Consumer Theory

Motivation: The key limitation of Solow growth theory is that it abstracts from optimizing consumers. Economists widely believe that consumers and firms respond to incentives. Absent optimizing consumers, the theory offers little in the way of

(1) an understanding of how the economy will react to shocks due to war, immigration, technological booms or important changes in tax policy

(2) a welfare analysis of different policies

Review of Static Consumer Theory:

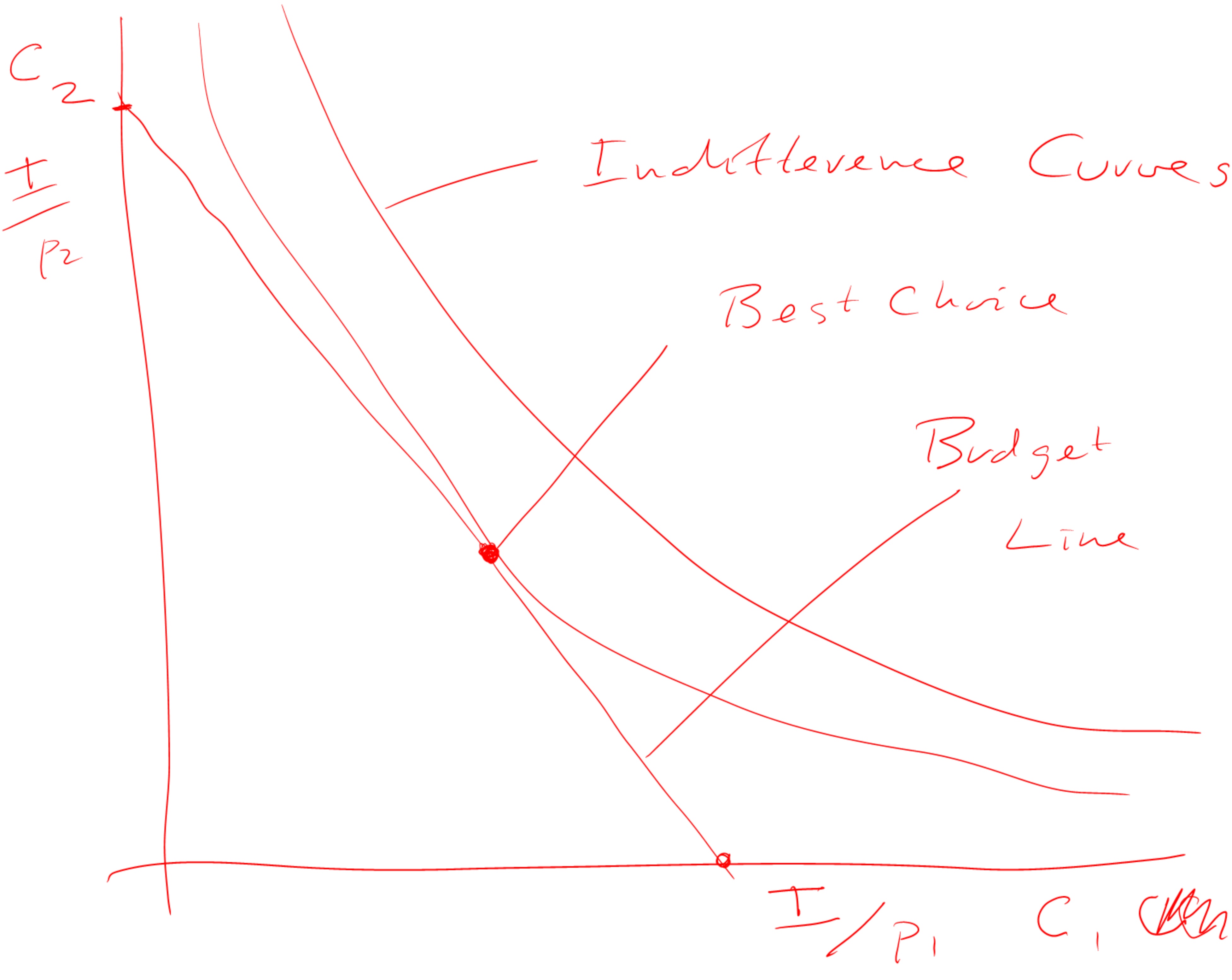
Assume:

