Kaldor's Growth "Facts"

1. Output per capita grows over time

2. Capital per capita grows over time

3. Capital-Output ratio is approx constant over time

4. Capital and Labor's share is approx constant over time

5. Return to Capital has no trend

6. Output per capita varies widely across countries at a point in time

Solow Growth Theory

Key Assumptions:

1. Agg. Production Function w/ Diminishing MPK

2. Can Accumulate Physical Capital

3. Technology Grows Exogenously

4. Constant Saving/Investment Rate.

Basic Solow Model

$C_t + I_t = Y_t = F(K_t, L_t)$ - CRS

$I_t = sF(K_t, L_t)$ - investment

$K_{t+1} = K_t(1 - \delta) + I_t$ - Capital Accumulation

$L_t = L$

Implication: $K_{t+1} = K_t(1 - \delta) + sF(K_t, L_t)$

Steady State: $\delta K = sF(K, L)$

Restate Model in Terms of $k \equiv K/L$

$c_t + i_t = y_t = F(k_t, 1)$ - CRS

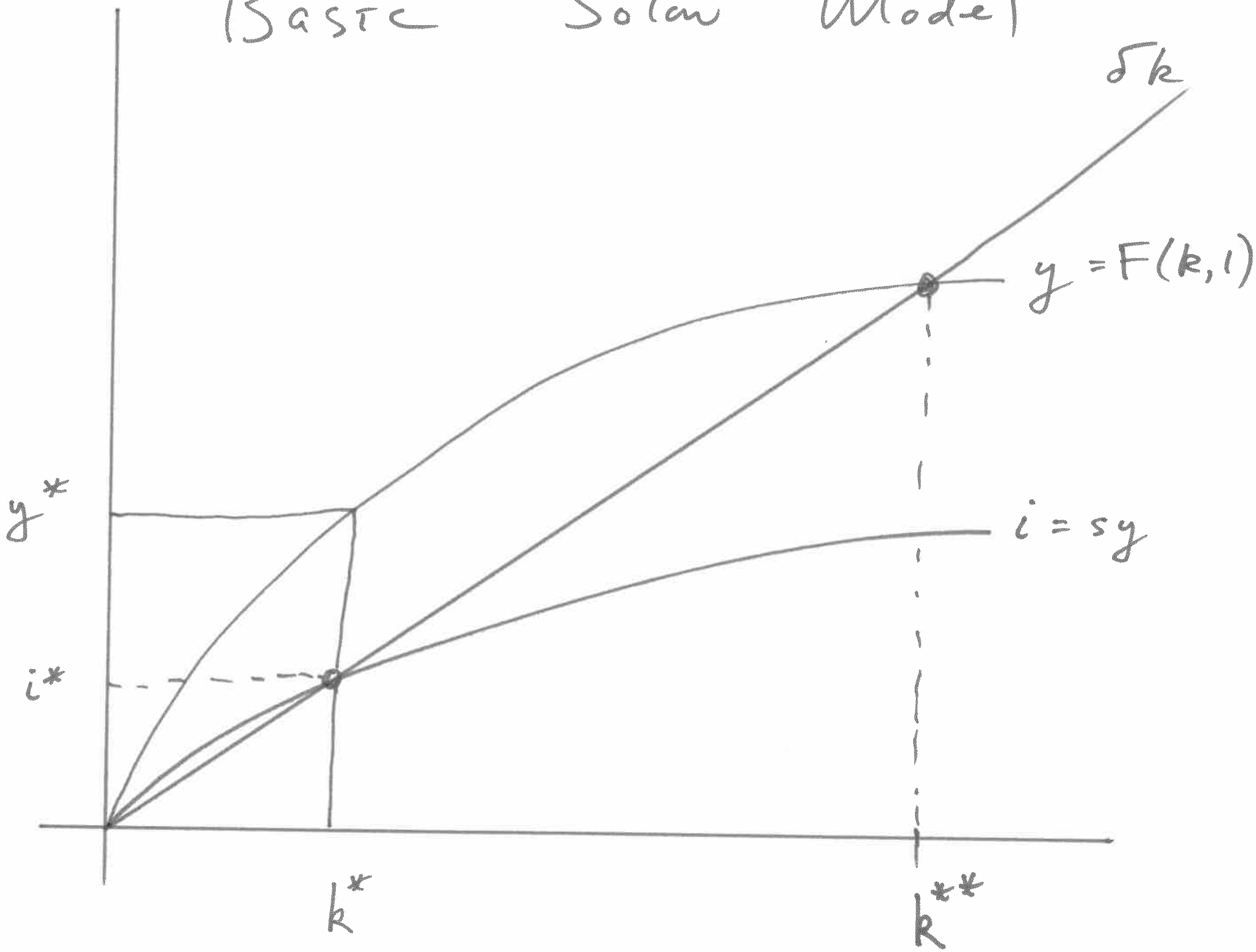
$i_t = sF(k_t, 1)$ - investment

$k_{t+1} = k_t(1 - \delta) + i_t$ - Capital Accumulation

Implication: $k_{t+1} = k_t(1 - \delta) + sF(k_t, 1)$

Steady State: $\delta k = sF(k, 1)$

Basic Solow Model



Main Properties of the Basic Solow Model

Define $k = \frac{K}{L}$

1. One positive capital steady state capital-labor ratio k^*.

2. Higher savings rate s implies a higher steady state value k^*.

3. The economy converges over time to the steady state k^*.

4. There is a max feasible steady state capital-labor ratio k^{**}.

How to Use the Model

1. Use the graph to get qualitative insights to two types of experiments: (1) exogenous one-time changes in capital or labor (e.g. war or disease) and (2) permanent changes in model parameters (e.g. change the savings rate s).

2. One can get insight into how factor prices move if one adopts competitive theory of factor prices. Factor prices are simply marginal products.

3. Make assumptions on the parametric form of the production function and choose all parameter values, then use the model for quantitative insights (e.g. how much does increasing the saving rate increase steady state output?).

Returning to Kaldor's Facts

1. In steady state y does not grow!

2. Thus, the only possibility to explain sustained growth is to have all countries BELOW steady state.

3. This is unsatisfactory as then growth should be slowing down over time. The data say that growth rates are increasing over long time periods! These facts lead to adding technological change to the basic Solow model.

"Full" Solow Model: (Use Trick $k_t \equiv \frac{K_t}{L_t A_t}$)

