an ambiguous effect on optimal insurance spending.

However, if the available budget increases above a certain threshold,  $R^*$ , optimal spending on public insurance abruptly jumps from  $M^*$  to  $R$  and the whole budget should be spent on transfers in the bad state. This is simply because with a large budget, transfer-inclusive income in the bad state can be

FIGURE 4. Marginal Cost and Marginal Benefit of an Increase in the Public Insurance Benefit



larger than income in the good state, so the bad state is preferred.<sup>22</sup> However, this possibility should be viewed only as a technical curiosity, since social protection budgets are extremely limited in most poor countries (likely leaving governments constrained at point A) and since if the budget was so large, the government would surely search for alternative ways to distribute it to the population, instead of just doing so in the bad state.

#### IV. SIMULATING THE EFFECTS OF MULTIDIMENSIONAL POVERTY

Several comparative static exercises can be contemplated within this framework, most of which require simulation methods.<sup>23</sup> The issue of most policy relevance—and pertinent to the discussion of vulnerability and multidimensional poverty—is the effect of correlation between income and risk. If income-poor people tend to face greater risk, how should policy respond in terms of the allocation of the budget to insurance and public goods? Because analytic answers to this kind of question are hard to come by, a simulation exercise is used below to develop some intuition.

##### *Specification*

Individuals are assumed to be distributed on  $\Omega = [y_{\min}, y_{\max}] \times [0, 1]$  according to the bivariate log-normal distribution with mean parameters  $\mu_y, \mu_p$ , dispersion parameters  $\sigma_y, \sigma_p$ , and correlation coefficient  $\rho$  (with the external probability distributed proportionately across the domain). The effect of the public good on risk is parameterized by assuming that the  $\pi$  function takes the form

$$(12) \quad \pi(G, p) = p + \beta(1 - p)(1 - e^{-kG})$$

for some  $\beta \in (0, 1)$ . This has the properties that  $\pi(0, p) = p$ ,  $\pi_p > 0$ ,  $\pi_G > 0$ , and  $\pi_{Gp} < 0$ .

All individuals have the same von Neumann–Morgenstern utility functions, specified by the constant relative risk aversion form

$$(13) \quad u(y) = \frac{y^{1-\sigma}}{1-\sigma}$$

where  $\sigma$  is the coefficient of relative risk aversion.

With these parameterizations, it is straightforward to show that for each underlying probability  $p$ , there is a cutoff income level  $\hat{y}(p; m, G)$  such that

22. The authors thank an anonymous referee for providing the intuition for this result.

23. These include, for example, variations in risk aversion, the effects of including the administrative costs of running public insurance systems, and changes in the within-state productivity of the public good (so far it has been assumed that  $G$  affects only the probability of different states occurring, but not the realized income in those states).

individuals with probability of the good state equal to  $p$  choose private insurance whenever  $y > \hat{y}(p; m, G)$ . The expression for  $\hat{y}(p; m, G)$  is

$$(14) \quad \hat{y}(p; m, G) = \frac{m}{z(\pi(G, p)) - \alpha}$$

where

$$(15) \quad z(\pi) = \left( \frac{[(\pi + \alpha(1 - \pi))^{(1-\sigma)} - \pi]}{1 - \pi} \right)^{1/(1-\sigma)}.$$

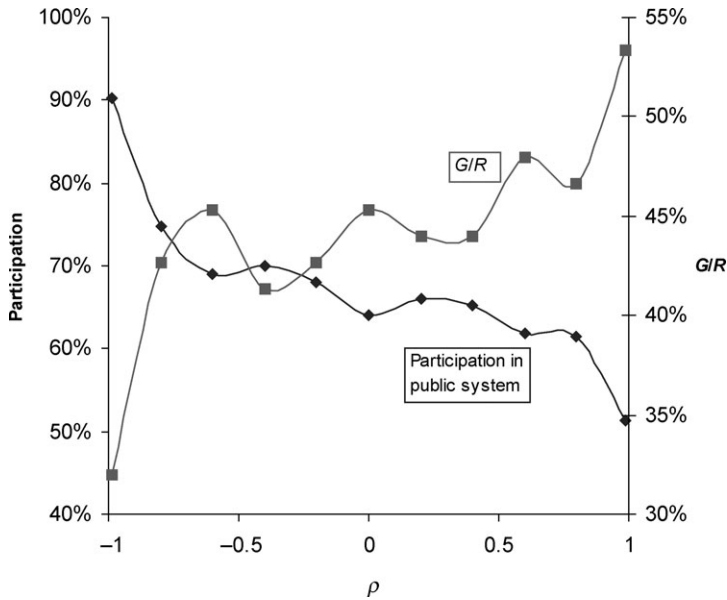
In the simulation a simple grid search is performed over  $(G, m)$  pairs. Because of the government's budget constraint, there is only one degree of freedom, so  $G$  is simply iterated over. For each  $G$ ,  $m$  is iterated over using a basic Newton's method until  $m(G)$  is found such that the budget constraint is satisfied. Welfare is calculated at each  $G$  to find the maximum.