1. Preferences over goods: $U(c_1, c_2)$

2. Budget set: $p_1 c_1 + p_2 c_2 \leq I$

Prices: (p_1, p_2)

Income: I


C2
I/P2
Indifference Curves
Best Choice
Budget Line
I/P1
C1

Consumer maximization implies:

(1) $MRS(c_1, c_2) = -\frac{p_1}{p_2}$

(2) $p_1 c_1 + p_2 c_2 = I$

Notation: $MRS(c_1, c_2)$ denotes the marginal rate of substitution between good 1 and good 2 at the allocation (c_1, c_2). Basic Micro: $MRS(c_1, c_2) = -\frac{U_1(c_1,c_2)}{U_2(c_1,c_2)}$

Cobb-Douglas Preferences:

$$U(c_1, c_2) = c_1^{\alpha} c_2^{1-\alpha}$$

$$U(c_1, c_2) = \alpha \log c_1 + (1 - \alpha) \log c_2$$

$$MRS(c_1, c_2) = -\frac{U_1(c_1,c_2)}{U_2(c_1,c_2)} = -\frac{\alpha c_2}{(1-\alpha)c_1}$$

Solve (1)-(2) w/ Cobb-Douglas Preferences:

(1) $MRS(c_1, c_2) = -\frac{\alpha c_2}{(1-\alpha)c_1} = -\frac{p_1}{p_2}$

(2) $p_1 c_1 + p_2 c_2 = I$

Result (Marshallian Demand Functions) :

$c_1 = \frac{\alpha I}{p_1}$

$c_2 = \frac{(1-\alpha)I}{p_2}$

Q: Is the consumer theory we have just reviewed applicable to decision making over time which is key in Macro?

A: With some reinterpretation of income and price concepts, standard vanilla consumer theory is a theory of optimal decision making over time.

Key Assumptions: perfect foresight and lifetime plans

History of Economic Thought:

Irving Fisher - early 1900's he applies stardard consumer theory to decision making over time

Modigliani and Brumberg - 1950's pursue this idea over the 'life cycle'

Friedman - 1950's pursues this idea but adds risky labor income

Dynamic Consumer Theory:

Maximize $U(c_1, c_2)$ subject to

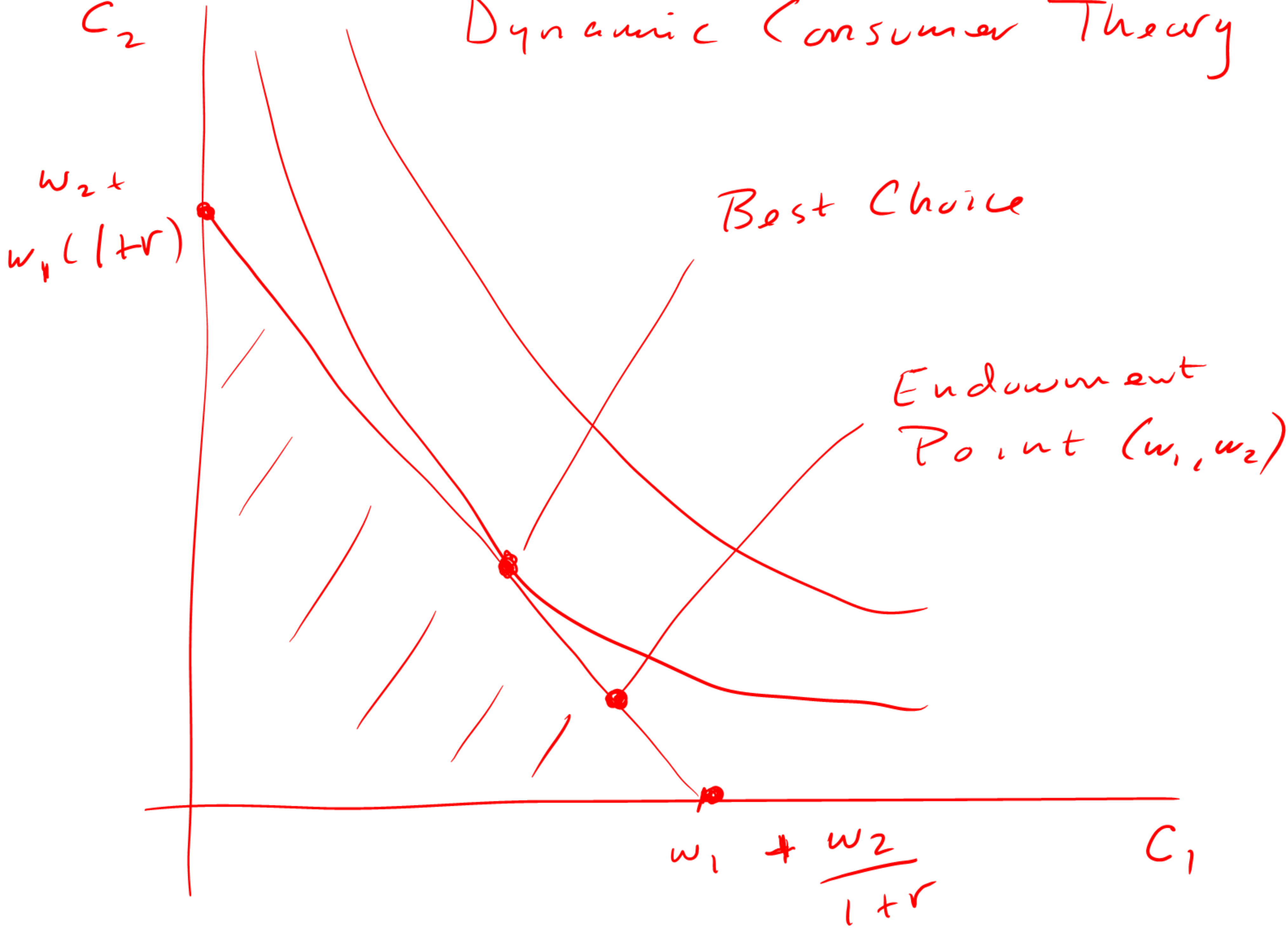
(1) $c_1 + a_2 \leq w_1$

(2) $c_2 \leq w_2 + a_2(1 + r)$

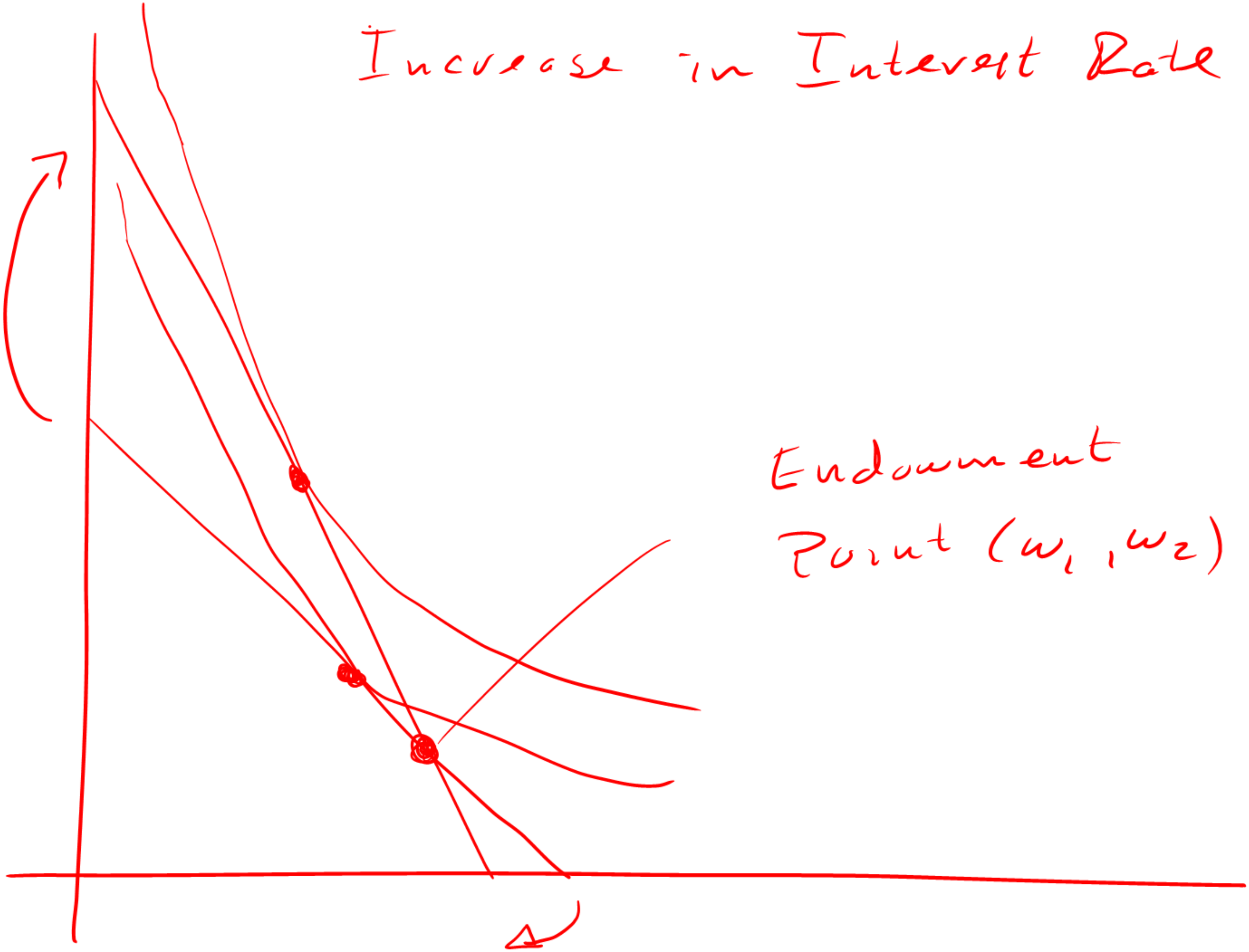
(w_1, w_2) - wages in period 1 and 2

r - real interest rate

Dynamic Consumer Theory



Increase in Interest Rate



Algebra:

1) $c_1 + a_2 \leq w_1$

(2) $c_2 \leq w_2 + a_2(1 + r)$

imply the "present value budget constraint":

$c_1 + \frac{c_2}{1+r} \leq w_1 + \frac{w_2}{1+r}$

Equivalence of present value and standard budget constraint:

$$c_1 + \frac{c_2}{1+r} \leq w_1 + \frac{w_2}{1+r}$$

$$p_1 c_1 + p_2 c_2 \leq I$$

Equivalent when

$p_1 = 1, p_2 = 1/(1+r)$ and $I = w_1 + \frac{w_2}{1+r}$

Cobb-Douglas Preferences - Again!:

[Use: $p_1 = 1, p_2 = 1/(1+r)$ and $I = w_1 + \frac{w_2}{1+r}$]

$$c_1 = \frac{\alpha I}{p_1} = \alpha[w_1 + \frac{w_2}{1+r}]$$

$$c_2 = \frac{(1-\alpha)I}{p_2} = \frac{(1-\alpha)[w_1+\frac{w_2}{1+r}]}{\frac{1}{1+r}}$$

$$a_2 = w_1 - c_1$$

Multi-Period Models:

Many applications require many time periods to interpret data

Utility - Generalized Cobb Douglas:

$$U(c_1, c_2, ..., c_J) = \alpha_1 \log c_1 + \alpha_2 \log c_2 + ... + \alpha_J \log c_J$$

Assume: weights add up to 1 (i.e. $\sum_j \alpha_j = 1$)

Budget in Multi-period Model:

(1) $c_1 + a_2 \leq w_1$ - period 1

(2) $c_2 + a_3 \leq w_2 + a_2(1 + r)$ - period 2 ...

