

# How Well Does the U.S. Social Insurance System Provide Social Insurance?

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## Abstract

We analyze the insurance provided by the U.S. social security and income tax system within a model where agents receive idiosyncratic, wage-rate shocks that are privately observed. We find large welfare gains to two reforms: a piecemeal reform that optimally chooses the social security benefit function and a reform which eliminates the entire social insurance system and replaces it with an optimal tax on lifetime earnings. When the wage shocks match properties found in U.S. data, an optimal lifetime earnings tax produces a welfare gain equivalent to a 4.3 percent increase in consumption each period and an optimal reform of the social security benefit function leads to a gain worth a 3.9 percent increase.

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*“From the point of view of insurance, there seem to me to be two compelling theoretical arguments for having the State rather than the market provide a wide range of insurance, for old-age pensions, disability and sickness, unemployment and low income: the first is that the market handles adverse selection badly. The second is that, even if adverse selection were not important, people should take out insurance at an age when they are incapable of doing so rationally, namely zero.”* - Mirrlees (1995, p. 384)

## 1 Introduction

One rationale for a government-provided, insurance system is the provision of insurance for risks that are not easily insured in private markets. One can find this rationale in textbooks, in public policy documents and in the work of prominent economists.<sup>1</sup>

An important risk that is often discussed in the context of social insurance is labor income risk. Individual workers experience substantial variation in wage rates which are not related to systematic life-cycle variation or to aggregate fluctuations.<sup>2</sup> A common view is that labor income is not easily insured because it is partly under an individual’s control by the choice of unobserved effort or unobserved labor hours and because a component of labor income risk is realized at a young age. It is often claimed that a progressive income tax system together with a progressive social security system may provide valuable insurance. The Economic Report of the President (2004, Ch. 6) claims that the progressive relationship between monthly social security benefit payments in the U.S. and a measure of lifetime labor income may be an important source of insurance.

We provide a benchmark analysis of how well a stylized version of the U.S. social insurance system provides social insurance. We do so by determining the maximum possible gain to superior insurance. We analyze only the retirement component of the social security system, treat social security together with income taxation as the entire social insurance system and focus only on a single but very important source of risk. The risk that is examined here is idiosyncratic, wage-rate risk.

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<sup>1</sup>See Rosen (2002, Ch. 9), The Economic Report of the President (2004, Ch. 6) and Mirrlees (1995).

<sup>2</sup>See Heathcote, Storesletten and Violante (2008) or Kaplan (2007).

Our methodology involves the analysis of two decision problems. One decision problem is that of a cohort of ex-ante identical agents. Each agent maximizes expected utility in the presence of the model social insurance system. It is assumed that asset markets transfer resources over time and that the social insurance system (i.e. social security and income taxation) is the only way to transfer resources across different histories of wage shocks. We then contrast the ex-ante expected utility in the model insurance system with the maximum ex-ante expected utility that a planner could deliver to this cohort. The planner uses no more resources in present-value terms than are used by a cohort in a solution to the model insurance system. The planner is also restricted to choose allocations that are incentive compatible. The incentive problem arises from the fact that the planner observes each agent's earnings but not an agent's hours of work or an agent's wage.

The model we analyze is closely related to the work of Kaplan (2007). He first estimates a process for male wages that accounts for the variation in mean wages and the idiosyncratic component of wages over the life cycle. He then estimates preference parameters to best match moments characterizing the distribution of consumption, hours and wages over the life cycle. The main deviation from Kaplan's model is that we replace the proportional tax rates on labor and capital income in his model with the structure of the U.S. social security system and the U.S. federal income tax system.

We analyze two versions of this model. The full model captures the pattern of permanent, persistent and purely temporary idiosyncratic wage variation estimated by Kaplan, whereas the permanent-shock model shuts down the variance in the persistent and temporary shock components. The analysis of the permanent-shock model is motivated in part because we can solve the planner's problem for this model but not for the full model. Thus, we calculate maximum welfare gains to superior insurance only for the permanent-shock model. However, we calculate optimal parametric policy reforms in both models.

We find that the maximum welfare gain to improved insurance in the permanent-shock model is large. The maximum welfare gain is equivalent to a 4.1 percent increase in consumption each model period. Important differences in time spent working are

behind this welfare gain. Specifically, high productivity agents work too little and low productivity agents work too much under the U.S. system as compared to the solution to the planning problem.

One reason for these differences in work time is that the pattern of intratemporal wedges in the planning problem differs markedly from the wedges under the U.S. system. In the planning problem, the wedge between the intratemporal marginal rate of substitution and the wage rate is zero for the highest wage agents at each age and increases as an agent's wage rate falls. Thus, the greatest wedge at each age is for the lowest productivity agent. In the U.S. system, the pattern of wedges is exactly the opposite because marginal income tax rates are progressive and because the social security benefit function is concave in a measure of lifetime earnings.<sup>3</sup>

We explore two main reforms. First, we conduct an optimal piecemeal reform by allowing the social security benefit function to be chosen optimally without changing the social security tax rate or the income tax system. This reform leads to almost no welfare gain in the permanent-shock model but an important welfare gain equivalent to a 3.9 percent consumption increase in the full model.

The second reform is more radical. We eliminate the model social insurance system and replace it with an optimal tax on the present value of labor earnings. This tax system imposes no wedge on the intertemporal consumption margin. It imposes an age-invariant wedge on any agent's intratemporal consumption-labor margin within the permanent-shock model. We find that an optimal present value earnings tax achieves nearly all of the maximum possible welfare gain in the permanent-shock model. Intuitively, the present-value tax performs so well because it approximates the wedges between marginal rates of substitution and transformation arising in a solution to the planning problem while allowing for a flexible relationship between lifetime earnings and lifetime consumption. In the full model this reform leads to a welfare gain worth a 4.35 percent increase in consumption. This gain exceeds the welfare gain achieved by optimally choosing the benefit function.

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<sup>3</sup>Average tax rates on lifetime earnings are substantially more progressive in a solution to the planning problem than in the model of the U.S. system. Thus, the large welfare gain originates both from too *little* progression in lifetime taxation and from the wrong pattern of marginal tax rates at each age.

Two literatures are most closely related to the analysis in this paper. First, there is the dynamic contract theory literature which analyzes optimal planning problems in which some key information is only privately observed.<sup>4</sup> Our work is similar in spirit to Hopenhayn and Nicolini (1998), Wang and Williamson (2002) and Golosov and Tsyvinski (2004). These papers analyze optimal planning problems and stylized social insurance systems. Second, there is the literature on social security systems with idiosyncratic risk (e.g. Imrohoroglu, Imrohoroglu and Joines (1995), Huggett and Ventura (1999) and Storesletten, Telmer and Yaron (1999)). Nishiyama and Smetters (2007) is one interesting paper from this literature. They consider various ways of partially privatizing the U.S. social security system. They find important efficiency gains when they abstract from idiosyncratic wage risk. When idiosyncratic risk is added, they find either no efficiency gains or very small gains for the reforms they analyze.

Our findings paint a different picture. We find that the maximum welfare gain to improved insurance substantially increases as the magnitude of idiosyncratic wage risk increases. Our work differs from Nishiyama and Smetters (2007) in at least two main ways. First, we focus on ex-ante welfare as is common in the contract theory literature rather than the ex-interim notion they use. This allows us to assess insurance provision over shocks realized early in life. Second, the methodology differs as we solve for allocations maximizing ex-ante welfare rather than trying particular reforms. This methodology allows one to determine if the maximum possible welfare gain is large or small and to determine which reforms are well focused. It also allows one to take steps towards designing superior insurance systems simply because properties of solutions to the planning problem are known in advance.

The paper is organized as follows. Section 2 presents the framework. Section 3 sets model parameters. Section 4 and 5 present the main results. Section 6 concludes.

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<sup>4</sup>This work builds upon Mirrlees (1971). Golosov, Tsyvinski and Werning (2006) review the recent theoretical literature.

## 2 Framework

### 2.1 Preferences

An agent's preferences over consumption and labor allocations over the life cycle are given by a calculation of ex-ante, expected utility.

$$E \left[ \sum_{j=1}^J \beta^{j-1} u(c_j, l_j) \right] = \sum_{j=1}^J \sum_{s^j \in S^j} \beta^{j-1} u(c_j(s^j), l_j(s^j)) P(s^j)$$

Consumption and labor allocations are denoted  $(c, l) = (c_1, \dots, c_J, l_1, \dots, l_J)$ . Consumption and labor at age  $j = 1, \dots, J$  are functions  $c_j : S^j \rightarrow R_+$  and  $l_j : S^j \rightarrow [0, 1]$  mapping  $j$ -period shock histories  $s^j \in S^j$  into consumption and labor decisions. The set of possible  $j$ -period histories is denoted  $S^j = \{s^j = (s_1, \dots, s_j) : s_i \in S, i = 1, \dots, j\}$ , where  $S$  is a finite set of shocks.  $P(s^j)$  is the probability of history  $s^j$ . An agent's labor productivity in period  $j$ , or equivalently at age  $j$ , is given by a function  $\omega(s_j, j)$  mapping the period shock  $s_j$  and the agent's age  $j$  into labor productivity - effective units of labor input per unit of time worked.

### 2.2 Incentive Compatibility

Labor productivity is observed only by the agent. The principal observes the earnings of the agent which equals the product of a wage rate, labor productivity and work time. In this context, the Revelation Principle implies that the allocations  $(c, l)$  that can be achieved between a principal and an agent are precisely those that are incentive compatible.<sup>5</sup>

We now define incentive compatible allocations. For this purpose, consider the report function  $\sigma \equiv (\sigma_1, \dots, \sigma_J)$ , where  $\sigma_j$  maps shock histories  $s^j \in S^j$  into  $S$ . The truthful report function  $\sigma^*$  has the property that  $\sigma_j^*(s^j) = s_j$  in any period for any  $j$ -period history. An allocation  $(c, l)$  is *incentive compatible* (IC) provided that the truthful report function always gives at least as much expected utility to the agent as

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<sup>5</sup>See Mas-Colell, Whinston and Green (1995, Prop. 23.C.1).

any other feasible report function.<sup>6</sup> The expected utility of an allocation  $(c, l)$  under a report function  $\sigma$  is denoted  $W(c, l; \sigma, s_1)$ .<sup>7</sup> Using this notation,  $(c, l)$  is IC provided  $W(c, l; \sigma^*, s_1) \geq W(c, l; \sigma, s_1), \forall s_1, \forall \sigma$ .

$$W(c, l; \sigma, s_1) \equiv \sum_{j=1}^J \sum_{s^j \in S^j} \beta^{j-1} u \left( c_j(\hat{s}^j), \frac{l_j(\hat{s}^j) \omega(\sigma_j(s^j), j)}{\omega(s_j, j)} \right) P(s^j | s_1)$$

$$\hat{s}^j \equiv (\sigma_1(s^1), \dots, \sigma_j(s^j))$$

## 2.3 Decision Problems

This paper focuses on two decision problems: the U.S. social insurance problem and the planning problem. These problems have the same objective but different constraint sets.  $V^{us}$  and  $V^{pp}$  denote the maximum ex-ante, expected utility achieved.

$$V^{us} \equiv \max_{(c, l) \in \Gamma^{us}} E \left[ \sum_{j=1}^J \beta^{j-1} u(c_j, l_j) \right]$$

$$\Gamma^{us} = \left\{ (c, l) : \sum_{j=1}^J \frac{c_j}{(1+r)^{j-1}} \leq \sum_{j=1}^J \frac{(w\omega(s_j, j)l_j - T_j(x_j, w\omega(s_j, j)l_j))}{(1+r)^{j-1}} \right.$$

$$\left. \text{and } x_{j+1} = F_j(x_j, w\omega(s_j, j)l_j, c_j), x_1 \equiv 0 \right\}$$

$$V^{pp} \equiv \max_{(c, l) \in \Gamma^{pp}} E \left[ \sum_{j=1}^J \beta^{j-1} u(c_j, l_j) \right]$$

$$\Gamma^{pp} = \left\{ (c, l) : E \left[ \sum_{j=1}^J \frac{(c_j - w\omega(s_j, j)l_j)}{(1+r)^{j-1}} \right] \leq Cost \text{ and } (c, l) \text{ is IC} \right\}$$

The constraint set  $\Gamma^{us}$  is specified by a tax function  $T_j$  and a law of motion  $F_j$  for a vector of state variables  $x_j$ . The tax function states the agent's tax payment at age  $j$  as a function of period earnings  $w\omega(s_j, j)l_j$  and the state variables  $x_j$ . Earnings equal the product of a wage rate  $w$  per efficiency unit of labor, labor productivity  $\omega(s_j, j)$  and work time  $l_j$ . Allocations in  $\Gamma^{us}$  have the property that the present value of consumption is no more than the present value of labor earnings less net taxes for

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<sup>6</sup>A report function  $\sigma$  is feasible for  $(c, l)$  provided (1)  $\omega(s_j, j)$  is always large enough to produce the output required by a report (i.e.  $0 \leq l_j(\hat{s}^j) \omega(\sigma_j(s^j), j) \leq \omega(s_j, j), \forall j, \forall s^j$ , where  $\hat{s}^j \equiv (\sigma_1(s^1), \dots, \sigma_j(s^j))$ ) and (2)  $\sigma$  maps true histories into reported histories that can occur with positive probability.

<sup>7</sup> $W(c, l; \sigma, s_1)$  is defined only for  $\omega(s_j, j) > 0$ . Later in the paper, we will set labor productivity to zero beyond a retirement age. It is then understood that labor supply is set to zero at those ages.

any history of labor-productivity shocks.<sup>8</sup> The next section demonstrates that this abstract formulation can capture important features of the U.S. social security and income tax system.

The constraint set  $\Gamma^{pp}$  for the planning problem has two restrictions. First, the expected present value of consumption less labor income cannot exceed some specified value, denoted  $Cost$ . We set  $Cost$  to the present value of resources extracted from a cohort in a solution to the U.S. social insurance problem:  $Cost \equiv E[\sum_{j=1}^J -\frac{T_j(x_j, w\omega(s_j, j)l_j^{us})}{(1+r)^{j-1}}]$ . As all shocks are idiosyncratic, a known fraction of agents  $P(s^j)$  in a cohort receives any shock history  $s^j \in S^j$ . Thus, while the resources extracted from a single agent over the lifetime is potentially random, the resources extracted from a large cohort is not random. Second, allocations  $(c, l)$  must be incentive compatible (IC).

Ex-ante expected utility can be ordered in these problems so that  $V^{pp} \geq V^{us}$ . The argument is based on showing that if the allocation  $(c^{us}, l^{us})$  achieves the maximum, then  $(c^{us}, l^{us})$  is also in  $\Gamma^{pp}$ . Since  $(c^{us}, l^{us})$  satisfies the present value condition in  $\Gamma^{us}$ , then it also satisfies the expected present value condition in  $\Gamma^{pp}$  by the choice of  $Cost$ . It remains to argue that  $(c^{us}, l^{us})$  is incentive compatible. However, the fact that  $(c^{us}, l^{us})$  is an optimal choice implies that it is incentive compatible.