$C_t + I_t = Y_t = F(K_t, L_t A_t)$ - CRS

$I_t = sF(K_t, L_t A_t)$ - investment

$K_{t+1} = K_t(1-\delta) + I_t$ - Capital Accumulation

$L_{t+1} = L_t(1+n)$

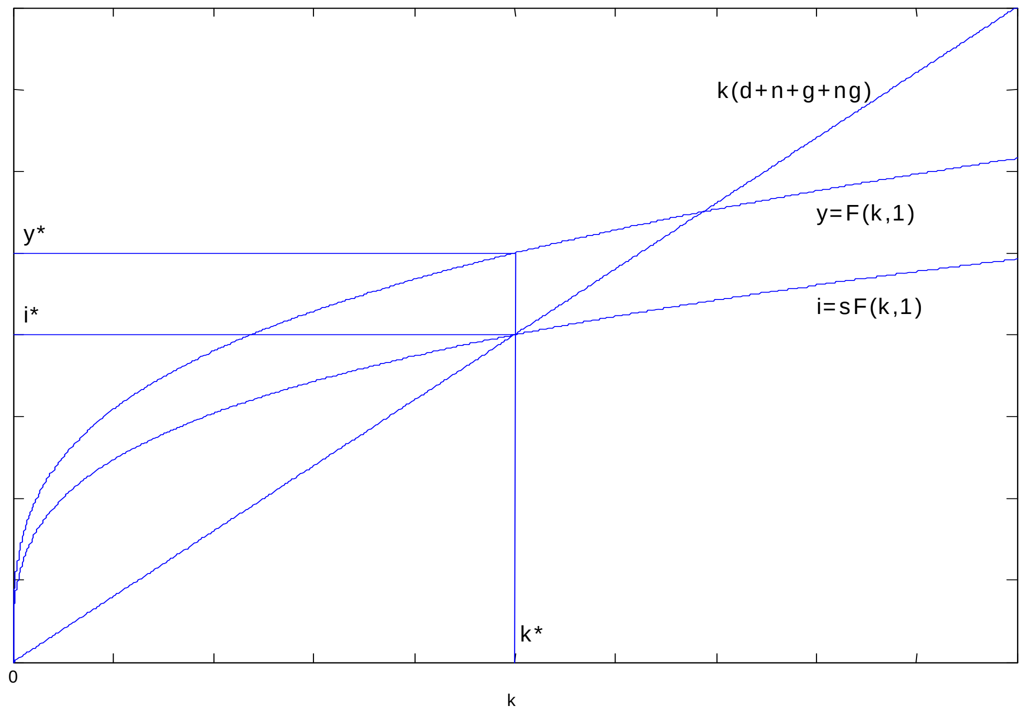
$A_{t+1} = A_t(1+g)$

Implication: $K_{t+1} = K_t(1-\delta) + sF(K_t, L_t A_t)$

$k_{t+1}(1+n)(1+g) = k_t(1-\delta) + sF(k_t, 1)$

Steady State: $\delta k[(1+n)(1+g) - (1-\delta)] = sF(k, 1)$

Figure 1: Solow Model—Steady State



Interpretation of Steady State: $k_t \equiv \frac{K_t}{L_t A_t}$