#### *Income-Risk Correlation and Public Policy*

The coefficient of relative risk aversion is fixed at  $\sigma = 1.5$ , and the correlation between risks and income is varied. Recall that  $p$  is the probability of the good state, so a positive correlation indicates an environment in which individuals with low incomes on average face a greater chance of being in the bad state. In changing the distribution of individuals in  $\Omega$ , as occurs when the correlation is varied, aggregate income in the economy is naturally altered. Holding public sector revenue constant in such a comparative statics exercise may not be appropriate. Therefore the budget is fixed as 20 percent of GDP (figure 5). The share of public expenditure devoted to the public good is higher at the extremes—correlations near  $+1$  and  $-1$ —but nearly constant within this range. Participation in the public system has a similar (but inverted) shape—lower rates at correlations near  $+1$  and  $-1$  but nearly constant for a wide range of subunitary correlations.

These simulation results suggest that the mix of public spending between risk reduction and insurance is not very sensitive to the correlation between risk and income heterogeneity. Of course, facing more risk (to the extent this is so) makes the poor poorer. The simulation suggests, however, that the impact this has on policy may be relatively small. The heuristic approach of section III is useful in providing intuition for the apparently small and ambiguous impact of changes in  $\rho$  on optimal public good spending. The initial impact of an increase in  $\rho$  is to increase the resources devoted to public insurance, due to its beneficial targeting properties. But the concomitant reduction in public good spending induces more individuals to take up public insurance, thereby lowering insurance benefits per beneficiary, which in turn mitigates the social benefit of the initial increase in the public insurance budget.

FIGURE 5. Relationship between Optimal Expenditure Policy and the Correlation of Incomes and Risks



Of course, the simulation does not definitively support this conclusion; no simulation can. The simulation does indicate that empirical observations about the correlation between income and risk do not automatically support a shift toward either public good spending or public insurance.

## V. CONCLUSIONS

This article presents a model of the allocation of budgetary resources in a risky environment. In particular, it examines the division of public expenditures between those that reduce underlying uncertainty and those that provide explicit insurance. In addition to deriving conditions for the optimal allocation of public resources, this formulation permits an evaluation of alternative incremental changes to each kind of expenditure when public expenditures are not necessarily optimal, in the spirit of cost-benefit analysis.

An important feature of this article is that it assumes an efficient private insurance market. Some implications of inefficient insurance markets are discussed below, but first it is noted that if insurance markets are efficient, it might be expected that there should be no role for public insurance and that all public spending should be directed toward risk reduction. This is correct in the absence of distributional concerns, but when individuals are heterogeneous

with regard to either income levels ( $y$ ) or risk exposure ( $p$ ), the kind of public insurance described here performs a redistributive function. In particular, its self-targeting properties—individuals with high incomes or low risk tend to opt out of the public system—make it a useful tool of social protection in the broad sense of the term.

Although this redistributive role underpins much of the support expressed for public insurance systems in the context of social protection, the somewhat complicated analytics of self-targeting are underappreciated in the literature. The exact nature of the targeting inherent in the system depends on the division of spending between the public good and the insurance program through their impact on the participation decision. Public good spending makes public insurance less valuable, thus focusing participation on individuals who are relatively poor and relatively high risk. But such spending must be financed with reductions in insurance benefits: so the better targeted public insurance scheme may provide less generous benefits for each person enrolled. The nonlinearities induced by changes in the participation decisions mean that characterization of the optimal spending allocation is nontrivial and that optimal allocations do not vary monotonically with underlying parameters (as shown, for example, in figures 4 and 5).

What changes should be expected with a more realistic view of the potential inefficiencies of the private insurance market? For instance, it has been assumed that risk heterogeneity does not lead to adverse selection and that insurance can be purchased at actuarially fair prices. Although the potential for adverse selection is clear, the qualitative features of the model would be expected to hold if introduced explicitly, for two reasons. First, while adverse selection leads to individuals with low risk opting out of private insurance markets, much practical experience (for example, in Chile) suggests that individuals with high risk tend to end up in the public system. This is exactly the pattern of participation the model here predicts.

Second, the model ignores issues of both hidden action and hidden information moral hazard. The social returns to public insurance might be expected to fall in the presence of moral hazard, so that public good provision might be more favored—a seemingly valid argument for the case of hidden information moral hazard. If hidden action moral hazard is thought to be important, then just as incentives for precautionary actions can be reduced by insurance, so too can public good provision crowd out private precautions, and so the net social productivity of  $G$  may fall as well. The net impact on the division between  $G$  and  $M$  would then be ambiguous.

Notwithstanding these shortcomings, the analysis in this article points to a role for publicly provided insurance that is distinct from its usual role as a correction for failures in the private market. In countries where governments have limited instruments for redistributing income, especially between low- and high-risk individuals, publicly provided insurance can go a long way toward achieving this welfare-enhancing redistribution.

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