## 2.4 Model Tax-Transfer System

The tax function and law of motion  $(T_j, F_j)$  are now specified to capture features of U.S. social security and federal income taxation. The tax function  $T_j$  is the sum of social security taxes  $T_j^{ss}$  and income taxes  $T_j^{inc}$ . The state variable  $x_j = (x_j^1, x_j^2)$  in  $T_j$  has two components:  $x_j^1$  is an agent's average earnings up to period  $j$  and  $x_j^2$  is an agent's asset holdings.

$$T_j(x_j, w\omega(s_j, j)l_j) = T_j^{ss}(x_j^1, w\omega(s_j, j)l_j) + T_j^{inc}(x_j^1, x_j^2, w\omega(s_j, j)l_j)$$

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<sup>8</sup>The constraint set can equivalently be formulated as a sequence of budget restrictions where the agent has access to a risk-free asset, starts life with zero units of this asset and must end life with non-negative asset holding.

### 2.4.1 Social Security

The model social security system taxes an agent's labor income before a retirement age  $R$  and pays a social security transfer at and after the retirement age. Specifically, taxes are proportional to labor earnings ( $w\omega(s_j, j)l_j$ ) for earnings up to a maximum taxable level  $e_{max}$ . The social security tax rate is denoted by  $\tau$ . Earnings beyond the maximum taxable level are not taxed. At and after the retirement age, a transfer  $b(x^1)$  is given that is a fixed function of an accounting variable  $x^1$ . The accounting variable is an equally-weighted average of earnings before the retirement age  $R$  (i.e.  $x_{j+1}^1 = [\min\{w\omega(s_j, j)l_j, e_{max}\} + (j-1)x_j^1]/j$ ). The earnings that enter into the calculation of  $x_j^1$  are capped at a maximum level  $e_{max}$ . After retirement, the accounting variable remains constant at its value at retirement.

$$T_j^{ss}(x_j^1, w\omega(s_j, j)l_j) = \begin{cases} \tau \min\{w\omega(s_j, j)l_j, e_{max}\} & : j < R \\ -b(x_j^1) & : j \geq R \end{cases}$$

The relationship between average past earnings  $x^1$  and social security benefits  $b(x^1)$  in the model is shown in Figure 1. Benefits are a piecewise-linear function of average past earnings. Both average past earnings and benefits are normalized in Figure 1 so that they are measured as multiples of average earnings in the economy. The first segment of the benefit function in Figure 1 has a slope of .90, whereas the second and third segments have slopes equal to .32 and .15. The bend points in Figure 1 occur at 0.21 and 1.29 times average earnings in the economy. The variable  $e_{max}$  is set equal to 2.42 times average earnings.

We set the bend points and the maximum earnings  $e_{max}$  equal to the actual multiples of mean earnings used in the U.S. social security system. We also set the slopes of the benefit function equal to actual values.<sup>9</sup> Figure 1 says that the social security

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<sup>9</sup>In the U.S. social security system, a person's monthly retirement benefit is based on a person's averaged indexed monthly earnings (AIME). For a person retiring in 2002, this benefit equals 90% of the first \$592 of AIME, plus 32% of AIME between \$592 and \$3567, plus 15% of AIME over \$3567. Dividing these "bend points" by average earnings in 2002 and multiplying by 12 gives the bend points in Figure 1. Bend points change each year based on changes in average earnings. The maximum taxable earnings from 1998-2002 averaged 2.42 times average earnings. All these facts, as well as average earnings data, come from the Social

retirement benefit payment is about 45 percent of mean earnings in the economy for a person whose average earnings over the lifetime equals mean earnings in the economy.

Two differences between the model system and the old-age component of the U.S. system are the following:<sup>10</sup>

- (i) The accounting variable in the U.S. system is an average of the 35 highest earnings years, where the yearly earnings measure which is used to calculate the average is capped at a maximum earnings level.<sup>11</sup> In the model, earnings are capped at a maximum level just as in the actual system, but earnings in all pre-retirement years are used to calculate average earnings.
- (ii) In the U.S. system the age at which benefits begin can be selected within some limits with corresponding actuarial adjustments to benefits. In the model the age  $R$  at which retirement benefits are first received is fixed.

### 2.4.2 Income Taxation

Income taxes in the model economy are determined by applying an income tax function to a measure of an agent's income. The empirical tax literature has calculated effective tax functions (i.e. the empirical relationship between taxes actually paid and income).<sup>12</sup> We use tabulations from the Congressional Budget Office (2004, Table 3A and Table 4A) for the 2001 tax year to specify the relation between average effective federal income tax rates and income. Figure 2 plots average effective tax rates for two types of households: head of household is 65 or older and head of household is younger than 65. The horizontal axis in Figure 2 measures income in 2001 dollars. Figure 2 shows that average federal income tax rates increase strongly in income.

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Security Handbook (2003). The retirement benefit above is for a single-person household. We abstract from spousal benefits.

<sup>10</sup>We acknowledge that we do not try to capture the degree to which the progressivity of the old-age component of social security is mitigated by a positive correlation between survival rates and earnings.

<sup>11</sup>The 35 highest years are calculated on an indexed basis in that indexed earnings in a given year equal actual nominal earnings multiplied by an index. The index equals the ratio of mean earnings in the economy when the individual turns 60 to mean earnings in the economy in the given year. In effect, this adjusts nominal earnings for inflation and real earnings growth.

<sup>12</sup>See, for example, Gouveia and Strauss (1994).

In the model economy, we choose income taxes  $T_j^{inc}(x_j^1, x_j^2, w\omega(s_j, j)l_j)$  before and after the retirement age  $R$  to approximate the average tax rates in Figure 2. We proceed in three steps. First, we approximate the data in 2001 dollars with a continuous function. Specifically, we use the quadratic function passing through the origin that minimizes the squared deviations of the tax function from data. This gives average tax functions before and after the retirement age. Second, we express model income in 2001 dollars.<sup>13</sup> Third, the average tax rates on model income are given by the function estimated in the first step after expressing model income in 2001 dollars. Model income equals the sum of labor income  $w\omega(s_j, j)l_j$ , asset income  $x_j^2r$  and social security transfer income  $b_j(x_j^1)$ , where initial assets are zero (i.e.  $x_1^2 = 0$ ).

### 3 Parameter Values

The results of the paper are based upon the parameter values in Table 1. Model parameters are principally set equal to the values estimated by Kaplan (2007). The goal of Kaplan’s work is to understand many dimensions of cross-sectional inequality from the perspective of a standard, incomplete-markets model with endogenous labor supply. Model parameters are estimated to account for the cross-sectional, variance-covariance patterns of hours, consumption and wages at different ages over the life cycle.<sup>14</sup>

One key departure from Kaplan’s model is that our tax-transfer system differs. We consider a tax-transfer system that captures features of social security and federal income taxation. Thus, net marginal tax rates will vary with an agent’s age and state. Capital and labor taxes in Kaplan’s work are proportional taxes that are age and state invariant.<sup>15</sup>

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<sup>13</sup>This is done using the ratio between the average U.S. earnings and average model earnings. The figure for average U.S. earnings is \$32,921. This comes from the benefit calculation section of the Social Security Handbook (2003).

<sup>14</sup>Heathcote, Storesletten and Violante (2008) analyze a related model with time-varying variances of different components of wages to account for the change in cross-sectional hours, wage, earnings and consumption inequality in the U.S. over time.

<sup>15</sup>There are two other departures. First, we do not allow for heterogeneity in the preference parameters. Second, the working lifetime is 40 years rather than the 38 years in Kaplan (2007). We thank Greg Kaplan

Table 1: Parameter Values

Definition	Symbol	Value	Source
Model Periods	$J$	$J = 56$	Age 25 - 80
Retirement Period	$R$	$R = 41$	Kaplan (2007)
Labor Productivity	$\omega(s_j, j)$	$\omega(s_j, j) = \mu_j \exp(s_j^1 + s_j^2 + s_j^3)$ $s_j^1 \sim N(-\sigma_1^2/2, \sigma_1^2)$ $s_j^2 = \rho s_{j-1}^2 + \eta_j, \eta_j \sim N(0, \sigma_2^2)$ $s_j^3 \sim N(-\sigma_3^2/2, \sigma_3^2)$ Permanent-Shocks Model $(\sigma_1^2, \sigma_2^2, \sigma_3^2, \rho) = (.056, 0, 0, 0)$ Full Model $(\sigma_1^2, \sigma_2^2, \sigma_3^2, \rho) = (.056, .019, .072, .946)$	Kaplan (2007)
Mean Productivity	$\mu_j$	Figure 3	Kaplan (2007)
Preferences	$u(c, l)$	$u(c, l) = \frac{c^{(1-\nu)}}{(1-\nu)} + \phi \frac{(1-l)^{(1-\gamma)}}{(1-\gamma)}$ $(\nu, \gamma, \phi) = (1.66, 5.55, 0.13)$	Kaplan (2007)
Social Security Tax	$\tau$	$\tau = .106$	OASI tax rate
Benefit Function	$b(x)$	Figure 1	SS Handbook (2003)
Income Tax	$T^{inc}$	Figure 2	CBO (2004)
Interest Rate	$r$	$r = 0.042$	Siegel (2002)
Discount Factor	$\beta$	$\beta = .98803$	See Text

There are  $J = 56$  model periods in an agent's life. Retirement occurs at model period  $R = 41$ . At the retirement age labor productivity is zero and an agent starts collecting social security benefits. One model period corresponds to one year. Thus, we view the agent as starting the working life at a real-life age of 25, retiring at age 65 and dying after age 80.

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for providing his estimates of the mean productivity profile based upon 40 working years.

An agent's labor productivity is  $\omega(s_j, j) = \mu_j \exp(s_j^1 + s_j^2 + s_j^3)$ . One's wage at age  $j$  is determined by a fixed wage rate  $w$  per efficiency unit of labor and by one's labor productivity  $\omega(s_j, j)$ . Labor productivity is given by a deterministic component  $\mu_j$  and by an idiosyncratic shock component  $s_j = (s_j^1, s_j^2, s_j^3)$  which captures permanent, persistent and temporary sources of productivity differences. The permanent component  $s^1$  stays fixed for an agent over the life cycle and is distributed  $N(-\sigma_1^2/2, \sigma_1^2)$ . The persistent component follows an autoregressive process  $s_j^2 = \rho s_{j-1}^2 + \eta_j$ ,  $\eta_j \sim N(0, \sigma_2^2)$ . The temporary component  $s_j^3$  is independent across periods and is distributed  $N(-\sigma_3^2/2, \sigma_3^2)$ .

We consider a benchmark model with only permanent shocks as well as a full model with all three stochastic components. The parameters are set to estimates from Kaplan (2007). A one standard deviation permanent shock leads to about a 24 percent permanent change in wages, whereas a one standard deviation innovation to the persistent component changes wages by about 14 percent. The persistent shock is set to zero for each agent at the beginning of the working life cycle. The deterministic wage component  $\mu_j$  is given in Figure 3. This component implies that wages approximately double over the life cycle. We approximate each productivity process with a discrete number of shocks.<sup>16</sup>

The period utility function in the model is additively separable  $u(c, l) = \frac{c^{(1-\nu)}}{(1-\nu)} + \phi \frac{(1-l)^{(1-\gamma)}}{(1-\gamma)}$ . Utility function parameters are set equal to Kaplan's estimates. The coefficient of relative risk aversion is  $\nu = 1.66$ . The coefficient  $\gamma = 5.55$  governs the Frisch elasticity of labor (i.e.  $\epsilon_{labor} = \frac{1}{\gamma} \frac{(1-l)}{l}$  so that the Frisch elasticity is 0.27 evaluated at  $l = .4$ ). These values lie well within a range of values estimated in the literature based upon micro-level consumption and labor data - see Browning et. al. (1999). The value  $\phi = 0.13$  is the mean value estimated by Kaplan.

One important restriction on the utility function  $u(c, l)$  is the assumption of additive separability. Much of the literature on dynamic contract theory with a labor decision

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<sup>16</sup>We approximate the permanent component with 5 equally-spaced points in logs on the interval  $[-\sigma_1^2/2 - 3\sigma_1, -\sigma_1^2/2 + 3\sigma_1]$ . Following Tauchen (1986), probabilities are set to the area under the normal distribution, where midpoints between the approximating points define the limits of integration. The persistent component is approximated with 3 equally-spaced points on the interval  $[-4\sigma_2, 4\sigma_2]$ . Transition probabilities are calculated following Tauchen (1986). The temporary component is approximated with 2 values.

employs this assumption. We make use of this assumption when we design a procedure to compute solutions to the planning problem.<sup>17</sup>

The parameters of the model tax-transfer system are set to capture features of social security and federal income taxation in the U.S. Thus, the social security tax rate  $\tau$  is set to equal 10.6 percent of earnings. This is the combined employee-employer tax for the old-age and survivor's insurance component of social security. The social security benefit function  $b(x)$  and the income tax function  $T_j^{inc}$  are given by Figure 1 and Figure 2, which were discussed in the previous section.

The model is explicitly a partial equilibrium model in that wage  $w$  per efficiency unit of labor and the real interest rate  $r$  are exogenous. They do not vary as we consider alternative social insurance arrangements. Nevertheless, we choose the value of the agent's discount factor  $\beta$  so that a steady state of a general equilibrium version of the full model produces the interest rate  $r = .042$  in Table 1. This interest rate is the average of the real return to stock and to long-term bonds over the period 1946-2001 (see Siegel (2002, Tables 1-1 and 1-2)). The value of the wage  $w$  in the model is then set to the value consistent with the factor inputs that produce this real return as explained in the Appendix.<sup>18</sup>

Figure 4 displays the evolution of the variance of (log) wages, earnings, work hours and consumption within the full model. The dispersion in wages early in life reflects the sum of the permanent and temporary components of productivity. The rise in wage dispersion with age reflects the role of persistent shocks. The dispersion in earnings over the life cycle closely mimics the pattern for wages. One reason for this is that, absent preference heterogeneity, the model produces little dispersion in work hours. The rise in consumption dispersion over the life cycle reflects mainly the role of persistent shocks. The levels of consumption, earnings and wage dispersion are lower at all ages within the full model compared to the U.S. facts documented in Heathcote, Storesletten and

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<sup>17</sup>It is used in Theorem A1 in the Appendix to establish which incentive constraints bind and to reduce dimensionality when we compute solutions to the permanent-shock problem.

<sup>18</sup>The notion of a steady state and how to compute it is standard and follows Huggett (1996). This involves choosing an aggregate production function and setting factor prices to marginal products. The Appendix describes in detail how this is carried out.

Violante (2005). This is because Kaplan (2007) analyzes residual dispersion - dispersion after controlling for observable sources of variation such as those related to differences in education - rather than total dispersion. Although the estimate of the permanent wage shock variance is reduced compared to the estimates in Heathcote, Storesletten and Violante (2008), the parameters of the wage process related to persistent and transitory shocks are not greatly affected.

## 4 Analyzing Welfare Gains

### 4.1 Maximum Welfare Gains

This section quantifies the maximum welfare gain for agents in the permanent-shock model. The welfare gain is measured by the percentage increase  $\alpha$  in consumption in the allocation  $(c^{us}, l^{us})$  solving the U.S. social insurance problem so that ex-ante expected utility is the same as in an allocation  $(c^{pp}, l^{pp})$  solving the planning problem.<sup>19</sup> These allocations use the same expected present value of resources. This calculation is shown below. The results of this section are based on computing solutions to each problem. Computational methods are described in the Appendix.

$$E \left[ \sum_{j=1}^J \beta^{j-1} u(c_j^{us}(1 + \alpha), l_j^{us}) \right] = E \left[ \sum_{j=1}^J \beta^{j-1} u(c_j^{pp}, l_j^{pp}) \right] \equiv V^{pp}$$

Figure 5 highlights the maximum welfare gains attainable for a range of values of the variance of the permanent component of wage shocks. Figure 5 shows that the welfare gain is increasing in this variance. This is true both when the model social insurance system only includes social security and when the model social insurance system includes both social security and income taxation.

