

# Golden Rule

Q 1. In the context of growth theory, which allocations are clearly bad allocations?

Q 2. What are the observable implications of these bad allocations?

The literature on the Golden Rule adopts a weak notion of what constitutes a bad allocation. Specifically, a feasible allocation is bad if there is another feasible allocation that at all dates allows for a greater amount of aggregate consumption.

## Golden Rule Steady State: $k_{GR}$

If one could live in any steady state of the Solow model, then which steady state would that be? Arguably, the one with the largest party! The size of the party is measured by aggregate steady state consumption.

$$\max F(k, 1) - k[(1 + n)(1 + g) - (1 - \delta)]$$

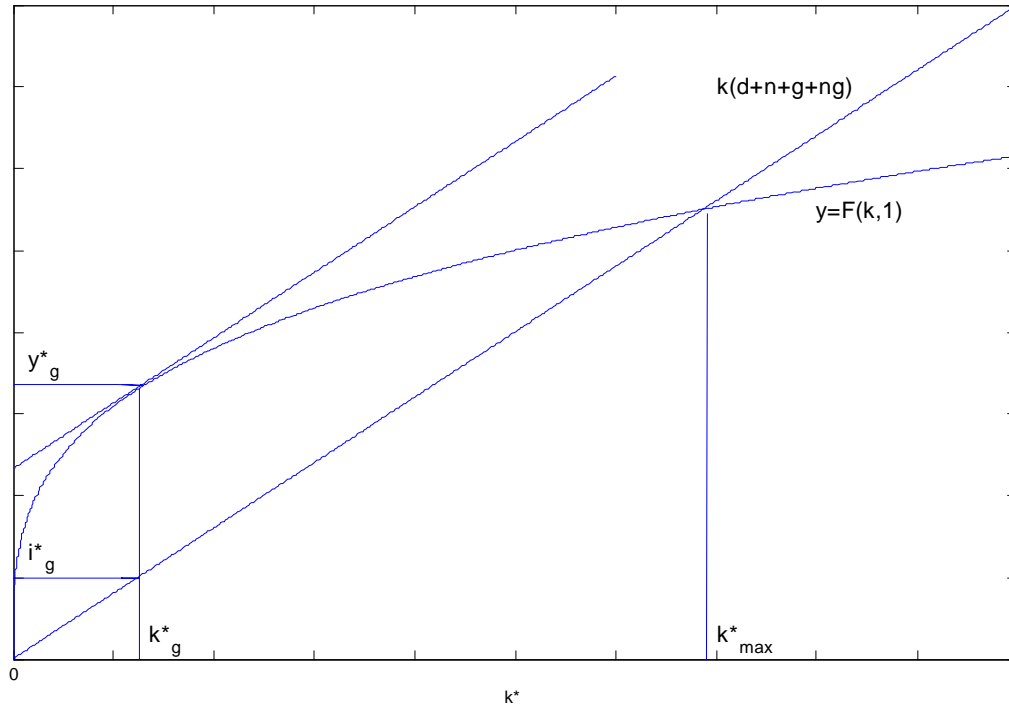
$$\Rightarrow F_k(k, 1) - [(1 + n)(1 + g) - (1 - \delta)] = 0$$

$$\Rightarrow F_k(k_{GR}, 1) = [(1 + n)(1 + g) - (1 - \delta)]$$

## Geometry:

The previous slide tells one that the Golden Rule steady state capital-labor ratio is precisely the ratio  $k$  where the marginal product of capital equals the slope of the line determining steady state investment. This result will be expressed geometrically on the next slide.

Figure 2: Solow Model—Golden Steady State



Q1: In the context of growth theory, which allocations are clearly bad allocations?

Answer: Any steady state  $k$  above the Golden Rule steady state is a BAD allocation. This is because in any such steady state there is something feasible to do which will INCREASE consumption in each future date.

Steady states below the Golden Rule are not BAD allocations by this criteria.

Q2: Observable Implications of these bad allocations?

$$F_k(k_{GR}, 1) = [(1 + n)(1 + g) - (1 - \delta)] - \text{Golden Rule}$$

$$F_k(k, 1) < [(1 + n)(1 + g) - (1 - \delta)] - \text{Above Golden Rule}$$

$$1 + F_k(k, 1) - \delta < (1 + n)(1 + g)$$

$$kF_k(k, 1) < k(\delta + n + g + ng)$$

$$1 + F_k(k, 1) - \delta < (1 + n)(1 + g)$$

LHS:  $1 +$  real interest rate

RHS:  $1 +$  growth rate of GDP in steady state

$$kF_k(k, 1) < k(\delta + n + g + ng)$$

LHS payment to capital

RHS is steady state investment

Abel, Mankiw, Summers and Zeckhauser:

They pushed a more sophisticated version of this line of argument in both theoretical and empirical directions.

Empirically: They conclude that it is best to look at the implications coming from comparing investment and the payment to capital. They looked at data from the US and other advanced countries to determine whether or not investment always exceeds the payment to capital. It does not!

Table: Investment and Capital Payments as a Fraction of U.S. GNP 1929- 1985

Year	$\frac{\text{Capital Payment}}{\text{GNP}}$	$\frac{\text{Investment}}{\text{GNP}}$
1929	.326	.161
1930	.317	.116
1931	.286	.077
1932	.264	.019
1933	.246	.029
1934	.261	.053
1935	.271	.091
1936	.264	.105
1937	.269	.133
1938	.265	.078
1939	.267	.104
1940	.282	.133
1941	.294	.146
1942	.290	.065
1943	.272	.032
1944	.252	.036
1945	.232	.053
1946	.240	.148
1947	.256	.149
1948	.278	.180
1949	.273	.140
1950	.284	.191
1951	.280	.181
1952	.269	.152
1953	.264	.148
1954	.269	.145
1955	.282	.172
1956	.276	.170
1957	.274	.158
1958	.270	.139
1959	.278	.162
1960	.271	.152
1961	.271	.144
1962	.274	.152
1963	.276	.153
1964	.277	.153
1965	.282	.165
1966	.278	.167
1967	.272	.154
1968	.266	.153
1969	.257	.159
1970	.246	.147
1971	.252	.156
1972	.256	.167
1973	.256	.176
1974	.251	.163
1975	.262	.137
1976	.264	.156
1977	.271	.173
1978	.276	.185
1979	.276	.181
1980	.274	.160
1981	.281	.169

Source: Abel, Mankiw, Summers and Zeckhauser (1989, Table 1)