1. (k_t, y_t, i_t, c_t) constant BUT $(K_t/L_t, Y_t/L_t, I_t/L_t, ...)$ grow

2. $(Y_t/L_t, K_t/L_t)$ grow at rate g and (Y_t, K_t) grow (approx) at rate $n + g$.

3. $Y_t = W_t L_t + R_t K_t$ implies (in steady state) W_t grows at rate g and R_t is constant.

4. In steady state, labor and capital's share of output are EXACTLY constant.

5. Note: Points 1-4 are Kaldor's Facts 1-5!

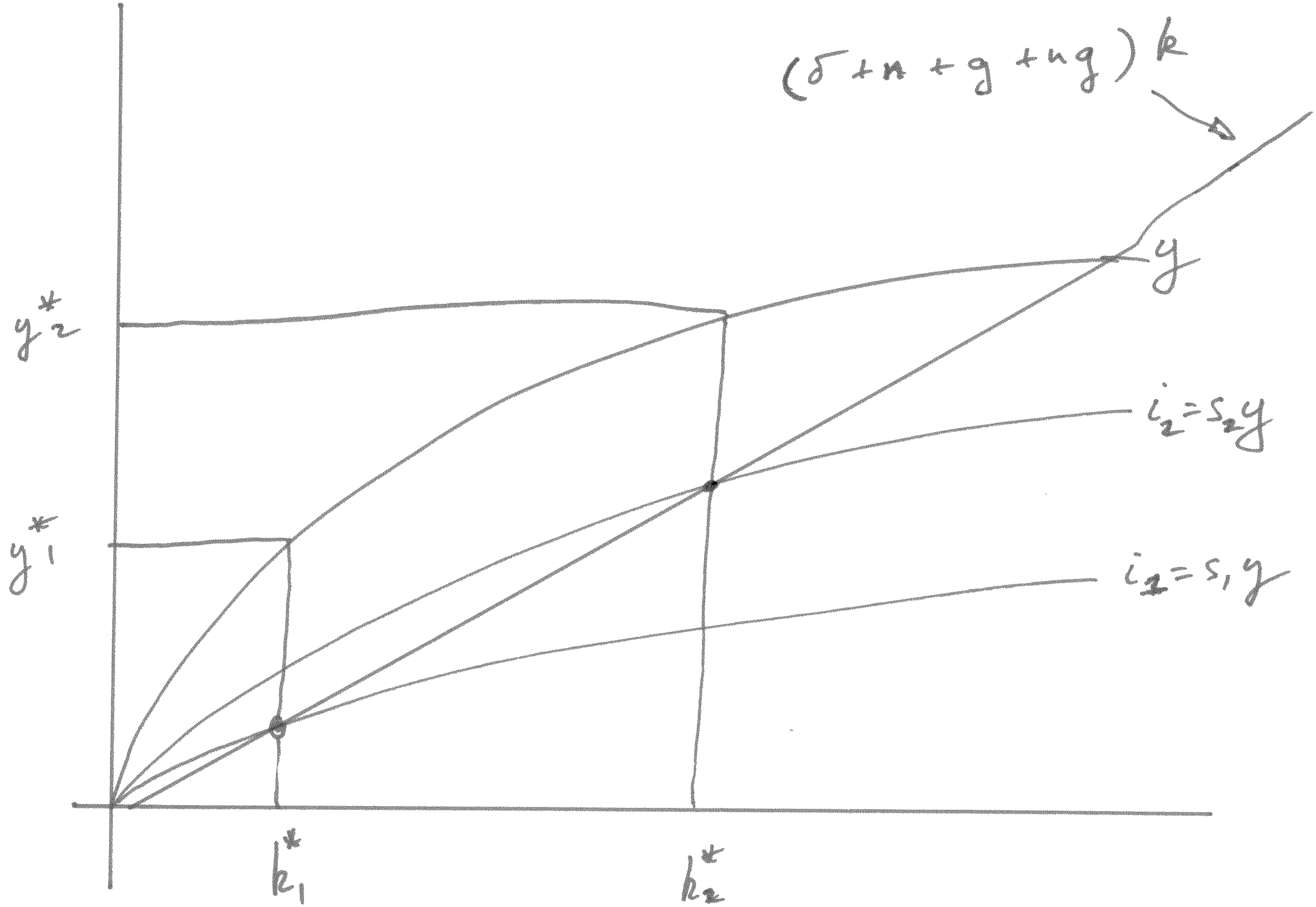
What Are the Effects of Increasing the Savings Rate?

