Aggregate Production Function

$$Y_t = A_t F(K_t, L_t)$$

Y_t - output at time t

A_t - technology level at time t

(K_t, L_t) - capital and labor at time t

Properties: constant returns to scale

A Production function $F(K, L)$ has constant returns to scale provided that whenever all inputs are scaled up or down by a given factor, then output is scaled by exactly the same factor.

$$\lambda Y = F(\lambda K, \lambda L), \forall \lambda > 0$$

Key Implication: No advantage to being a large country

$$\frac{Y}{L} = F(\frac{K}{L}, \frac{L}{L}) = F(\frac{K}{L}, 1)$$

Properties: diminishing marginal products

The marginal product of a factor of production is the extra output caused by increasing the input by one unit, other things equal. We will assume that marginal products are diminishing or falling as the input is increased.

Notation:

$F_K(K, L)$ - marginal product of capital

$F_L(K, L)$ - marginal product of labor

Properties: profit maximization

$$Profit = F(K, L) - wL - RK$$

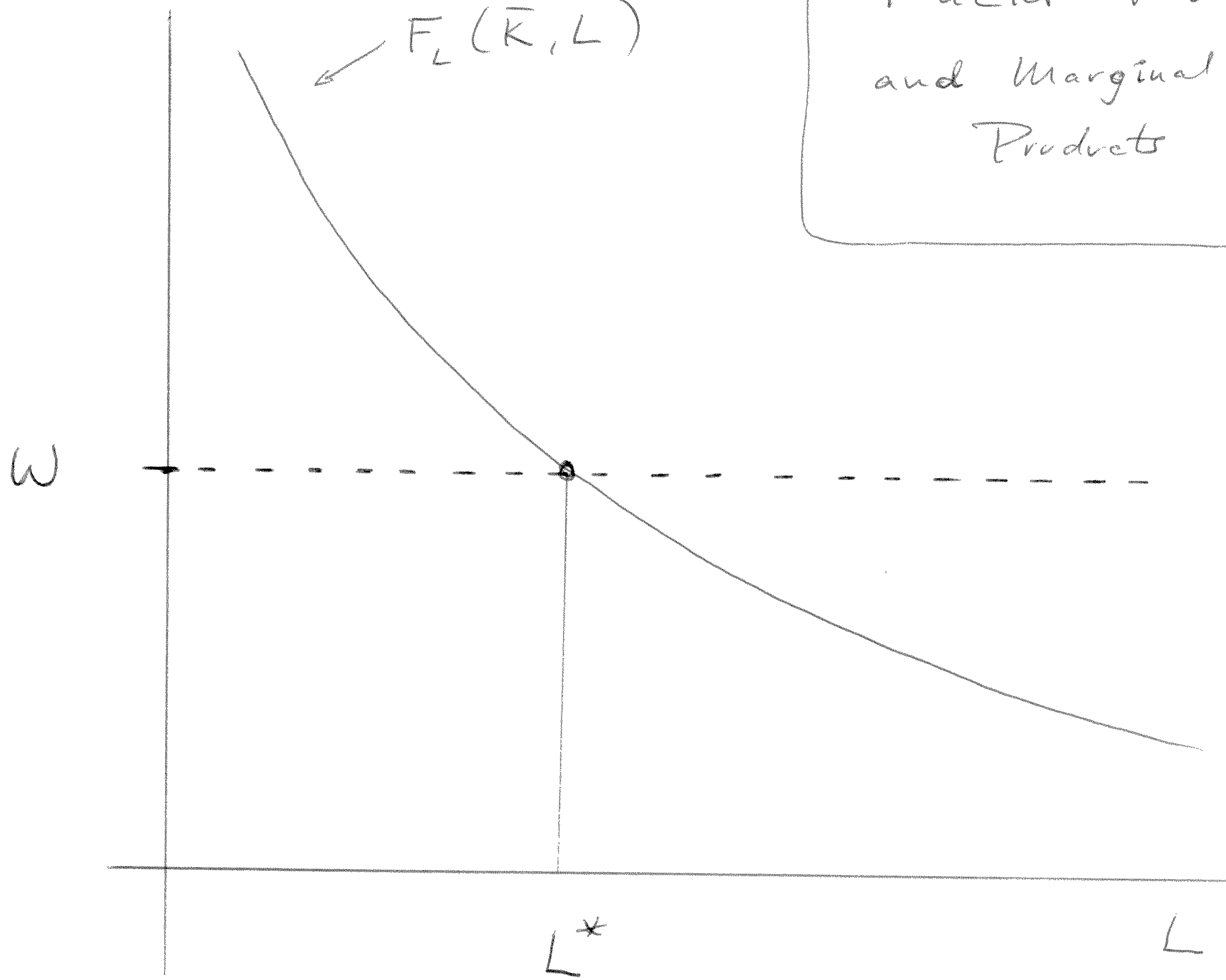
Necessary Conditions for Profit Maximization:

$F_L(K, L) = W$

$F_K(K, L) = R$

The theory asserts that factors are paid their marginal products, assuming competitive input markets.

Factor Prices and Marginal Products



Cobb-Douglas Production Function: $Y = AK^{\beta}L^{1-\beta}$

$F_L(K,L) = (1-\beta)AK^{\beta}L^{-\beta} = (1-\beta)A(\frac{K}{L})^{\beta}$

$F_K(K,L) = \beta AK^{\beta-1}L^{1-\beta} = \beta A(\frac{L}{K})^{1-\beta}$

$Y = F(K,L) = F_L(K,L)L + F_K(K,L)K$ - CRS

Important Property: Constant Factor Shares

$Labor's\ Share = \frac{F_L(K,L)L}{Y} = (1-\beta)$

$Capital's\ Share = \frac{F_K(K,L)K}{Y} = \beta$