To quantify the size of the maximum welfare gain, we need an estimate of this variance. Kaplan (2007) estimates that  $\sigma_1^2 = .056$  for permanent shocks. Thus, a one

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<sup>19</sup>When the range of the period utility function of consumption is not bounded from above, then there is always a value  $\alpha$  solving this equation. The utility to consumption is bounded above by zero for the period utility function in Table 1. Nevertheless, as Figure 4 highlights,  $\alpha$  is well defined for all the examples analyzed.

standard deviation shock increases wages permanently over the lifetime by about 24 percent. Heathcote et. al. (2008) estimate a wage process with a similar structure to Kaplan (2007) but find that  $\sigma_1^2 = .109$ . One reason for this difference is that in a first stage regression Kaplan controls for permanent differences in wages related to education whereas Heathcote et. al. do not. It is valuable to keep both estimates in mind in viewing Figure 5a. Using Kaplan's estimate, Figure 5a shows that the maximum welfare gain in the model of the combined social security and income tax system is equivalent to a 4.1 percent increase in consumption each period.

The analysis in Figure 5a is based upon the idea that while earnings are publicly observed both individual hours of work and individual wage rates are only privately observed. This implies that any mechanism determining consumption and labor over the lifetime must respect the incentive compatibility constraints. Figure 5b describes how important private information is for limiting the size of the gains to superior insurance. Figure 5b plots the maximum welfare gain in the economy with social security and income taxation when wage rates are private information and when they are public information. At the value  $\sigma_1^2 = .056$ , the maximum welfare gain under public information is equivalent to a 6.1 percent change in consumption at each age. This gain is achieved by having all agents of a given age consume the same amount despite large differences in earnings across agents with different productivities.

The remainder of section 4 develops an understanding of what lies behind the patterns in Figure 5. In doing so, the following questions are addressed: (1) How do patterns of lifetime taxation differ in the two problems?, (2) To what degree can welfare be improved by reallocating consumption, fixing the labor allocation?, (3) How do marginal rates of substitution in the model insurance system differ from those in the planning problem? and (4) Why does the welfare gain increase as the shock variance increases?

## 4.2 Patterns of Lifetime Taxation

To get a preliminary idea of the economics behind the maximum welfare gains, it is useful to examine patterns of lifetime taxation. Figure 6 graphs the present value of

earnings and consumption for agents at each of the five values of the permanent shock. This is done both in the model social insurance system and in the planning problem for the benchmark variance of  $\sigma_1^2 = .056$ . Figure 6 shows that lifetime taxation is progressive in both allocations in that the ratio of the present value of consumption to the present value of earnings falls as lifetime earnings increase. Furthermore, there is much more progression in lifetime average tax rates in the planning allocation than in the allocation under the model social insurance system. One additional feature of Figure 6 is that both allocations involve extracting resources in present-value terms from a cohort. This last point is clear as the lifetime tax patterns under the model social insurance system is below the 45 degree line for agents at all permanent shock levels.<sup>20</sup>

A quick look at Figure 6 reveals that the labor allocation must be quite different across these two allocations as the present value of earnings differs sharply. To see this we plot work time over the life cycle. Figure 7a shows that in the planning problem the highest productivity shock agents work the greatest fraction of time and the lowest productivity shock agents work the least. In the U.S. system this pattern of work time is exactly reversed with the lowest productivity agents working the most, as shown in Figure 7b.

One issue raised by Figure 6 and 7 is the extent to which the maximum welfare gains arise from simply reallocating consumption across agents with different permanent shocks, holding the labor allocation fixed. The remaining gains are related to changing work time. Thus, if it were possible to raise the consumption of low shock agents and lower that of high shock agents, how far would such a reallocation go to improving welfare? While such a reallocation would improve ex-ante utility because the utility function is concave in consumption, this reallocation can only be pushed up to the point where the incentive constraints bind.

To answer this question, we calculate the new allocation  $(c^*, l^{us})$  which maximizes

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<sup>20</sup>Intuitively, a pay-as-you-go social security system alone should extract resources from current and future birth cohorts to pay for “free” benefits to previous cohorts. Fullerton and Rogers (1993, Table 4-14) calculate that lifetime average tax rates in the U.S. are roughly progressive in lifetime income and that resources are extracted in present value terms from the cohorts they analyze.

ex-ante utility, holding labor fixed at  $l^{us}$ , while imposing incentive compatibility and the present value resource constraint. We find that at the benchmark value  $\sigma^2 = .056$  the new allocation  $(c^*, l^{us})$  increases welfare over  $(c^{us}, l^{us})$  by 2.9 percent, compared to a maximum 4.09 percent achieved in the planning problem. Thus, important parts of the maximum welfare gain are due both to reallocating consumption and changing the labor allocation.

### 4.3 Sources of Welfare Gains

To further understand what lies behind the results in Figure 5, we examine marginal rates of substitution.

#### 4.3.1 No Labor-Productivity Risk

In the absence of risk, marginal rates of substitution and transformation are equated in a solution to the planning problem. In the model social insurance system, an optimizing agent equates marginal rates of substitution to the after-tax wage and after-tax gross interest rate. Thus, absent risk, non-zero marginal tax rates on capital or labor income lead to welfare losses. Standard intuition (e.g. Feldstein (1996) and Auerbach and Hines (2002)) is that maximum welfare gains are increasing in the magnitude of distortionary taxation, absent risk. The results in Figure 5 are consistent with this intuition. Specifically, the intercept in Figure 5a shows that without risk the maximum welfare gain is equivalent to a .05 percent gain in consumption when only a social security system is present but a .75 percent gain when both social security and income taxation are present.

While the welfare gains without risk are not large compared to those with risk, it is useful to understand the patterns of marginal tax rates that are behind these gains. Thinking about these marginal tax rates will help us develop an understanding of the large differences in marginal tax rates on earnings across agents in the presence of permanent wage risk.

Figure 8 shows that the marginal tax rate on earnings decreases with age over the

life cycle in the model without income taxation.<sup>21</sup> Why is this? The marginal tax rate equals the social security tax rate  $\tau$  less the present value of marginal social security benefits incurred from an extra unit of earnings. Thus, the marginal tax rate decreases with age because the present value of marginal benefits incurred increases as an agent ages. This occurs for two reasons. First, since the retirement benefit in the model is based on average earnings, a one unit increase in earnings in any period raises the social security retirement payment by the same amount.<sup>22</sup> Second, since the real interest rate in the model is positive, the present value of these marginal benefits incurred is greater towards the end of the working life cycle than at the beginning.

Now consider the model with both social security and income taxation. Figure 8 shows that the income tax substantially increases the marginal tax rate on labor earnings. In fact, the marginal tax rate on earnings is now hump-shaped over the working life cycle. Intuitively, this occurs when income over the life cycle is hump-shaped since average and marginal income tax rates increase with income (see Figure 2).

To understand the results in the next section, it is important to keep in mind how the marginal earnings taxes would differ for agents with low or high productivity shocks. Differences in the slope of the concave social security benefit function in Figure 1 are important. Agents with low productivity will have low lifetime earnings and will be on the steep portion of the social security benefit function, whereas agents with high productivity will have high lifetime earnings and will be on the flat part of the social security benefit function. This implies that the model social security system produces higher marginal tax rates over the life cycle on high productivity agents than on low

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<sup>21</sup>Tax rates are calculated using the social security tax rate, the slope of the benefit function, the interest rate and length of life. With income taxation, we also use computed income and marginal income tax rates. Calculating the marginal rate of substitution and wages from the model produces “noisy” versions of Figure 8. Clearly, introducing other features of the U.S. social security system (e.g. the spousal benefit or the fact that benefits are based on the 35 highest earnings years) would affect the patterns in Figure 8. The results in Figure 8 are similar to the marginal social security tax rates calculated by Feldstein and Samwick (1992, Table 1).

<sup>22</sup>We abstract from growth in average, economy-wide earnings and the indexing of individual earnings to economy-wide earnings.

productivity agents.<sup>23</sup> The model income tax system strengthens this pattern as in any period average and marginal income tax rates increase with period income.

### 4.3.2 Labor-Productivity Risk

We now try to better understand the maximum welfare gains in economies with labor-productivity risk. Solutions to the planning problem will involve some incentive compatibility constraint binding. As a consequence, at a solution it will not be true that all marginal rates of substitution are equated to marginal rates of transformation. To understand the sources of the maximum welfare gain it will be important to understand which marginal rates of substitution differ from marginal rates of transformation in the planning problem and by how much they differ.

In a solution to the planning problem the intertemporal marginal rate of substitution of consumption must equal the marginal rate of transformation  $(1 + r)$ . If not, then it is possible to deliver both the same expected utility and the same ex-post utilities at lower expected present value cost, without changing the labor allocation. This can be done by equating this marginal rate of substitution and transformation. The extra resources saved can then be used to make a uniform increase in utility to agents receiving all shocks while preserving incentive compatibility.<sup>24</sup>

Now consider the marginal rate of substitution between consumption and labor. This marginal rate of substitution will differ from an agent's wage rate in a solution to the planning problem depending on which incentive constraints bind. It turns out that only the local downward incentive constraints hold with equality in a solution. These constraints require that an agent with a given permanent shock weakly prefers his/her own allocation to the allocation received by pretending to have the next lowest shock. An important consequence of this (see Theorem A1 in the Appendix) is that the marginal rate of substitution between consumption and labor is then strictly below

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<sup>23</sup>Clearly, to the degree that high productivity agents live longer than low productivity agents, this counteracts the progressive marginal tax rates arising from the shape of the benefit function.

<sup>24</sup>Rogerson (1985) and Golosov et. al. (2003) present necessary conditions on this margin in planning problems with a more general structure of shocks. Their main result is the “inverse” Euler equation. The result stated in the text is a special case of their result as this Euler equation reduces to the claim made above, absent period-by-period shocks.

the wage rate  $w\omega(s, j)$  in all periods for all agents except the agent receiving the highest shock.<sup>25</sup> For the agent with the highest shock, there is no gain to distorting the consumption-labor margin at any age. The reason is that no other agent envies the consumption and output allocation of this agent. All other agents get strictly lower lifetime utility by pretending to be the high shock agent and, thus, allocating enough labor time to produce the higher output required.

Next, we examine the size of the wedges between the intratemporal marginal rate of substitution and transformation. Figure 9 graphs the ratio of the marginal rate of substitution to the agent's wage rate at each age for each of the five possible values of the permanent shock.<sup>26</sup> A ratio equal to one means that the wedge is zero. Figure 9a shows that in the planning problem this marginal rate of substitution is below an agent's wage for all agents but the agent with the highest permanent shock. Furthermore, within an age group the magnitude of this wedge decreases as an agent's wage increases.

In the context of the permanent-shock model, we are not aware of any existing theoretical results which describe how the wedge at each age moves as productivity increases. However, for the static Mirrlees model there are theoretical and computational results on this issue (see, for example, Tuomala (1990), Saez (2001) and the references cited in these papers). In the Mirrlees model the lognormal distribution of productivity is important for wedges to decline as productivity increases. We have computed the nature of wedges in the permanent-shock model when we replace the lognormal distribution with a Pareto distribution. The literature has argued that the upper tail of the earnings distribution has fat tails which are more in line with a Pareto distribution. For the Pareto distribution with the same mean and variance, we find that wedges do not decrease as productivity increases.<sup>27</sup>

The pattern of the wedges in the model social insurance system is quite different

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<sup>25</sup>A similar result holds in the one-period model studied by Mirrlees (1971).

<sup>26</sup>Recall from section 3 that the wage rate in the permanent-shock model is  $w\omega(s, j) = w\mu_j \exp(s^1)$  and that there are five equally-spaced shock values  $s_1^1 < s_2^1 < \dots < s_5^1$ .

<sup>27</sup>Following Tauchen (1986), we approximate a Pareto distribution with five equally-spaced points one standard deviation apart. The resulting wedge is positive and displays little variation across ages. The wedge for the lowest four shock levels averages approximately .12, .10, .16 and .20, in order of increasing productivity. The wedge for the highest productivity level is approximately zero.

from the pattern in the planning problem. First, the intertemporal marginal rate of substitution is below the gross interest rate because of income taxation, whereas in the planning problem this margin is not distorted. Second, the intratemporal marginal rate of substitution is below an agent's wage. Figure 9b shows that within age groups this wedge increases as an agent's wage increases. This is precisely the opposite of the pattern in the planning problem.

The wedge in Figure 9b is smallest for low productivity agents for two reasons. First, these agents have relatively low incomes and marginal income tax rates are relatively low at low income levels. This pattern is implied by the average federal tax rates in Figure 2. Second, these low productivity shock agents know that they will be on the steep part of the social security benefit function but face the same social security tax rate on earnings as do all agents who are below the maximum taxable earnings level. Thus, these agents face low marginal net tax rates on earnings coming from social security compared to agents who will be on the flatter portions of the social security benefit function. As earnings increase both the marginal income tax rate and the marginal net social security tax rate increase. This will hold until earnings hit the maximum taxable earnings under social security.

We conjecture that this difference in the nature of wedges in the model social insurance system and in the planning problem is a key reason why the maximum welfare gains increase as labor productivity risk increases.<sup>28</sup> Specifically, as risk increases from the no risk case, the intratemporal wedge in the planning problem tends to increase for all shock realizations but the highest. In contrast, this wedge in the model social insurance system tends to decrease for agents with low shock realizations.

## 5 Improving the Social Insurance System

We examine two ways to reform the model social insurance system. Reform 1 is a piecemeal reform in that a component of the social insurance system is changed

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<sup>28</sup>Another reason is that the pattern of lifetime taxation is wrong. As wage risk increases, the model social insurance system does not allocate enough consumption to low productivity agents and too much to high shock agents.

while maintaining the remainder of the system. In Reform 1 we change the social security benefit function without changing income taxation or the social security tax rate. Reform 2 is a radical reform as social security and income taxation are eliminated and are replaced with a tax on the present value of earnings.

Reform 1 and 2 are optimal parametric reforms. In each case we search over the parameters of the respective tax functions to find the parameter vector which maximizes ex-ante expected utility of the cohort of agents.<sup>29</sup> In each reform the same present value of resources is extracted from the cohort as in the original social insurance system. The Appendix describes computational methods. The Appendix is also useful for understanding how to achieve a tax on the present value of earnings using a period-by-period tax system. Intuitively, a given present-value tax can be achieved with very different timings of taxes and transfers over the lifetime.

## 5.1 Motivation

The policy literature is full of discussions of piecemeal reforms. In the social security literature, it is common to find the suggestion that the value of marginal social security benefits incurred by extra earnings should be more closely linked with marginal taxes paid in order to improve efficiency or a welfare measure. These considerations motivate the analysis of Reform 1 which is an optimal piecemeal reform that flexibly changes the benefit function.

The motivation for Reform 2 comes from our analysis of the solution to the planning problem with permanent shocks. We know the nature of wedges between various marginal rates of substitution and transformation both from general theoretical considerations and from computing solutions to the planning problem. Specifically, there is never an intertemporal wedge between the marginal rate of substitution of consumption intertemporally and the marginal rate of transforming consumption goods over

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<sup>29</sup>Our analysis of optimal parametric reforms is similar in some respects to the work of Conesa, Kitao and Krueger (2009). They choose the parameters of a labor income tax function and a linear capital income tax to maximize ex-ante lifetime utility in steady state. Our use of partial equilibrium analysis allows us to make direct comparisons to the Mirrleesian optimal taxation literature, which is not possible with a steady-state analysis.

time. However, there is a wedge between an agent's intratemporal marginal rate of substitution and an agent's wage. At a quantitative level, this wedge for a given agent is approximately age invariant but does differ strongly across agents with different permanent productivity realizations.