Distinguish between the “steady state” or long-run effects and the effects in “transition”.

Steady State: increasing savings rate does NOT change the long-run growth rate

Transition: increasing savings rate increases the growth rate of output for any finite time period

Two Savings Rates



Assessing the Solow Model: Cross-Country Differences

1. Are countries with high capital-labor ratios and high savings rates rich?

2. Do observed differences in savings rates imply large steady state GDP per worker differences within the Solow model?

3. Is the technology level the same across countries?

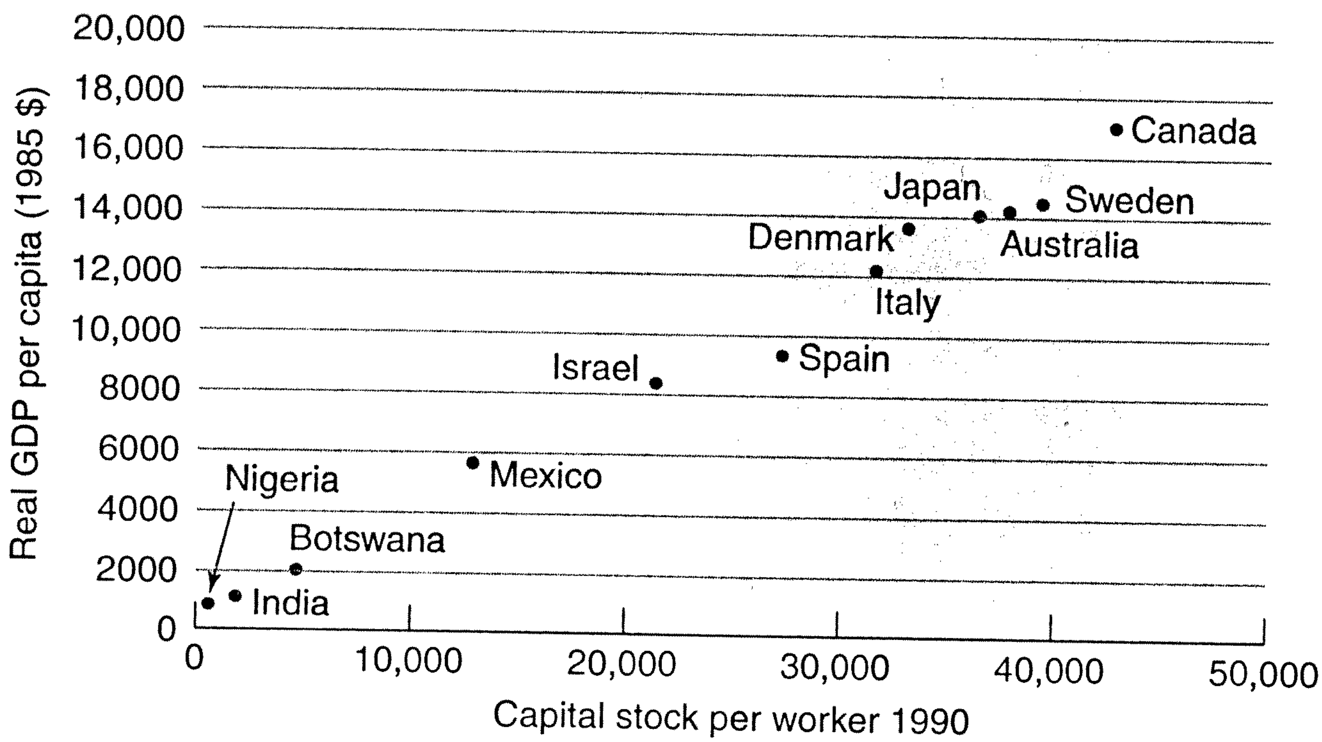
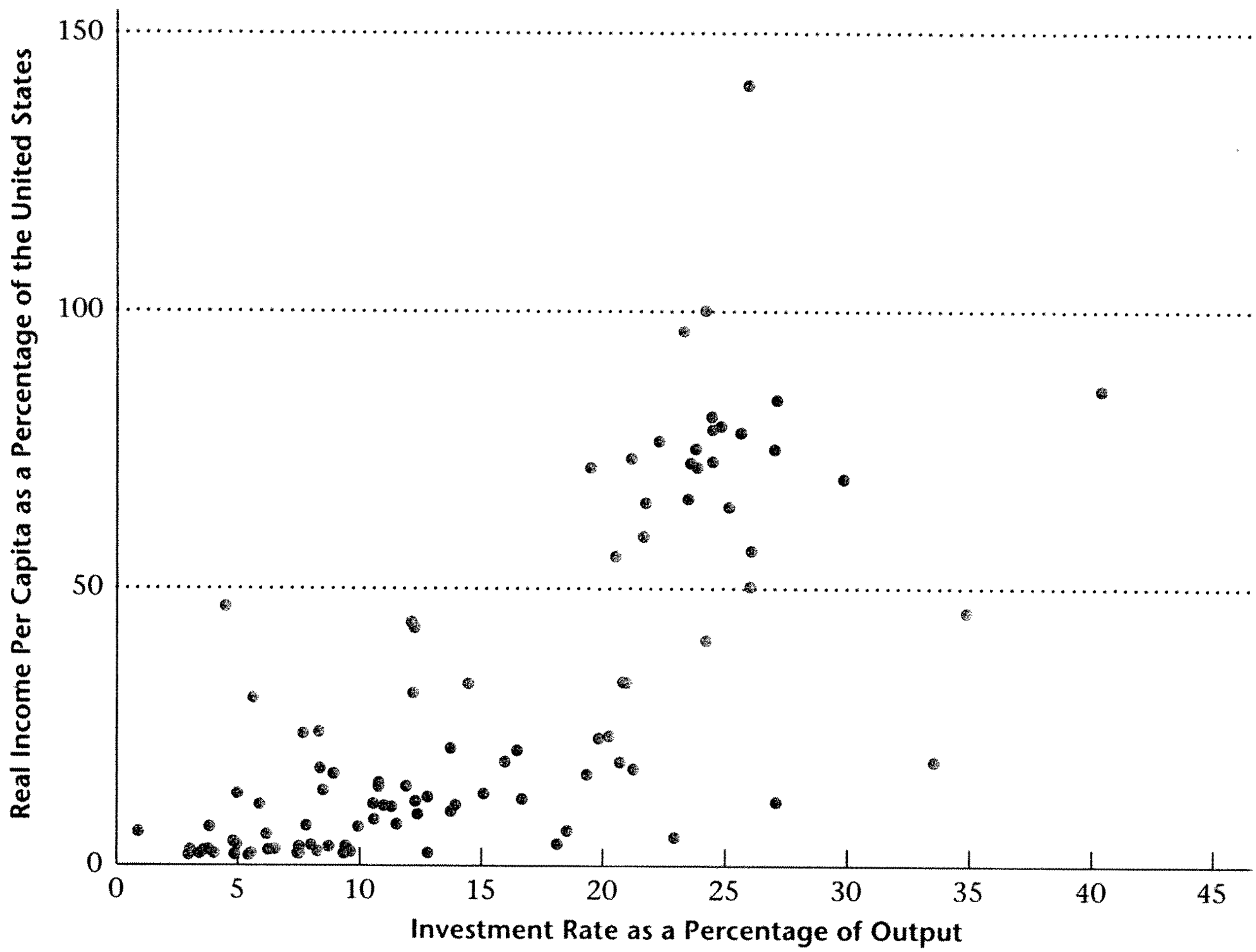


