Macro III

Homework 3- Euler Equation Methods

In this homework you will AGAIN consider the cake-eating problem. The approach to solving the problem will be similar to homework 2 BUT we will make use of the Euler equation.

Some Theory:

$$v_j(x) = \max_{0 \leq x' \leq x} u(x - x') + \beta v_{j+1}(x')$$

$$v_j'(x) = u'(x - x^*)$$

Nec. and Sufficient Condition (using v concave and differentiable):

$$u'(x - x^*) \geq \beta v_{j+1}'(x^*); \text{ “} = \text{”} \; for \; x' > 0$$

Algorithm:

1. Set $v_J'(x) = u'(x - 0)$ and $y_J(x) = 0$ at gridpoints.
2. Given v_{j+1}', solve Euler Eq. at gridpoints x. This gives the computed function $y_j(x)$ at gridpoints. Set $v_j'(x) = u'(x - y_j(x))$ at gridpoints x.
3. Repeat step 2 until $j = 1$.
4. To compute $v_1(x)$ at gridpoints, simulate consumption paths using $y_j(x)$, starting from gridpoints x in period 1. Then sum of the discounted utility to get $v_1(x)$.

Do the following:

1. Graph the optimal value function for this problem in period 1 and the computed value function. Do the same for the optimal decison rule in period 1. Do this for $N = 101$ grid points.
2. Include a copy of your computer code with your homework.

SUGGESTIONS:

1. Create an array $DV(NX, NJ)$ for the derivative of the value function, where NX is the number of grid points on the state and $NJ = J$ is the number of periods. Initialize the derivative of the value function in period J as indicated in step 1 of the Algorithm. We will need to discuss how to handle the difficulty related to handling the “lowest” gridpoint.
2. Your algorithm to solve this homework should be two DO LOOPS with a solution of a non-linear equation (Euler Eq) operation sandwiched in between.
3. To EVALUATE the Euler Eq, you will have to interpolate the derivative of the value function between gridpoints. Do this by linear interpolation. There

are two steps. First, find the two gridpoints points that lie on each side of the point to be interpolated. This can be done in some computer languages with a single command or by a subroutine in others- this is essentially "searching an ordered table" as described in Numerical Recipes. Second, hit the value function at the two interpolating gridpoints with the correct weights.

4. To SOLVE the Euler Eq, you can use a built-in one dimensional non-linear eq solver routine in some computer languages. Of course, you can write your own. For this problem this is not very difficult.