A tax on the present value of earnings does at least two important things in light of these properties. First, it imposes no intertemporal wedge. Second, it imposes an age-invariant wedge on the intratemporal margin that can be made to flexibly differ across agents. These two facts motivate the consideration of Reform 2.<sup>30</sup> Of course, there may be other ways of achieving this pattern of wedges involving consumption taxation.

## 5.2 Analysis

The welfare gain to each reform is given in Table 2. Welfare gains are stated in terms of the permanent percentage increase in consumption in the allocation in the model without the reform which is equivalent to the expected utility delivered under the optimal reform. Welfare gains are calculated for both the full model (i.e the model with permanent, persistent and temporary shocks) and the permanent-shock model. In both of these cases the model social insurance system captures both income taxation and social security.

We first discuss the results for the permanent-shock model. For Reform 1, we calculate the best constant benefit, the best linear benefit and the best quadratic benefit as a function of average lifetime earnings. These possibilities allow for quite different relationships between lifetime earnings and lifetime consumption on the one hand and for marginal earnings tax rates at different ages and states on the other hand. The best constant benefit function in the permanent-shock model leads to a welfare gain of 0.13 percent. A constant social security benefit increases the progressivity of lifetime earnings taxation but also increases marginal earnings taxes across earnings

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<sup>30</sup>Werning (2007) shows that a present-value tax system is optimal in some contexts. Specifically, he shows that such a tax implements a solution to a planning problem in the context of an infinitely-lived agent model where labor productivity takes on two possible values, labor productivity is private information and preferences are of the constant Frisch elasticity of labor form.

levels. The best linear benefit function has a positive intercept and a negative slope and leads to a welfare gain of 0.18 percent. The best quadratic benefit function that we find does not improve welfare over the best linear function.

Table 2: Welfare Gains to Optimal Parametric Reforms

Type of Reform	Permanent-Shock Model	Full Model
Reform 1: Change the Benefit Function	0.18	3.91
Reform 2: Present Value of Earnings Tax	3.95	4.35
Reform 3: Eliminate Capital Income Taxation	0.22	-0.22
Maximum Possible Gain	4.09	unknown

NOTE: The benefit function is  $b(x; \alpha) = \sum_{i=1}^3 \alpha_i x^{i-1}$ , where  $x$  is average lifetime earnings.

The present-value tax function  $T(pv; \alpha)$  is a class of step functions in the permanent-shock model.

In the full model  $T(pv; \alpha)$  is a class of piecewise-linear functions. See the Appendix.

For Reform 2, we find that an optimal present-value tax produces a larger welfare gain than an optimal piecemeal reform of the benefit function. The best present-value tax leads to a gain worth a 3.95 percent increase in consumption. We obtain this result when the class of tax functions are increasing step functions. We also examined this issue using quadratic and piecewise-linear tax functions and find slightly smaller welfare gains. We note that the best present-value tax achieves nearly all of the maximum possible welfare gain in the permanent-shock model.

We highlight two reasons why the optimal present-value earnings tax works well in the permanent-shock model. First, it allows for a flexible choice of lifetime taxation. Indeed, the graph of the present value of consumption as a function of the present value of earnings which turns out to be optimal is essentially the pattern in the planning problem - previously displayed in Figure 6. Second, the present-value tax is able to closely approximate the pattern of wedges between marginal rates of substitution and

transformation in the planning problem. Thus, it induces agents to choose an allocation not too far from the planning solution.<sup>31</sup>

We now discuss results for the full model. For Reform 1, the best constant, linear, and quadratic benefit functions lead to gains worth 3.33, 3.83 and 3.91 percent, respectively. A constant benefit level produces the bulk of the welfare gain produced by these reforms. The best quadratic has a positive intercept, but negative values for the coefficients on the slope and quadratic terms. Thus, the piecemeal reform that maximizes ex-ante welfare does not involve more closely linking the value of marginal benefits received to marginal taxes paid. We find that this holds within the full model as well as within the permanent-shock model.

For Reform 2 we find that in the full model an optimal present-value tax leads to a gain worth a 4.35 percent increase in consumption. We obtain this result when we examine a class of increasing, piecewise-linear functions.<sup>32</sup> Figure 10 displays the lifetime tax function which achieves the maximum ex-ante utility. Agents with low realized lifetime earnings receive a present-value transfer, whereas agents with high lifetime earnings pay a present-value tax. Lifetime tax rates are progressive in that the ratio of the present value of taxes paid to lifetime earnings typically increases as realized lifetime earnings increases. At a qualitative level, this pattern also holds for our analysis of present-value taxation within the permanent-shock model.

In Figure 10 we find that the slopes of the present-value-tax function start from a low of 0.24 for agents with the lowest earnings. These slopes tend to increase gradually to a slope of 0.37 for the agents with lifetime earnings equal to 47, which is within the top 1 percent of earners.<sup>33</sup> Beyond this level the slope is nearly zero. Unlike the

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<sup>31</sup>At a deeper level, a present-value tax may work well in these economies for two quite different reasons. First, one might conjecture that interior solutions to the planning problems with (i) constant Frisch elasticity of labor preferences (i.e.  $u(c_j, l_j) = u(c_j) + \phi \frac{l_j^{1+\gamma}}{1+\gamma}$ ) and (ii) permanent proportional productivity differences have the property that only local downward incentive constraints bind. If so, such allocations can always be implemented by a present-value tax system. A key property of such a solution, given assumptions (i)-(ii), is that the intratemporal wedge between marginal rates of substitution and transformation is age invariant - see the proof of Theorem A1(iii) in the Appendix. Second, the preferences used in Table 1 may effectively be quite close to those with constant Frisch elasticity of labor.

<sup>32</sup>The class of piecewise-linear functions we examine has 18 linear segments.

<sup>33</sup>The 10, 50 and 90th percentile of the lifetime earnings distribution occur at earnings levels equal to 11.4,

analysis of the permanent-shocks model, the slopes of the lifetime tax function do not immediately imply the nature of wedges between the intratemporal marginal rate of substitution and an agent's wage. However, towards the end of the working lifetime, the relationship between wedges and slopes of the present-value tax function tightens as an agent better understands where he or she will finally end up on this tax function at the end of the working lifetime.

Table 2 also presents results for Reform 3. Reform 3 is a piecemeal reform that maintains social security and income taxation but exempts capital income from entering into taxable income. An additional proportional labor income tax is added to satisfy the present-value resource constraint. Reform 3 is analyzed as a way to determine if an important part of the welfare gain obtained by lifetime earnings taxation comes simply from eliminating capital income taxation and the associated intertemporal wedge. Table 2 shows that eliminating capital income taxation in this way produces a welfare gain of 0.22 percent in the permanent-shock model and a welfare loss of  $-0.22$  percent in the full model. Thus, simply eliminating intertemporal wedges in this crude way, without substantially increasing the progressivity of lifetime taxation or altering the pattern of intratemporal wedges, does not go very far towards producing the welfare gains achieved by lifetime earnings taxation in either model.

In closing this section, we think that finding parametric tax systems that work well within the full model is a useful applied problem. This problem connects the policy literature to the literature on optimal taxation. We doubt that the tax systems that we have explored come close to exhausting the maximum possible welfare gains.<sup>34</sup> Further theoretical and computational work will likely be key for finding parametric tax systems that may allow for larger welfare gains. Our view is that there is currently little knowledge at hand that seems to be immediately useful for a wise choice of a parametric tax system to investigate, even for this simple problem.<sup>35</sup>

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17.5 and 26.8.

<sup>34</sup>Reform 1 puts a wedge on the intertemporal consumption margin after retirement, whereas Reform 2 violates the inverse Euler equation before retirement. Thus, neither system can exhaust the maximum possible welfare gains.

<sup>35</sup>One might ask whether results from the implementation literature, e.g. Kocherlakota (2005), are useful for the exercises carried out in Table 2. While such results are valuable in a number of regards, they are not

## 6 Conclusion

The question of whether to or how to fundamentally redesign social security systems has been and continues to be a major policy issue in the U.S. and in many other countries. One's position on this issue is likely to depend upon one's view of the rationale for social security and for social insurance more broadly. One standard rationale is the provision of insurance for risks that are not easily insured in private markets. For this reason, it would seem to be important to develop a quantitative understanding of how well social insurance systems serve such an insurance role.

This paper examines social insurance systems within an incomplete-markets model with idiosyncratic sources of labor productivity variation. When there are only permanent differences in productivity realized by the start of the working lifetime, the maximum welfare gain to improved insurance is equivalent to a 4.1 percent increase in consumption for a cohort. The model of the U.S. social insurance system has too little progressivity in lifetime earnings taxation compared to the pattern in the solution to the planning problem. The model places higher marginal earnings taxes on high productivity agents than on low productivity agents, whereas the pattern in the relevant wedge in the planning problem is the opposite. Additionally, the model has positive marginal tax rates on capital income, whereas the solution to the planning problem has no intertemporal wedge.

We examine two reforms of the model social insurance system. We find that changing the social security benefit function optimally, without changing the rest of the social insurance system, leads to only a small welfare gain in the permanent-shock model. In contrast, a system which eliminates social security and income taxation and replaces it with an optimal tax on lifetime earnings leads to a large welfare gain not far from the maximum possible welfare gain. The analysis of the same two reforms within the full model finds large gains to both types of reforms. An optimal lifetime earnings tax leads to larger gains than an optimal change in the social security benefit function.

We mention three directions to pursue in future work. First, it would be valuable to know quantitative properties of the solution to the planning problem within the full

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designed to provide parametric tax functions for the type of applied exercise conducted in this section.

model. This would require important theoretical and/or computational advances.<sup>36</sup> The structure of this paper reflects the difficulties of applying contract theory to leading positive models of inequality. Second, this paper treats labor productivity as being unaffected by the social insurance system. We expect that human capital models (e.g. Huggett, Ventura and Yaron (2007)) will be central both as positive models of inequality and as models for the analysis of social insurance issues. Because skill acquisition responds to policy in human capital models, labor productivity will not be policy invariant. Whether the gains to adopting superior systems are even larger within such models is an open question. Third, future work might expand the analysis of the social insurance system to go beyond income taxation and social security.

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<sup>36</sup>Fernandes and Phelan (2000) provide a recursive formulation of a planning problem with persistent shocks. Such a formulation is not computationally viable for the full model described in Table 1.

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# A Appendix

The Appendix contains two sections. Section A.1 describes our methods for computing solutions to the planning problem, the social insurance problem and the parametric planning problems. Section A.2 proves Theorem A1. In the Appendix the labor-productivity function is sometimes set to  $\omega(s_j, j) = s_j$  solely to shorten and simplify expressions. FORTRAN programs that compute solutions to all the problems analyzed in the paper are available upon request.

## A.1 Computation

### A.1.1 Social Insurance Problem

The social insurance problem is stated below as a dynamic programming problem. This involves reformulating the present value budget constraint as a sequence of budget constraints where resources are transferred across periods with a risk-free asset. Risk-free asset holding must then always lie above period and shock specific borrowing limits:  $\underline{a}_j(s)$ .<sup>37</sup> The state variable is  $x = (a, s, z)$ , where  $a$  is asset holdings,  $s$  is the period shock vector determining productivity and  $z$  is average past earnings. The functions  $T_j$  and  $F_j$  describe the tax system and the law of motion for average past earnings. The shock is Markovian with transition probability  $\pi(s'|s)$ .

$$\begin{aligned}
 V_j(a, s, z) &= \max_{(c, l, a')} u(c, l) + \beta \sum_{s'} V_{j+1}(a', s', z') \pi(s'|s) \\
 (1) \quad &c + a' \leq a(1+r) + w\omega(s, j)l - T_j(x, w\omega(s, j)l) \\
 (2) \quad &c \geq 0, a' \geq \underline{a}_j(s); l \in [0, 1] \\
 (3) \quad &z' = F_j(z, w\omega(s, j)l)
 \end{aligned}$$

This problem is solved computationally by backwards induction. The value function  $V_j$  is computed at selected grid points  $(a, s, z)$  by solving the right-hand-side of Bellman's equation. We use the simplex method (see Press et al (1994)). Evaluating the right-hand-side of Bellman's equation involves a bi-linear interpolation of the function  $V_{j+1}(a', s', z')$  over the asset and average past earnings dimensions:  $(a', z')$ . We set the borrowing limit to a fixed value  $\underline{a}$  in each period. We then relax this value so that it is not binding. This is a device for imposing period and state specific limits  $\underline{a}_j(s)$ . To use this device, penalties are imposed for states and decisions implying negative consumption.<sup>38</sup>

We compute ex-ante, expected utility  $V^{us}$  and the expected cost, denoted  $Cost$ , of running the social insurance system by simulation, under the assumption that an agent starts out with no assets. Specifically, we draw a large number (100,000) of lifetime labor-productivity profiles, compute realized utility and realized cost for each profile, using the computed optimal decision rules, and then compute averages. The same 100,000 histories are used in the calculation of expected utility and expected cost in the analysis of reforms.

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<sup>37</sup>These limits are the maximum present value of labor earnings plus social security benefits in the worst labor-productivity history. This assumes that one can borrow against future social security benefits. This is implicitly assumed in the benchmark model.

<sup>38</sup>The backward induction procedure takes as given a value for average earnings in the economy. This value is used to determine the tax function  $T_j$ . Thus, an additional loop is needed so that guessed and implied values of average earnings coincide.

### A.1.2 Steady State Calibration

We calibrate the discount factor  $\beta$  using the algorithm below. This algorithm is based on computing a stationary equilibrium. To set up this framework, we assume that (i) there is an aggregate production function  $Y = F(K, L) = K^\alpha L^{1-\alpha}$  stated in terms of aggregate capital  $K$  and labor  $L$ , (ii) physical capital depreciates at rate  $\delta$  and (iii) population growth is  $n$ .

We define an equilibrium using the recursive language - see Huggett (1996). To keep track of agent heterogeneity, we use probability measures  $\psi_j$  to describe the fraction of age  $j$  agents that have a state vector  $x = (a, s, z)$  lying in particular subsets of the state space  $X$ . The relative size of different age cohorts is given by  $\phi_j$ , where  $\phi_{j+1} = \phi_j/(1+n)$  and  $\sum_j \phi_j = 1$ . Denote aggregate capital, labor and government spending and consumption  $(K, L, G, C)$ :  $K \equiv \sum_j \phi_j \int a d\psi_j$ ,  $L \equiv \sum_j \phi_j \int \omega(s, j) l(x, j) d\psi_j$  and  $C \equiv \sum_j \phi_j \int c(x, j) d\psi_j$ . The probability measures must be consistent with one another. This is captured by the recursion  $\psi_{j+1} = \Gamma_j(\psi_j)$ , where  $\Gamma_j(\psi_j)(\cdot) \equiv \int P(x, j, \cdot) d\psi_j$ , and  $P$  is a transition function induced by the transition probabilities on shocks and by the period  $j$  decision rules. We do not write down all the details associated with the construction of this transition function partly because the algorithm below calculates the relevant integrals by simulating a large number of histories rather than by calculating probability measures on a rich collection of subsets of the state space and then integrating. However, details of how to do so are in Huggett (1996).