(J) $c_J \leq w_J + a_J(1 + r)$ - period J

imply a present-value budget constraint

$$c_1 + \frac{c_2}{(1+r)} + \frac{c_3}{(1+r)^2} + ... + \frac{c_J}{(1+r)^{J-1}} \leq$$

$$PV \equiv w_1 + \frac{w_2}{(1+r)} + \frac{w_3}{(1+r)^2} + ... + \frac{w_J}{(1+r)^{J-1}}$$

Solution:

Consumption: $c_j = \frac{\alpha_j PV}{(1+r)^{j-1}}, \forall j$

Asset Holding: $a_{j+1} = w_j + a_j(1 + r) - c_j$

Asset holding is determined as a ‘residual’

Stylized Numerical Example:

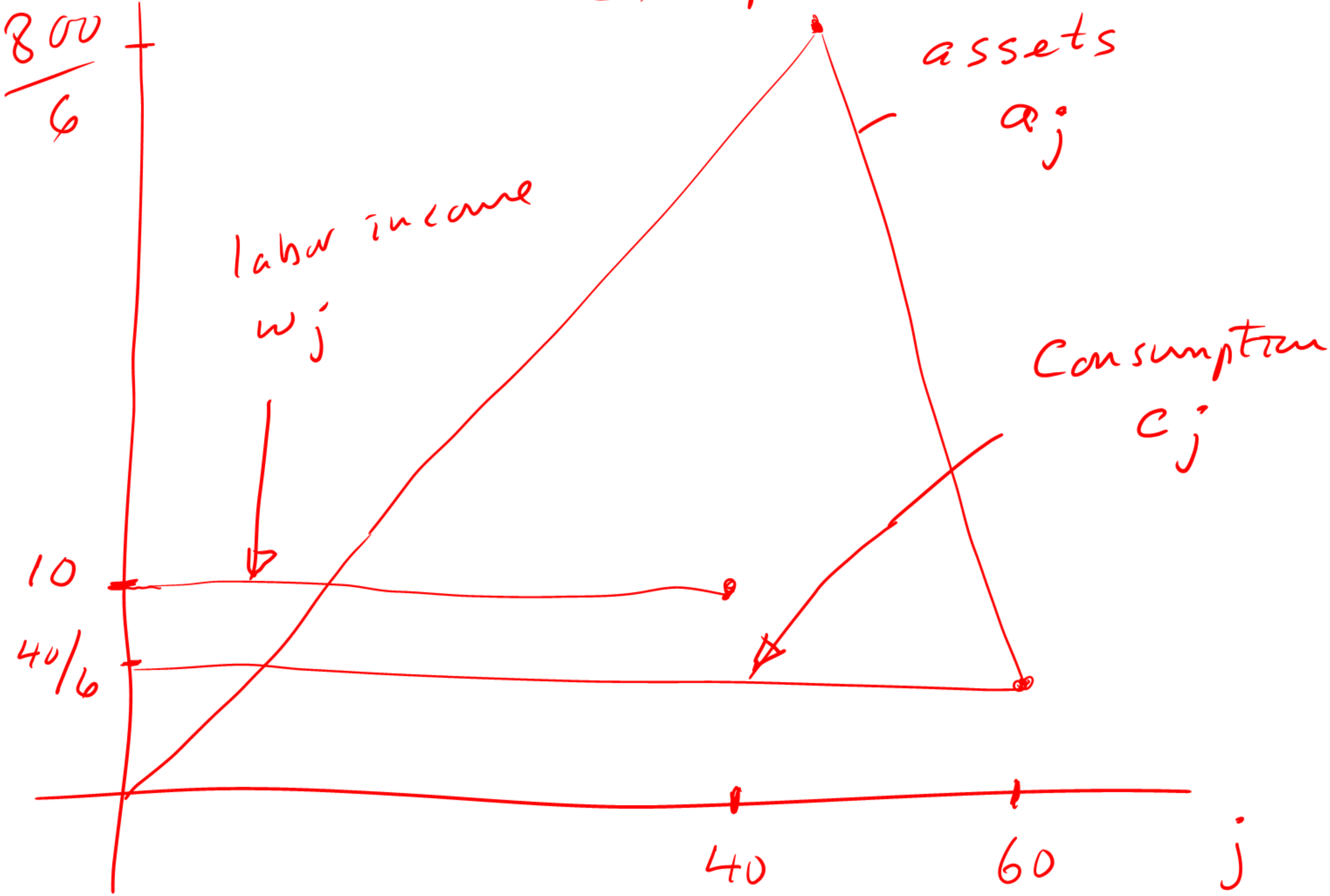
0. Lifetime - $J = 60$

1. Utility - as specified and $\alpha_j = 1/J$ at all ages

2. Interest rate: $r = 0$

3. Labor Income: constant (w_j = 10 j=1 -40) then zero for $j > 40$

Numerical Example



Empirically Motivated Questions:

1. Why is (mean) consumption profile hump shaped over the life cycle?

2. Why do high income households save a higher fraction of income than low income households?