FIGURE 5.1a GDP per capita versus capital stock per worker in 1990. *Source*: Summers and Hest dataset, Penn World Tables 5.5, http://pwt.econ.upenn.edu

Figure 6.2 **Real Income Per Capita vs. Investment Rate.**

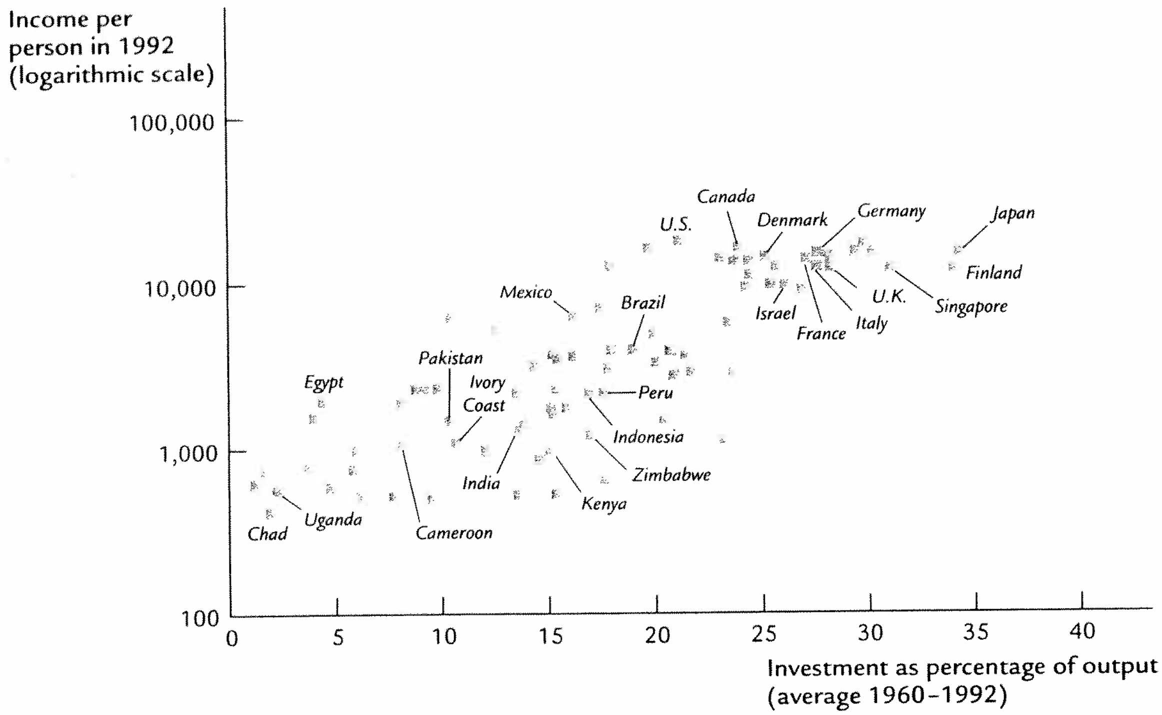
The figure shows a positive correlation across the countries of the world between output per capita and the investment rate.



Source: A. Heston, R. Summers, and B. Aten, *Penn World Table Version 6.1*, Center for International Comparisons at the University of Pennsylvania (CICUP), October 18, 2002, available at pwt.econ.upenn.edu.

Between 1800 and 1950, there was a divergence between living standards in the richest and poorest countries of the world.[5]