*Definition: A stationary equilibrium is  $(c(x, j), l(x, j), a(x, j), w, r, G)$ , tax-transfer functions  $(T_1, \dots, T_j)$  and probability measures  $(\psi_1, \dots, \psi_j)$  such that*

1.  $(c, l, a)$  solve Bellman's equation (Appendix A.1.1), given  $(w, r)$  and  $T_j$ .
2.  $w = F_2(K, L)$  and  $r = F_1(K, L) - \delta$
3.  $\psi_{j+1} = \Gamma_j(\psi_j), \forall j$
4.  $G = \sum_j \phi_j \int T_j(x, w\omega(s, j)l(x, j)) d\psi_j$
5.  $C + K(n + \delta) + G = F(K, L)$

Algorithm:

1. Fix  $(\alpha, \delta, n) = (.33, .06, .01)$ .
2. Set  $r = .042$  and  $w = 1.19461$ . Given  $(r, \alpha, \delta)$ , equilibrium condition 2 pins down the wage  $w$  at the value stated and pins down the capital-labor ratio  $K/L$ .
3. Guess the discount factor and average earnings  $(\beta, \bar{e})$ .
4. Compute decision rules  $(c, l, a)$  solving Bellman's equation, given the information in steps 1-3 using the procedures described in Appendix A.1.1.
5. Calculate implied values of aggregates  $(K', L', \bar{e}', \sum_j \phi_j \int T_j(x, w\omega(s, j)l(x, j)) d\psi_j)$  via simulation using the decision rules.
6. If  $K'/L' = K/L$ ,  $\bar{e}' = \bar{e}$  and  $\sum_j \phi_j \int T_j(x, w\omega(s, j)l(x, j)) d\psi_j > 0$ , then stop. Otherwise, update  $(\beta, \bar{e})$  and repeat steps 4-5.

Comments:

1. We compute  $\beta$  for the full model at the parameters listed in Table 1 and fix this value for all subsequent analysis.

2. The initial value of  $\beta$  in step 3 is set to  $\beta = 1/(1+r)$ . In carrying out this algorithm we first adjust average earnings  $\bar{e}$  in steps 3-6 until  $\bar{e}' = \bar{e}$ . The value of  $\beta$  is increased until step 6 approximately holds. We choose  $\bar{e}$  in step 3 because the tax-transfer function is only specified once  $\bar{e}$  is known - see section 2.4.1.

### A.1.3 Planning Problem

We show how to compute  $V^{pp}$  for the case of permanent shocks, given the value of  $Cost$ . The strategy is to analyze the Relaxed Problem. The Relaxed Problem is the same as the planning problem with permanent shocks except that only the local downward incentive constraints are imposed rather than all the incentive constraints. The local downward incentive constraints are the constraints stating that truth telling from shock  $s$  dominates claiming to be one shock lower, denoted  $s^-$ . There are  $N$  shock values that are ordered  $s_1 < s_2 < \dots < s_N$ . Below, we let  $\omega(s_j, j) = s_j$  solely to shorten and simplify expressions.

Relaxed Problem:  $\max_{(l_j(s), c_j(s))} \sum_s [\sum_j \beta^{j-1} (u(c_j(s)) + v(l_j(s)))] P(s)$   
subject to

$$\begin{aligned} \text{(i)} \quad & \sum_s [\sum_j (c_j(s) - w l_j(s) s) / (1+r)^{j-1}] P(s) \leq Cost \\ \text{(ii)} \quad & \sum_j \beta^{j-1} (u(c_j(s)) + v(l_j(s))) \geq \sum_j \beta^{j-1} (u(c_j(s^-)) + v(l_j(s^-) s^- / s)), \forall s > s_1 \end{aligned}$$

The strategy is to compute solutions to the Relaxed Problem and to verify that at the computed solution all incentive constraints hold. We compute solutions to the Relaxed Problem by solving the Equivalent Problem below. The Equivalent Problem is useful as it reduces the dimension of the control variables. The claimed equivalence follows from several facts about solutions to the Relaxed Problem. Specifically, at a solution (i) the cost constraint must hold with equality, (ii) consumption is chosen without intertemporal distortion (i.e.  $u'(c_j(s)) = \beta(1+r)u'(c_{j+1}(s)), \forall j, s$ ) and (iii) all local downward incentive constraints bind. As the first result is straightforward, we only formally state the last two in Theorem A1. Theorem A1 also provides an additional theoretical insight. Specifically, since the Lagrange multipliers on the incentive constraints are strictly positive, the Kuhn-Tucker conditions imply that at a solution the intratemporal marginal rate of substitution is strictly below labor productivity for all agents at any age except for the agent with the highest productivity shock. This is a generalization of a standard result for the one-period Mirrlees problem.

*Theorem A1: Assume  $u(c, l) = u(c) + v(l)$ ,  $u$  and  $v$  are concave and differentiable,  $u$  and  $v$  are strictly increasing and decreasing respectively. At an interior solution to the Relaxed Problem the following hold:*

- (i) all local downward incentive constraints bind,
- (ii)  $\frac{u'(c_j(s))}{\beta u'(c_{j+1}(s))} = 1 + r, \forall j, \forall s$

$$(iii) \quad -\frac{v'(l_j(s))}{u'(c_j(s))} < ws, \forall j, \forall s < s_N \quad \text{and} \quad -\frac{v'(l_j(s))}{u'(c_j(s))} = ws, \forall j \quad \text{and for } s = s_N.$$

Proof: See Appendix A.2.

In the Equivalent Problem the choice variables are labor and the lifetime utility of consumption  $u(s)$ . The cost constraint makes use of the function  $COST$ .  $COST(u)$  describes the resource cost of obtaining lifetime utility  $u$  from consumption, given that  $u'(c_j(s)) = \beta(1+r)u'(c_{j+1}(s))$ .<sup>39</sup> As all constraints are equality constraints, it is also possible to reduce dimensionality further by solving these constraints to express lifetime utility of consumption  $u(s)$  as a function of all labor profiles  $l$  and  $Cost$  as follows:  $u(s) = g(l, s, Cost)$ .

We use the simplex method from Press et al (1994) to solve the Equivalent Problem. This involves maximizing over  $(l_1(s), \dots, l_{R-1}(s))$ . These choices lie in an  $(R-1) \times N$  dimensional space as there are  $R-1$  labor periods and  $N$  possible permanent shocks.

$$\text{Equivalent Problem: } \max \sum_s [u(s) + \sum_j \beta^{j-1} v(l_j(s))] P(s)$$

subject to

- (i)  $\sum_s [COST(u(s)) - \sum_j w s l_j(s) / (1+r)^{j-1}] P(s) = Cost$
- (ii)  $u(s) + \sum_j \beta^{j-1} v(l_j(s)) = u(s^-) + \sum_j \beta^{j-1} v(l_j(s^-) s^- / s), \forall s > s_1$

#### A.1.4 Optimal Parametric Planning Problems

We examine a number of parametric tax systems. For any parametric tax system we choose the parameters of these tax systems to maximize ex-ante utility, given that agents behave optimally and that the present value resource constraint cannot be violated. We describe how we compute the optimal parametric tax system for the case of a tax on the present value of earnings. The computation of other optimal parametric tax systems is similar.

The agent's problem and the planner's problem are described below. The agent's state variable is  $x = (a, s, pv)$ , where  $pv$  is the present value of earnings earned from previous periods. The tax function  $T_j$  maps the present value of earnings from previous periods and earnings in period  $j$  into the tax paid or transfer received in period  $j$ .  $T_j$  depends upon a parameter vector  $\alpha$ . Solutions to the agent's problem are computed using the methods from section A.1.1.

$$V_j(a, s, pv; \alpha) = \max_{(c, l, a')} u(c, l) + \beta \sum_{s'} V_{j+1}(a', s', pv'; \alpha) \pi(s' | s)$$

$$(1) \quad c + a' \leq a(1+r) + w\omega(s, j)l - T_j(pv, w\omega(s, j)l; \alpha)$$

$$(2) \quad c \geq 0, a' \geq \underline{a}_j(s); l \in [0, 1]$$

$$(3) \quad pv' = pv + \frac{w\omega(s, j)l}{(1+r)^{j-1}}$$

$$\text{Parametric Planning Problem: } \max_{\alpha} E[V_1(0, s, 0; \alpha)]$$

subject to

$$E\left[\sum_j \frac{c_j(s^j; \alpha) - w\omega(s_j, j)l_j(s^j; \alpha)}{(1+r)^{j-1}}\right] \leq Cost$$

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<sup>39</sup>When  $u(c) = c^{1-\rho}/(1-\rho)$  and  $\rho \neq 1$ , then  $COST(u) = (\sum_j a^{j-1})[\frac{(1-\rho)u}{\sum_j b^{j-1}}]^{1/(1-\rho)}$ , where  $a = \frac{[\beta(1+r)]^{1/\rho}}{1+r}$  and  $b = \beta[\beta(1+r)]^{(1-\rho)/\rho}$ .

In the planner's problem the only constraint facing the planner, given agent's choices to any tax system are optimal, is the cost constraint. This is because the allocation induced by a solution to the agent's problem is incentive compatible. We compute solutions to the planner's problem by (i) drawing  $\alpha$ , (ii) computing optimal decision rules solving the agent's problem, given  $\alpha$  and (iii) simulating these decision rules to determine whether or not the resource constraint is violated at the allocation induced by  $\alpha$ . We use the simplex method to search over the space of parameters describing the tax function to maximize the objective function. The objective function is ex-ante utility less a penalty term when the cost constraint is violated.

We now describe how we choose the tax function  $T_j$  in the agent's problem. Start with a tax function  $T(pv; \alpha)$  mapping the present value of realized earnings over the lifetime into the present value of taxes paid over the lifetime. Define the period tax function  $T_j(pv, w\omega(s, j)l; \alpha)$  as indicated below. The tax paid in period  $j$  is based on the increment added to the present value of earnings. By the end of the working lifetime, the present value of taxes paid is simply  $T(pv; \alpha)$ , where  $pv$  is the realized present value of earnings over the working lifetime. This is one way to carry out a present-value tax  $T$  with a period-by-period tax system  $T_j$  for  $j = 1, \dots, J$ .<sup>40</sup>

$$T_j(pv, w\omega(s, j)l; \alpha) = \begin{cases} [T(pv + \frac{w\omega(s, j)l}{(1+r)^{j-1}}; \alpha) - T(pv; \alpha)](1+r)^{j-1} & : j \geq 2 \\ T(w\omega(s, j)l; \alpha) & : j = 1 \end{cases}$$

In our numerical implementation, we focus on two classes of parametric functions  $T$ . We use the class of piecewise-linear functions for the full model and the class of increasing step functions  $T$  for the permanent-shocks model. We choose as many steps as there are permanent shocks.

## A.2 Theorem A1

*Theorem A1: Assume  $u(c, l) = u(c) + v(l)$ ,  $u$  is strictly concave and differentiable,  $v$  is concave and differentiable,  $u$  and  $v$  are strictly increasing and decreasing respectively. At an interior solution to the Relaxed Problem the following hold:*

- (i) all local downward incentive constraints bind,
- (ii)  $\frac{u'(c_j(s))}{\beta u'(c_{j+1}(s))} = 1 + r, \forall j, \forall s$
- (iii)  $-\frac{v'(l_j(s))}{u'(c_j(s))} < ws, \forall j, \forall s < s_N$  and  $-\frac{v'(l_j(s))}{u'(c_j(s))} = ws, \forall j$  and for  $s = s_N$ .

Proof:

(i) We study the Lagrange function below. Let  $\gamma(s)$  denote multipliers on the local downward incentive constraints and  $\lambda$  denote the multiplier on the resource constraint. A superscript + or - denotes one higher or lower shock, respectively.

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<sup>40</sup>Vickery (1939) discusses some mechanics for a period-by-period tax system where taxes paid are based upon an average of past years incomes.

$$L = \sum_s [\sum_j \beta^{j-1} (u(c_j(s)) + v(l_j(s)))] P(s) + \lambda [Cost - \sum_s [\sum_j (c_j(s) - w l_j(s)) / (1+r)^{j-1}] P(s)] + \sum_{s > s_1} \gamma(s) \sum_j \beta^{j-1} [u(c_j(s)) + v(l_j(s)) - u(c_j(s^-)) - v(l_j(s^-)) s^- / s]$$

At an interior solution the Kuhn-Tucker conditions  $dL/dc_j(s) = 0$  and  $dL/dl_j(s) = 0$  hold:

$$\frac{dL}{dc_j(s)} = \begin{cases} \beta^{j-1} u'(c_j(s)) [P(s) - \gamma(s^+)] - \lambda P(s) / (1+r)^{j-1} & s = s_1 \\ \beta^{j-1} u'(c_j(s)) [P(s) - \gamma(s^+) + \gamma(s)] - \lambda P(s) / (1+r)^{j-1} & s_1 < s < s_N \\ \beta^{j-1} u'(c_j(s)) [P(s) + \gamma(s)] - \lambda P(s) / (1+r)^{j-1} & s = s_N \end{cases} \quad (1)$$

$$\frac{dL}{dl_j(s)} = \begin{cases} \beta^{j-1} v'(l_j(s)) \left[ P(s) - \gamma(s^+) \frac{v'(l_j(s)s/s^+)s/s^+}{v'(l_j(s))} \right] + \frac{\lambda w s P(s)}{(1+r)^{j-1}} & s = s_1 \\ \beta^{j-1} v'(l_j(s)) \left[ P(s) - \gamma(s^+) \frac{v'(l_j(s)s/s^+)s/s^+}{v'(l_j(s))} + \gamma(s) \right] + \frac{\lambda w s P(s)}{(1+r)^{j-1}} & s_1 < s < s_N \\ \beta^{j-1} v'(l_j(s)) [P(s) + \gamma(s)] + \frac{\lambda w s P(s)}{(1+r)^{j-1}} & s = s_N \end{cases} \quad (2)$$

Claims 1-4 establish that in a solution to the Kuhn-Tucker conditions all multipliers on incentive constraints are strictly positive:  $\gamma(s) > 0, \forall s$ . Theorem A1(i) follows from this result.

Claim 1: For  $N \geq 2, \gamma(s_N) > 0$ .

Claim 2: For  $N > 2, \gamma(s^-) = \gamma(s) = 0$  for any  $s$  is impossible.

Claim 3: For  $N > 2, \gamma(s^-) > 0, \gamma(s) = 0$  for any  $s$  is impossible.

Claim 4: For  $N > 2, \gamma(s_2) = 0, \gamma(s_3) > 0$  is impossible.

Proof of Claim 1: If  $\gamma(s_N) = 0$ , then  $dL/dc_j(s) = 0$  and  $u$  strictly concave implies  $c_j(s_N) \leq c_j(s_{N-1}), \forall j$ . If  $\gamma(s_N) = 0$ , then  $dL/dl_j(s) = 0$  and  $v$  concave implies  $l_j(s_N) > l_j(s_{N-1}), \forall j$ . Thus, the downward incentive constraint for the agent with shock  $s_N$  is violated.

Proof of Claim 2: Suppose  $\gamma(s^-) = \gamma(s) = 0$  for some  $s$ . Let  $s$  be the greatest  $s$  such that this holds. Claim 1 implies that  $\gamma(s^+) > 0$ . Then  $dL/dc_j(s) = 0$  and  $u$  concave implies  $c_j(s^-) > c_j(s), \forall j$ .  $dL/dl_j(s) = 0$  and  $v$  concave implies  $l_j(s^-) < l_j(s), \forall j$ . Thus, the downward incentive constraint for the agent with shock  $s$  is violated.

Proof of Claim 3: Suppose  $\gamma(s^-) > 0, \gamma(s) = 0$  for some  $s$ . Let  $s$  be greatest  $s$  such that this holds. Claim 1 and 2 imply  $\gamma(s^+) > 0$ . Then  $dL/dc_j(s) = 0$  and  $u$  concave implies  $c_j(s^-) > c_j(s), \forall j$ .  $dL/dl_j(s) = 0$  and  $v$  concave implies  $l_j(s^-) < l_j(s), \forall j$ . Thus, the downward incentive constraint for the agent with shock  $s$  is violated.

Proof of Claim 4: Suppose  $\gamma(s_2) = 0, \gamma(s_3) > 0$ . Then  $dL/dc_j(s) = 0$  and  $u$  concave implies  $c_j(s_1) > c_j(s_2), \forall j$ .  $dL/dl_j(s) = 0$  and  $v$  concave implies  $l_j(s_1) < l_j(s_2), \forall j$ . This violates the downward incentive constraint for the agent with shock  $s_2$ .

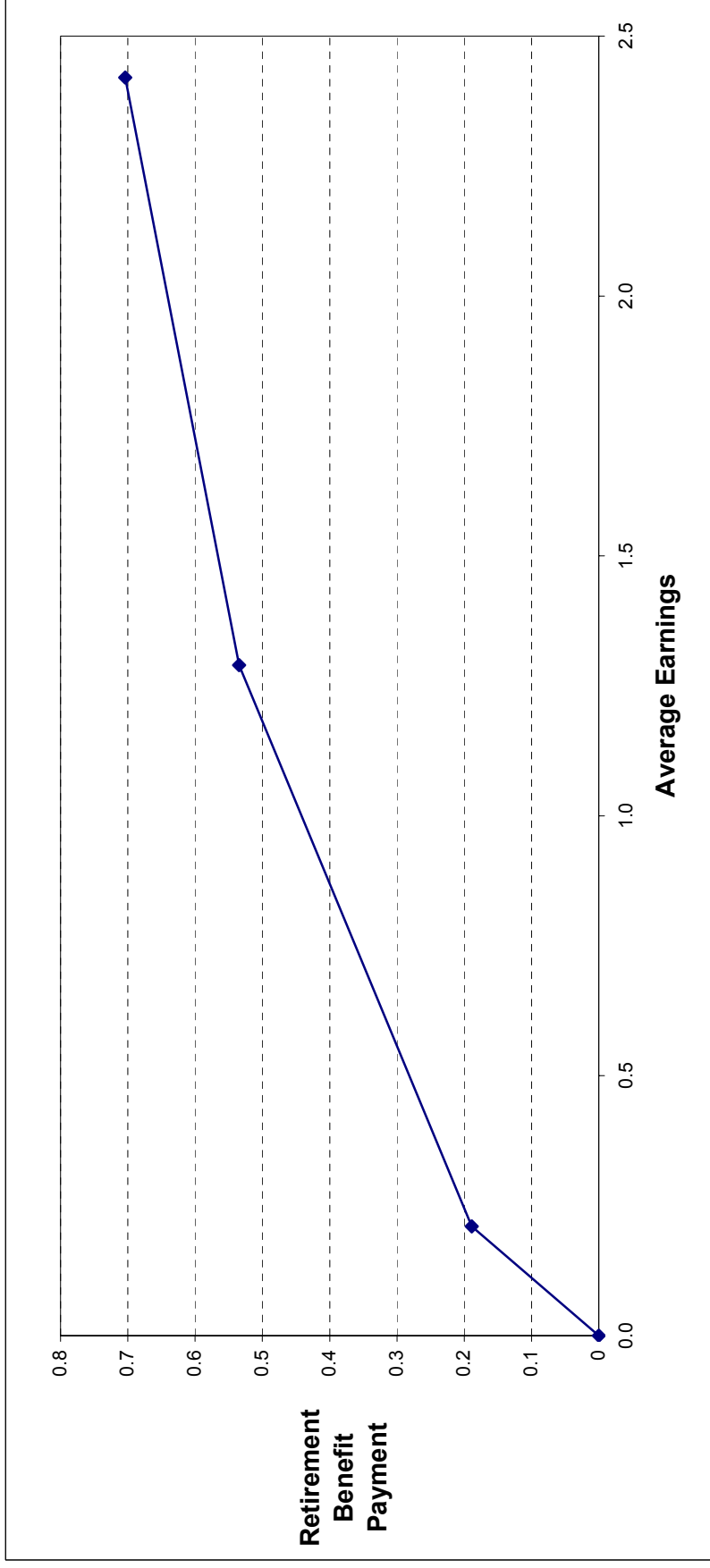
(ii) This is implied by  $dL/dc_j(s) = 0, \forall j$ .

(iii)  $dL/dl_j(s) = 0$  and  $dL/dc_j(s) = 0$  imply the line below. The result then follows from the fact that  $\gamma(s) > 0$  (Theorem A1(i)) and from the concavity of  $v$ . The result for the case  $s = s_N$  is obvious.

$$-\frac{v'(l_j(s))}{u'(c_j(s))} = \begin{cases} ws \frac{[P(s) + \gamma(s) - \gamma(s^+)]}{\left[ P(s) + \gamma(s) - \gamma(s^+) \frac{v'(l_j(s) \frac{s}{s^+}) \frac{s}{s^+}}{v'(l_j(s))} \right]} & : s_1 < s < s_N \\ ws \frac{[P(s) - \gamma(s^+)]}{\left[ P(s) - \gamma(s^+) \frac{v'(l_j(s) \frac{s}{s^+}) \frac{s}{s^+}}{v'(l_j(s))} \right]} & : s = s_1 \end{cases}$$

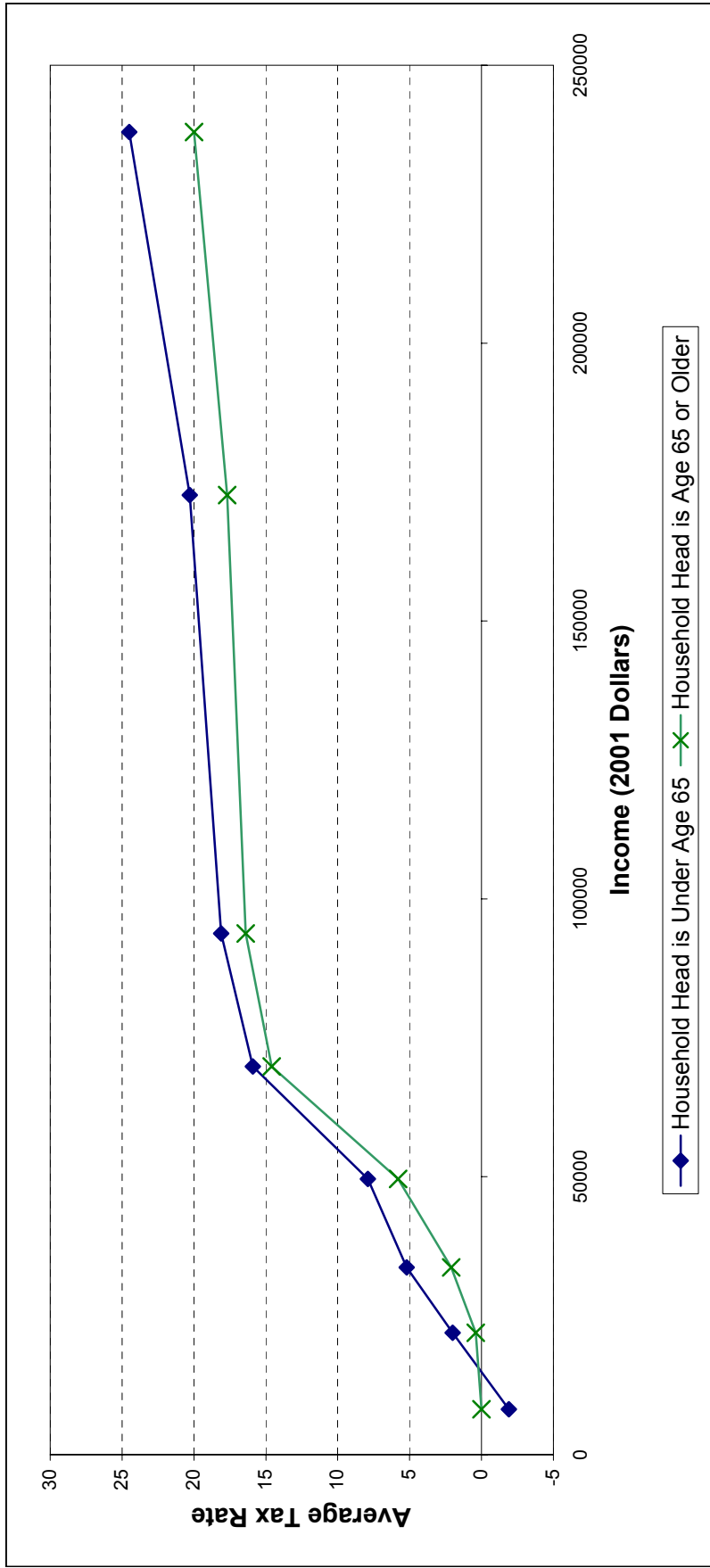
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**Figure 1: US Social Security Benefit Formula**



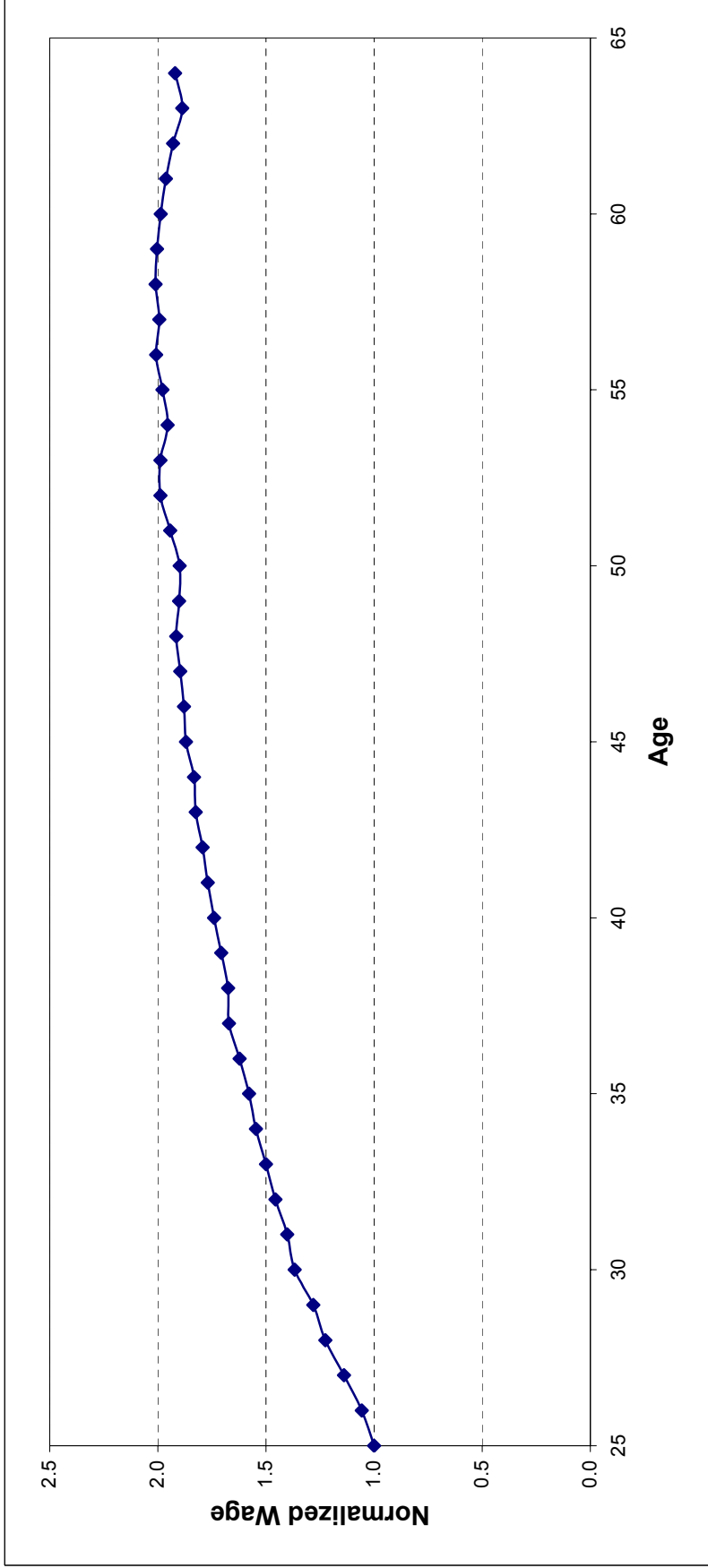
Source: Social Security Handbook (2003).  
Average earnings and benefit payments are both expressed as a multiple of average economy wide earnings.