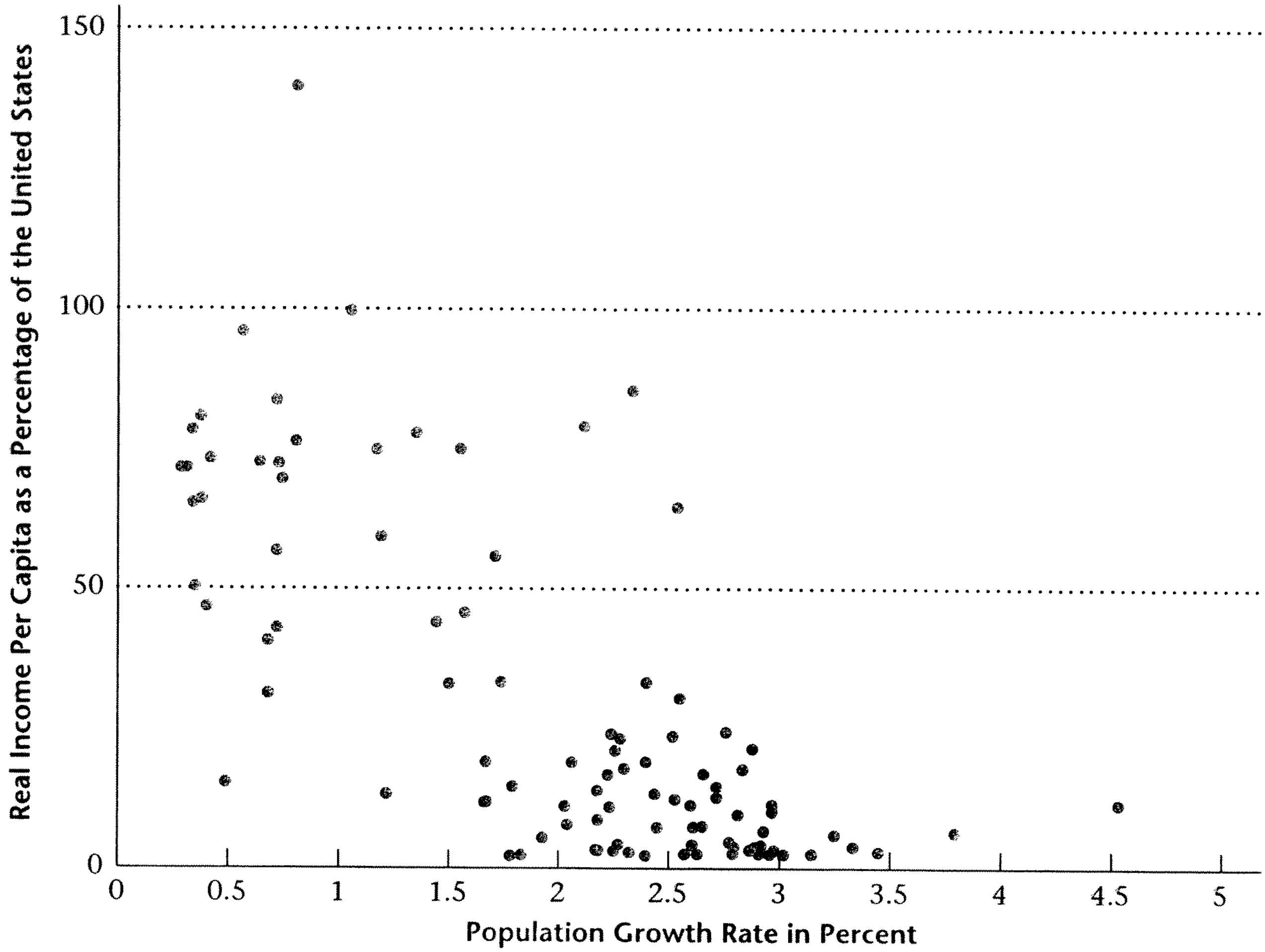
6. *There is essentially no correlation across countries between the level of output per capita in 1960 and the average rate of growth in output per capita for the years 1960–2000.* Standards of living would be converging across countries if income (output) per capita were converging to a common value. For this to happen, it would have to be the case that poor countries (those with low levels of income per capita) are growing at a higher rate than are rich countries (those with high levels of income per worker). Thus, if convergence in incomes per capita is occurring, we should observe a negative correlation between the growth rate in income per capita and the level of income per capita across countries. Figure 6.4 looks at data



International Evidence on Investment Rates and Income per Person This scatterplot shows the experience of 84 countries, each represented by a single point. The horizontal axis shows the country's rate of investment, and the vertical axis shows the country's income per person. High investment is associated with high income per person, as the Solow model predicts.

Source: Robert Summers and Alan Heston, Supplement (Mark 5.6) to "The Penn World Table (Mark 5): An Expanded Set of International Comparisons 1950–1988," *Quarterly Journal of Economics* (May 1991): 327–368.

Figure 6.3 Real Income Per Capita vs. the Population Growth Rate.
The figure shows a negative correlation across the countries of the world between output per capita and the population growth rate.