**Figure 2: Average Federal Income Tax Rates**



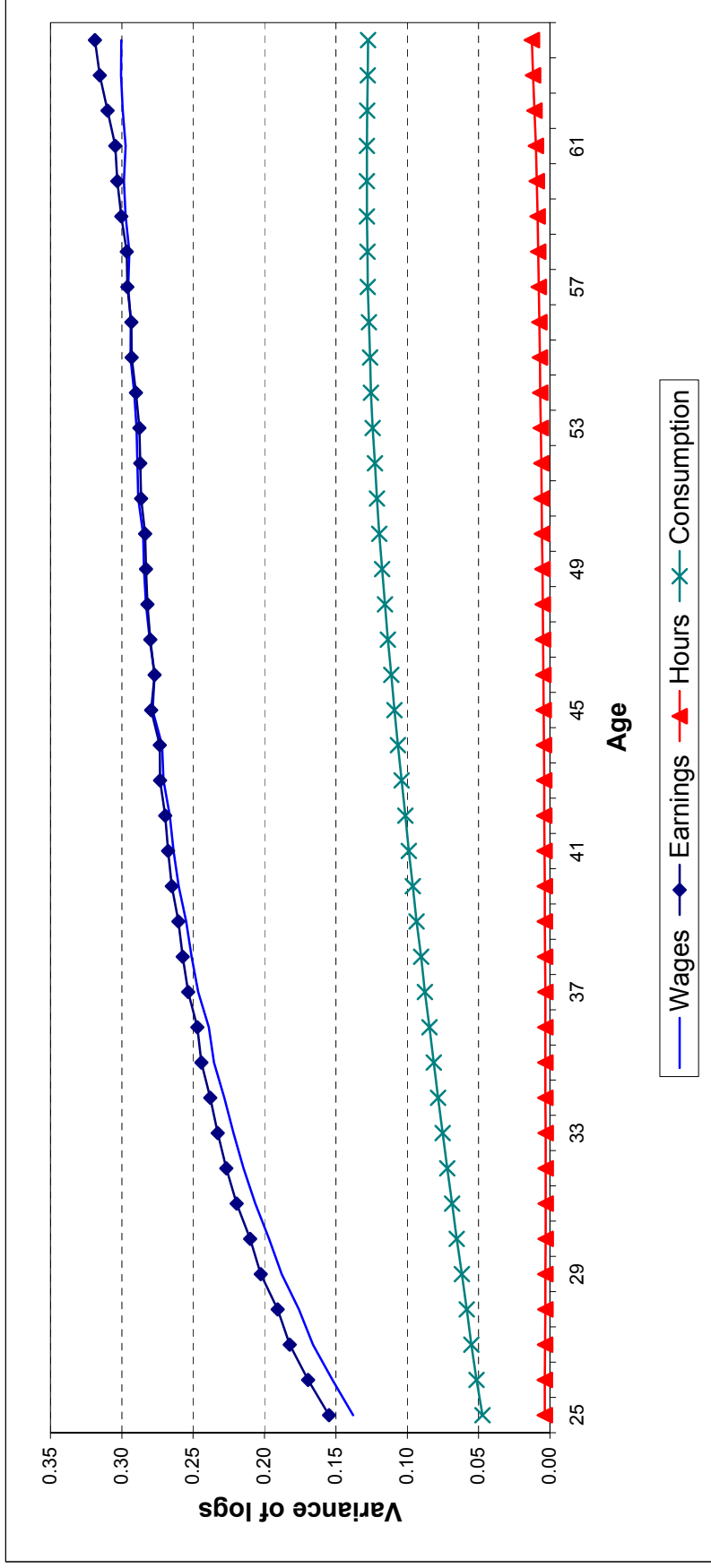
Source: Congressional Budget Office (2004)

Figure 3: US Wage Profile

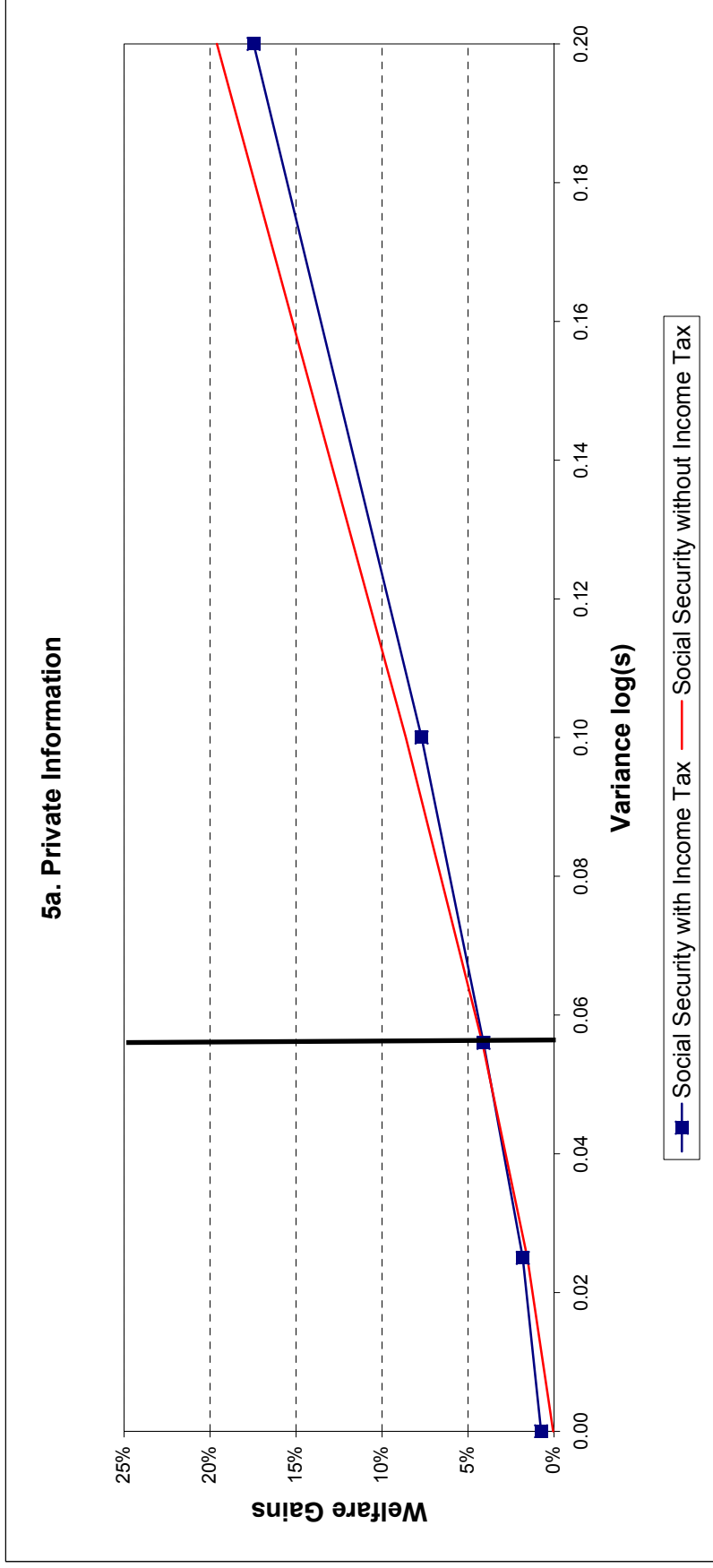


Source: Kaplan (2007).

Figure 4: Inequality Over the Life Cycle - Full Model

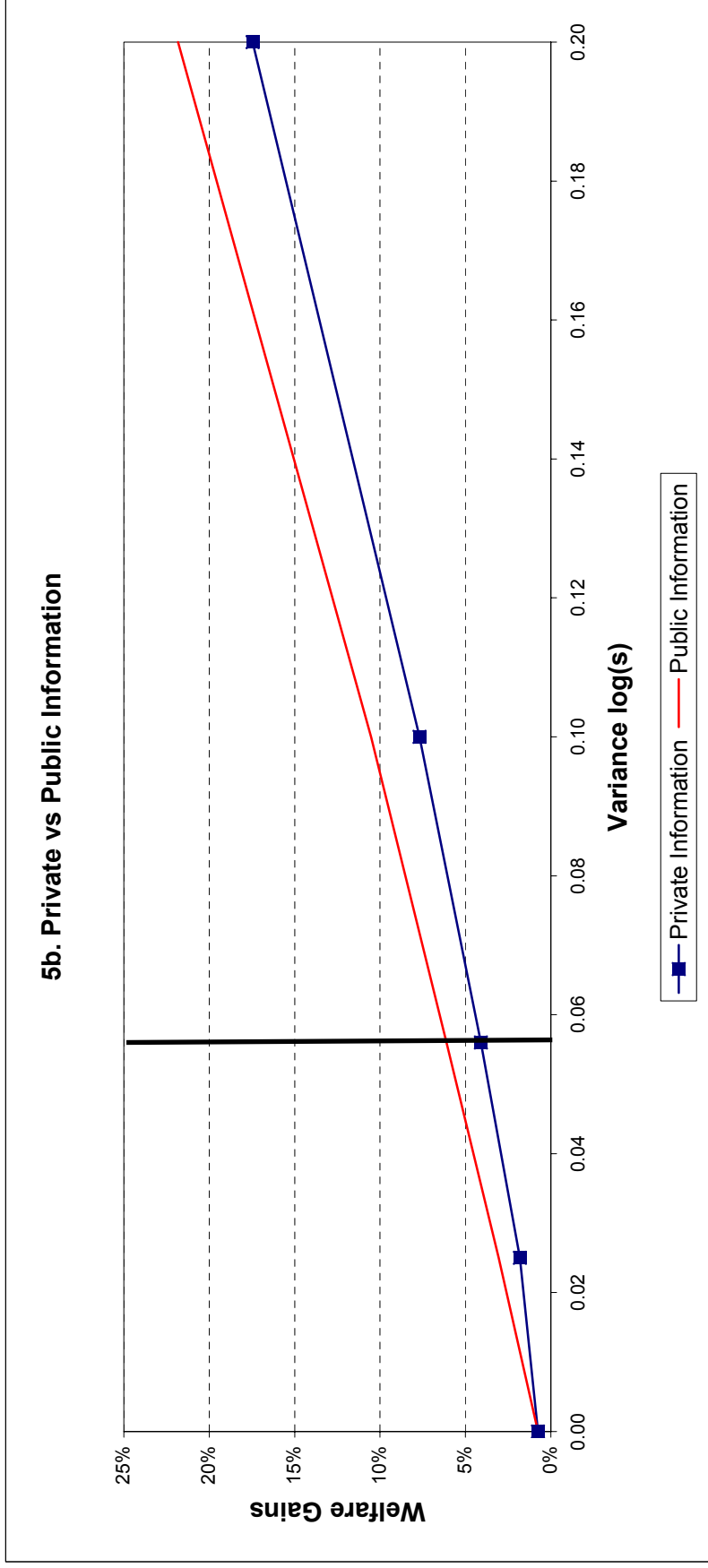


**Figure 5: Maximum Welfare Gains**



The bold vertical line in Figure 5 highlights the location of the point estimate of the variance described in the text.

**Figure 5: Maximum Welfare Gains**



The bold vertical line in Figure 5 highlights the location of the point estimate of the variance described in the text.

Figure 6: Lifetime Taxation

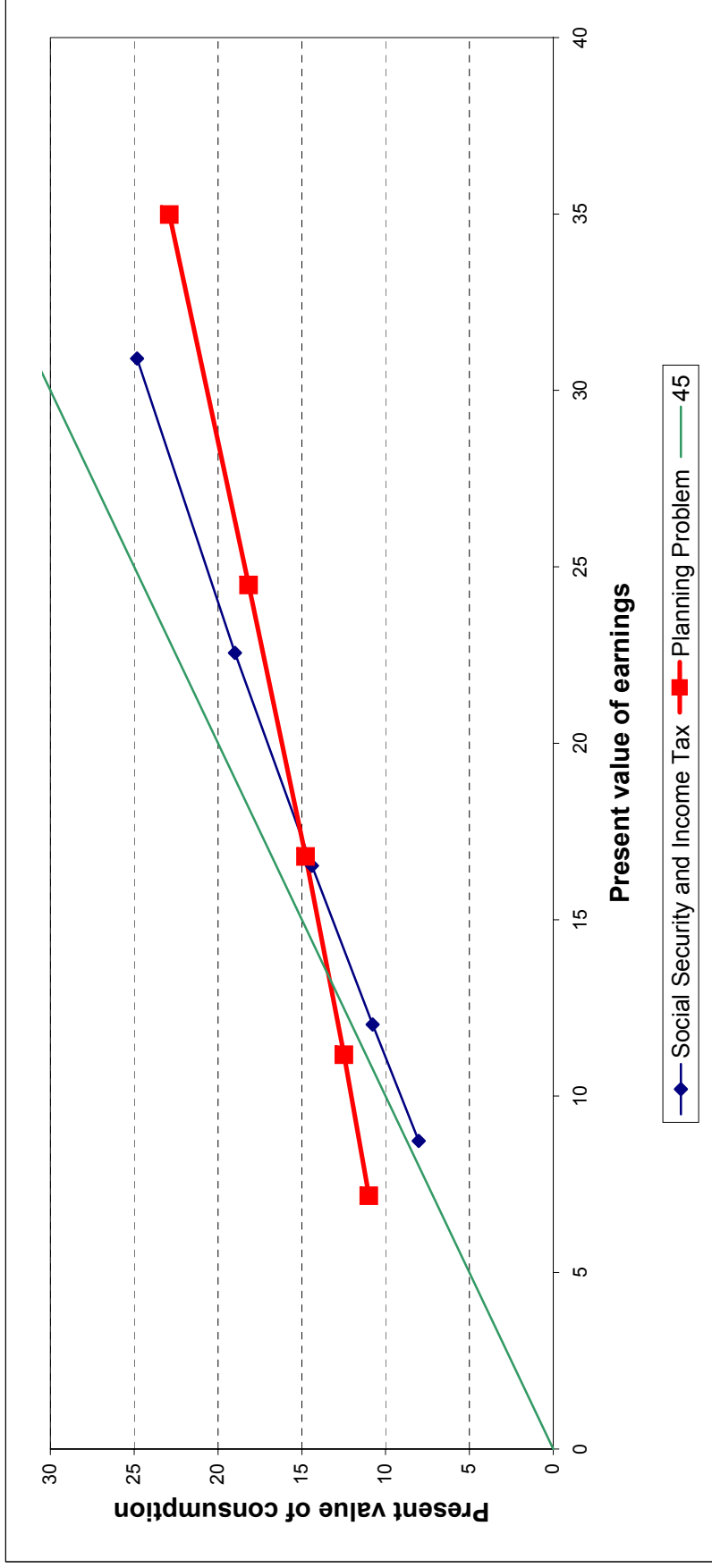
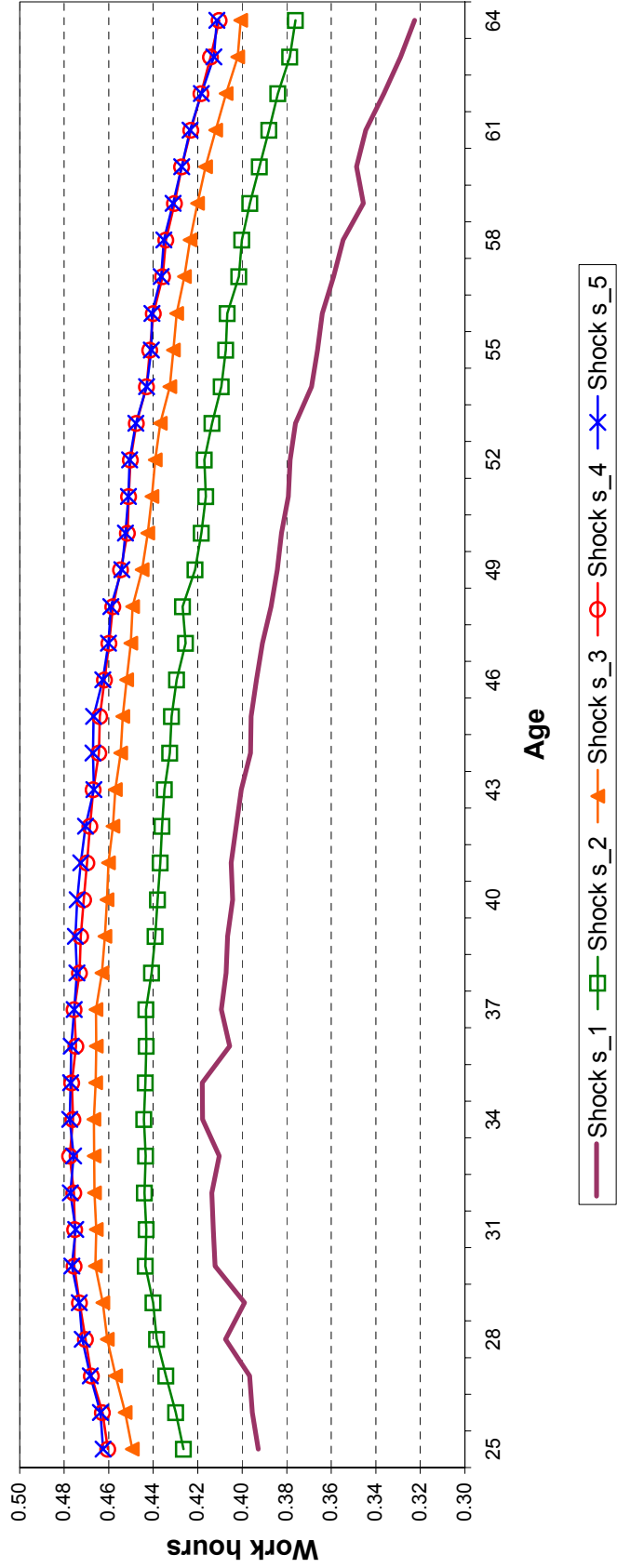


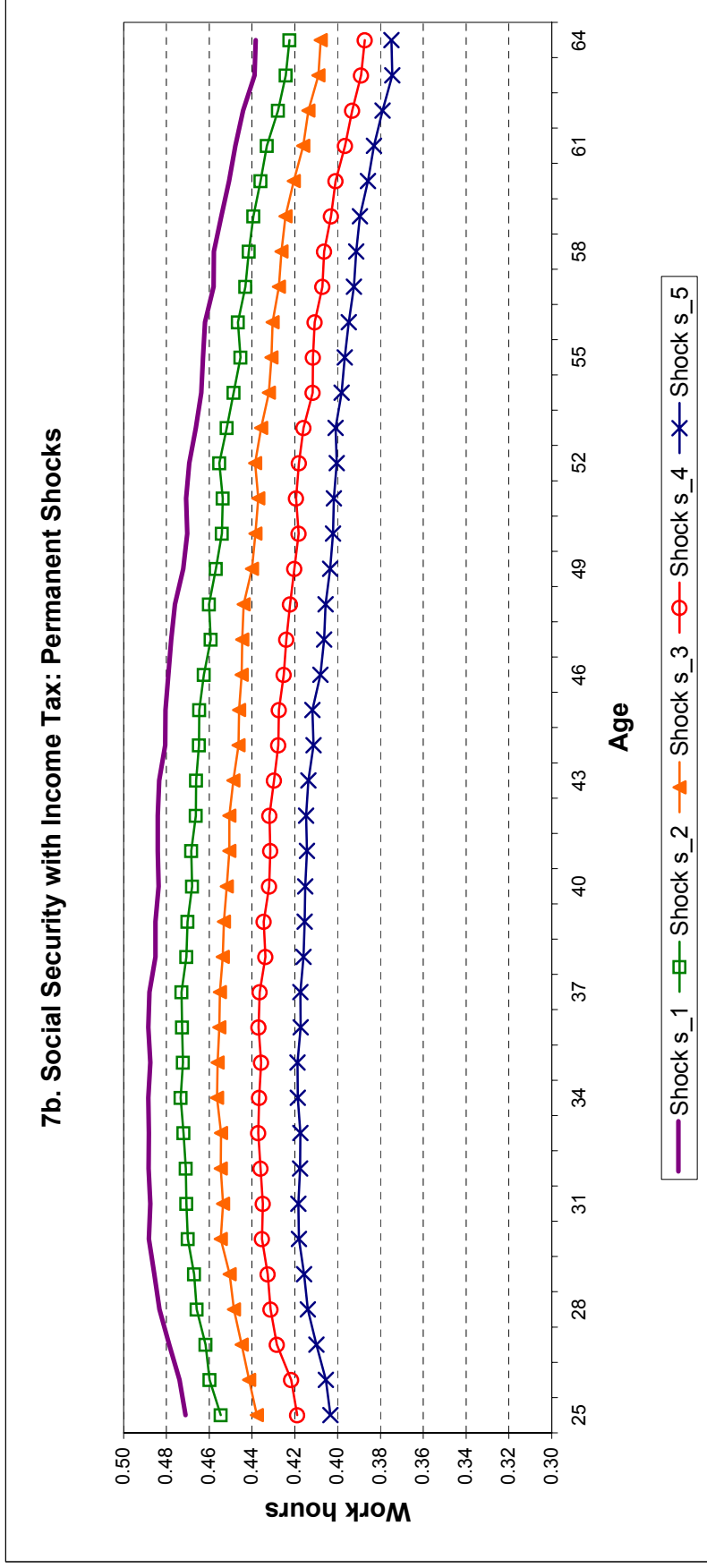
Figure 7: Work Hours Profiles

7a. Planning Problem: Permanent Shocks



Labor productivity  $w(s, j)$  increases in the shock  $s$ . There are five possible shock values  $s_1 < s_2 < s_3 < s_4 < s_5$ .

**Figure 7: Work Hours Profiles**



Labor productivity  $w(s, j)$  increases in the shock  $s$ . There are five possible shock values  $s_1 < s_2 < s_3 < s_4 < s_5$ .

Figure 8: Marginal Tax Rate on Earnings

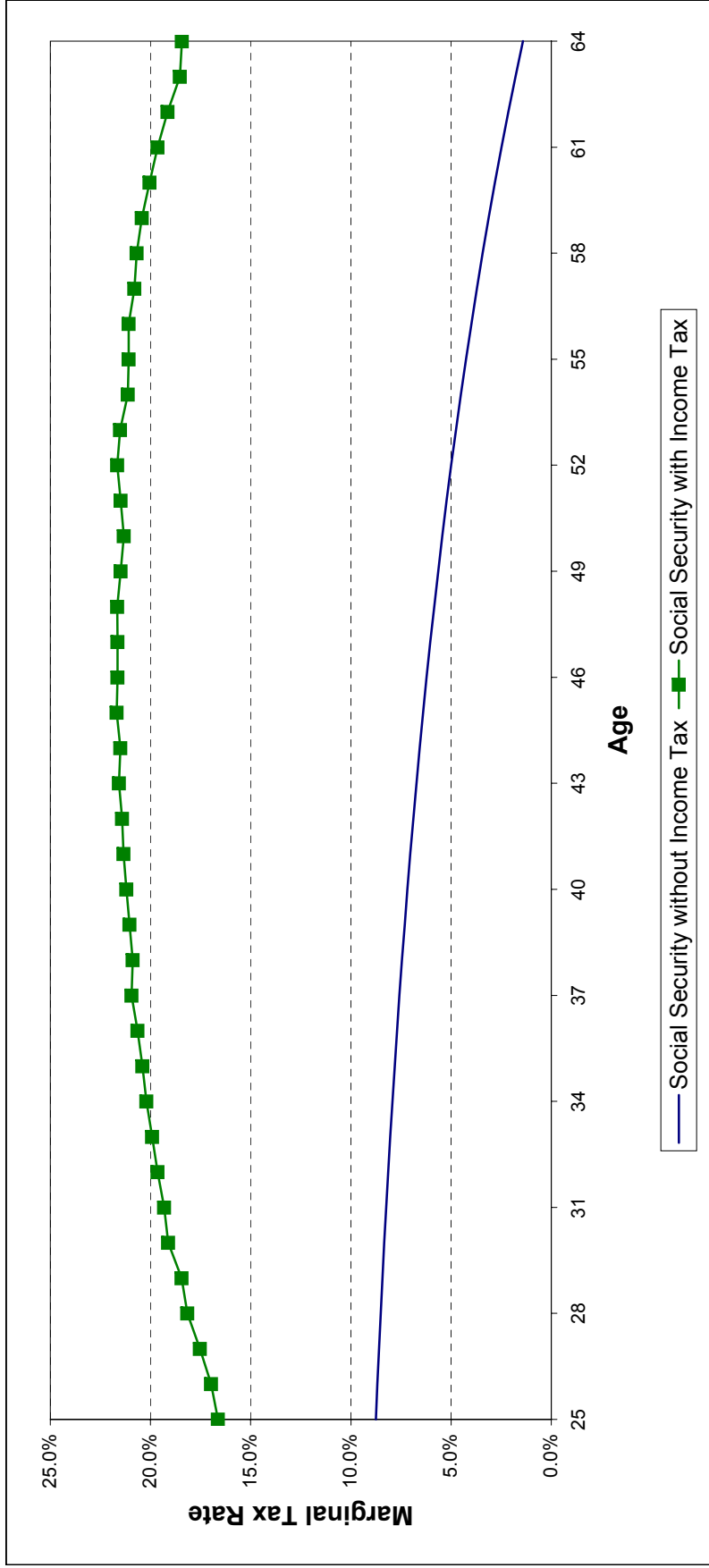
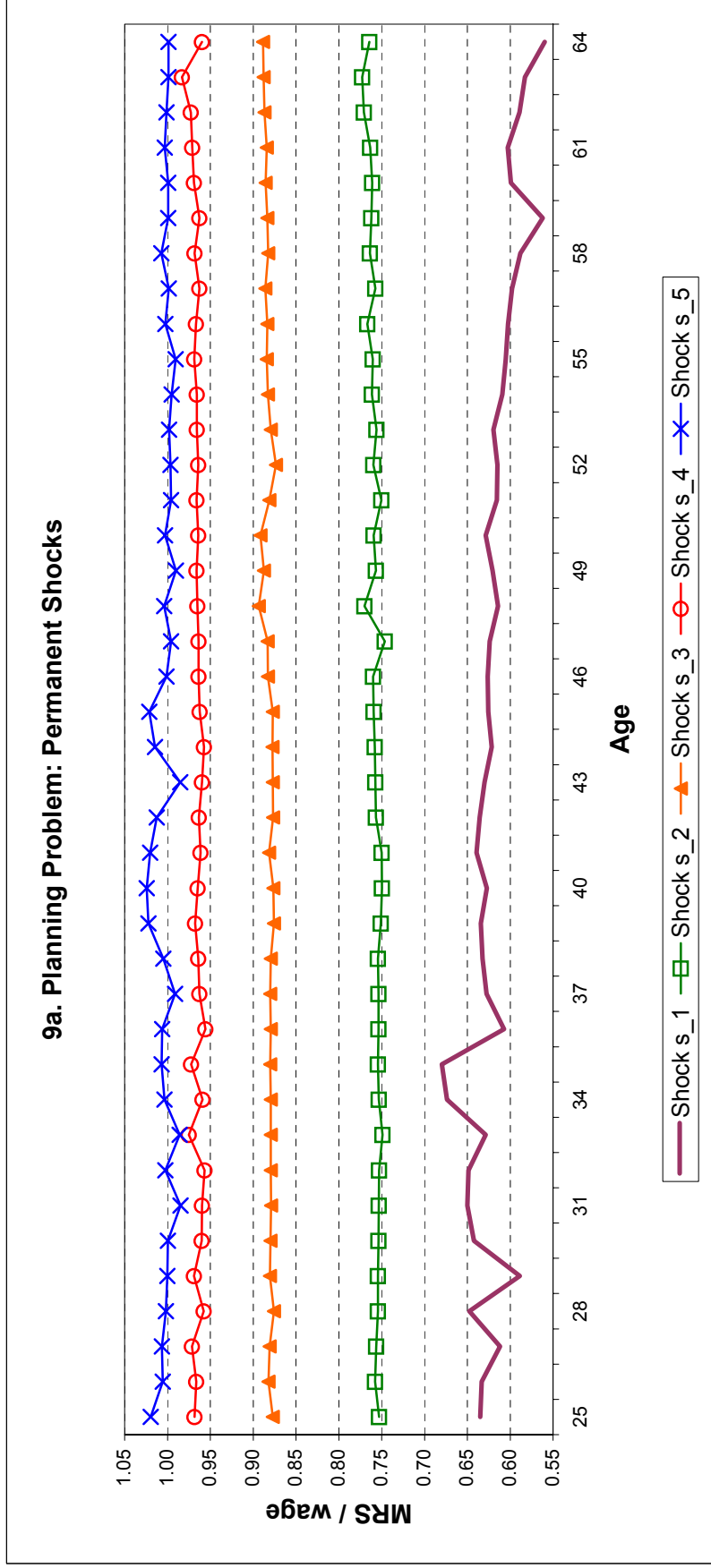
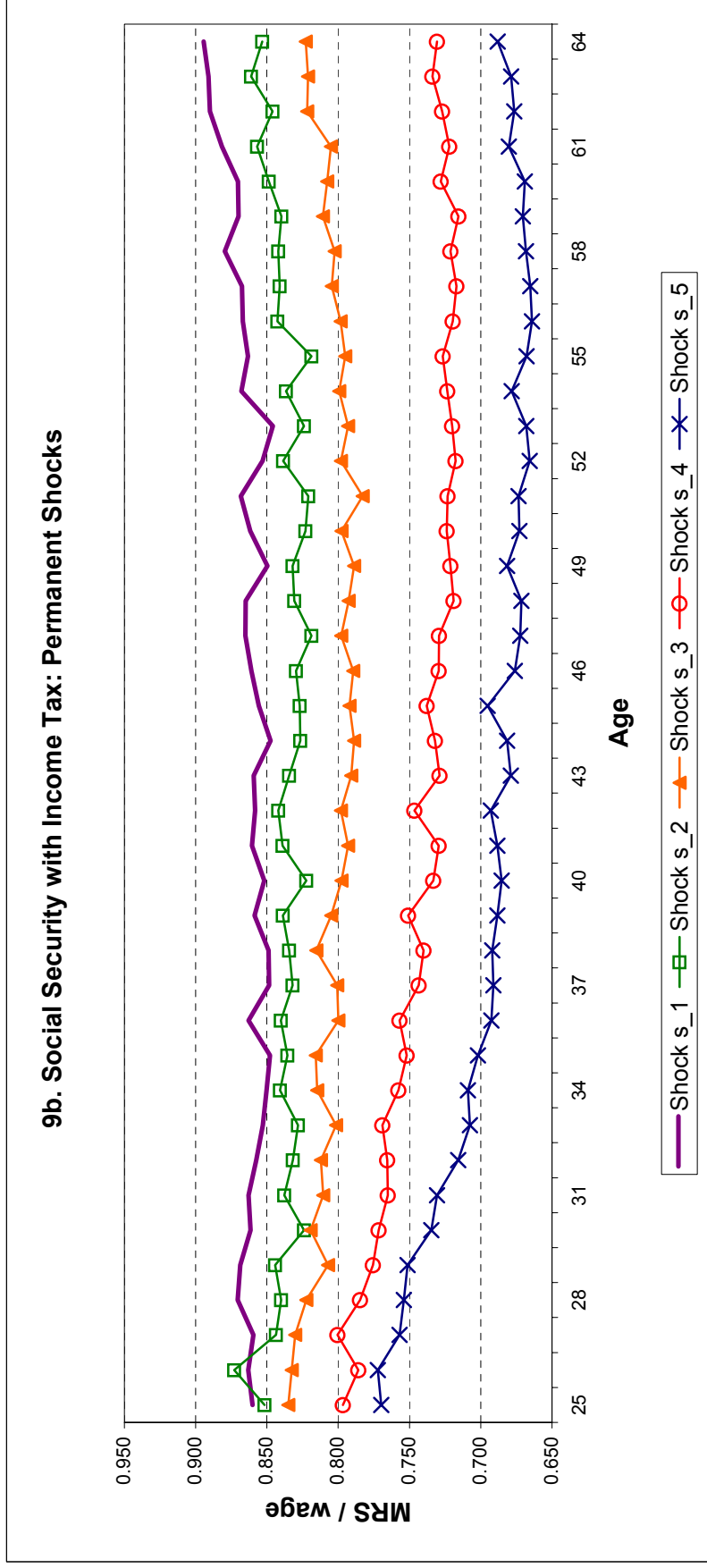


Figure 9: Consumption - Labor Wedge



Labor productivity  $w(s, j)$  increases in the shock  $s$ . There are five possible shock values  $s_1 < s_2 < s_3 < s_4 < s_5$ .

Figure 9: Consumption - Labor Wedge



Labor productivity  $w(s, j)$  increases in the shock  $s$ . There are five possible shock values  $s_1 < s_2 < s_3 < s_4 < s_5$ .

Figure 10: Optimal Present Value of Earnings Tax Function