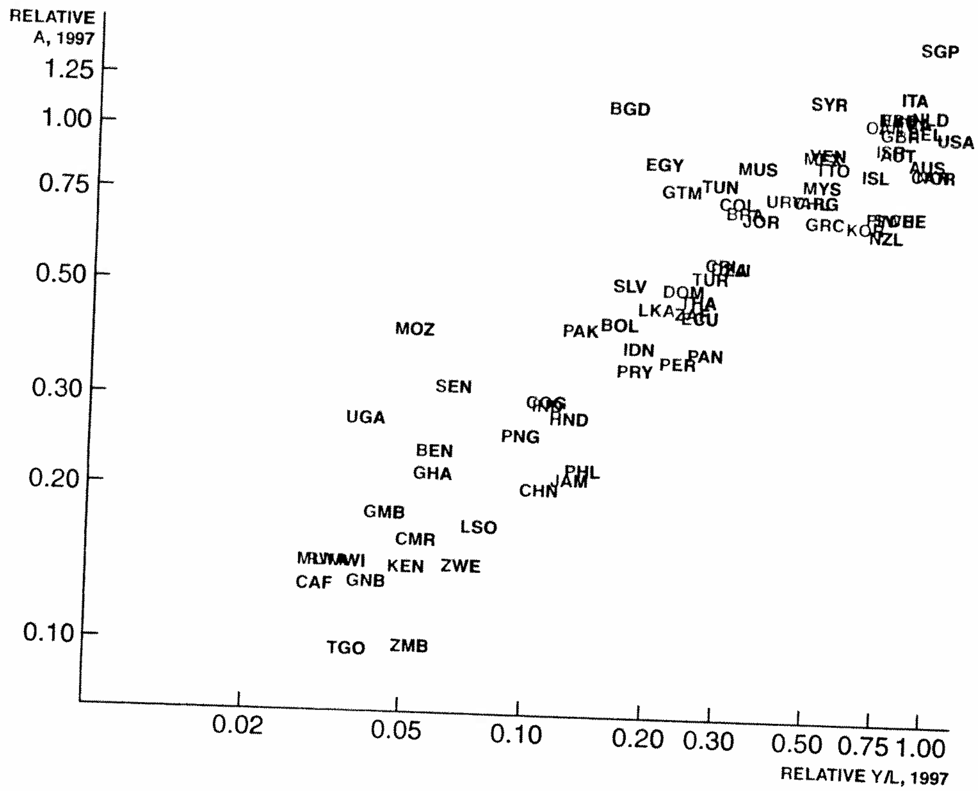
Source: A. Heston, R. Summers, and B. Aten, *Penn World Table Version 6.1*, Center for International Comparisons at the University of Pennsylvania (CICUP), October 18, 2002, available at pwt.econ.upenn.edu.

for 1960–2000, the period for which good data exists for most of the countries in the world. The figure shows the average rate of growth in output per capita for the period 1960 to 2000, versus the level of output per capita (as a percentage of output per capita in the United States) in 1960 for a set of 99 countries. There is essentially no correlation shown in the figure, which indicates that, for all countries of the world, convergence is not detectable for this period.

7. *Richer countries are much more alike in terms of rates of growth of real per capita income than are poor countries.* In Figure 6.4, we observe that there is a much wider vertical scatter in the points on the left-hand part of the scatterplot than on the right-hand side. That is, the variability in the real income growth rates is much smaller for rich countries than for poor countries.

In this chapter and Chapter 7, we use growth facts 1 to 7 to motivate the structure

FIGURE 3.2 **PRODUCTIVITY LEVELS, 1997**



Note: A log scale is used for each axis, and U.S. values are normalized to 1.

Assessing the Importance of Savings Rate Differences

Two countries:

same technology: $y = F(k, 1) = Ak^{\beta}$

savings rate differs: $s_H = .30, s_L = .05$

$\frac{y_H}{y_L} = \frac{F(k_H,1)}{F(k_L,1)} = \frac{k_H^{\beta}}{k_L^{\beta}} = (\frac{k_H}{k_L})^{\beta}$

In steady state: $sk^{\beta} = k(n+g+ng+\delta)$ implies $k = (\frac{s}{n+g+ng+\delta})^{1/(1-\beta)}$

$\frac{y_H}{y_L} = (\frac{k_H}{k_L})^{\beta} = (\frac{s_H}{s_L})^{\frac{\beta}{1-\beta}} = (\frac{.30}{.05})^{\frac{\beta}{1-\beta}}$ and $\frac{y_H}{y_L} \doteq 2.15$ if $\beta = .3$!!!!!!!