3. Why are consumption responses to permanent and temporary income changes quite different?

Attanasio (1999)
Handbook of Macroeconomics

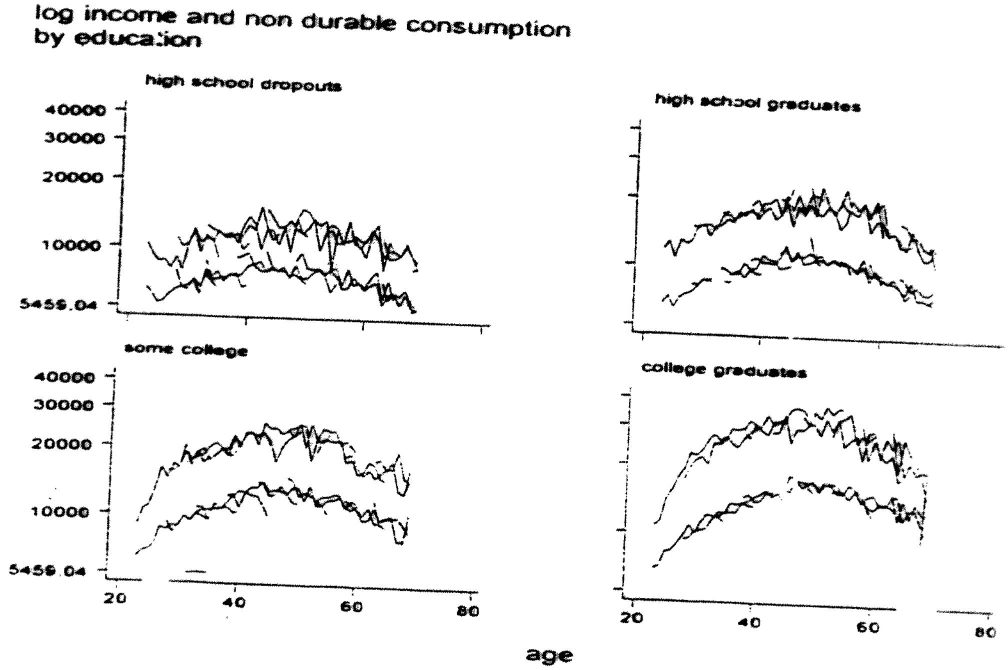


Fig. 7.

Table 6
Variability of consumption and income by educational group

Variable	Standard error (%)		
	High school dropouts	High school graduates	More than high school
Total consumption	2.88	2.74	2.85
Non-durable consumption	2.40	2.93	2.25
Durable consumption	22.85	16.58	9.66
Income	5.98	6.53	5.17

Why is Consumption Hump Shaped?

Possible Story: Income is hump shaped ... and somehow this implies the same for consumption.

Use Dynamic Consumer Theory: $c_j = \frac{\alpha_j PV}{(1+r)^{j-1}}, \forall j$

Does this work??????

Two Famous Savings Rate Facts:

1. The aggregate savings rate in the US is without trend

2. In any year, high income households in the US save a greater fraction of income than low income households.

What type of theory explains these facts?

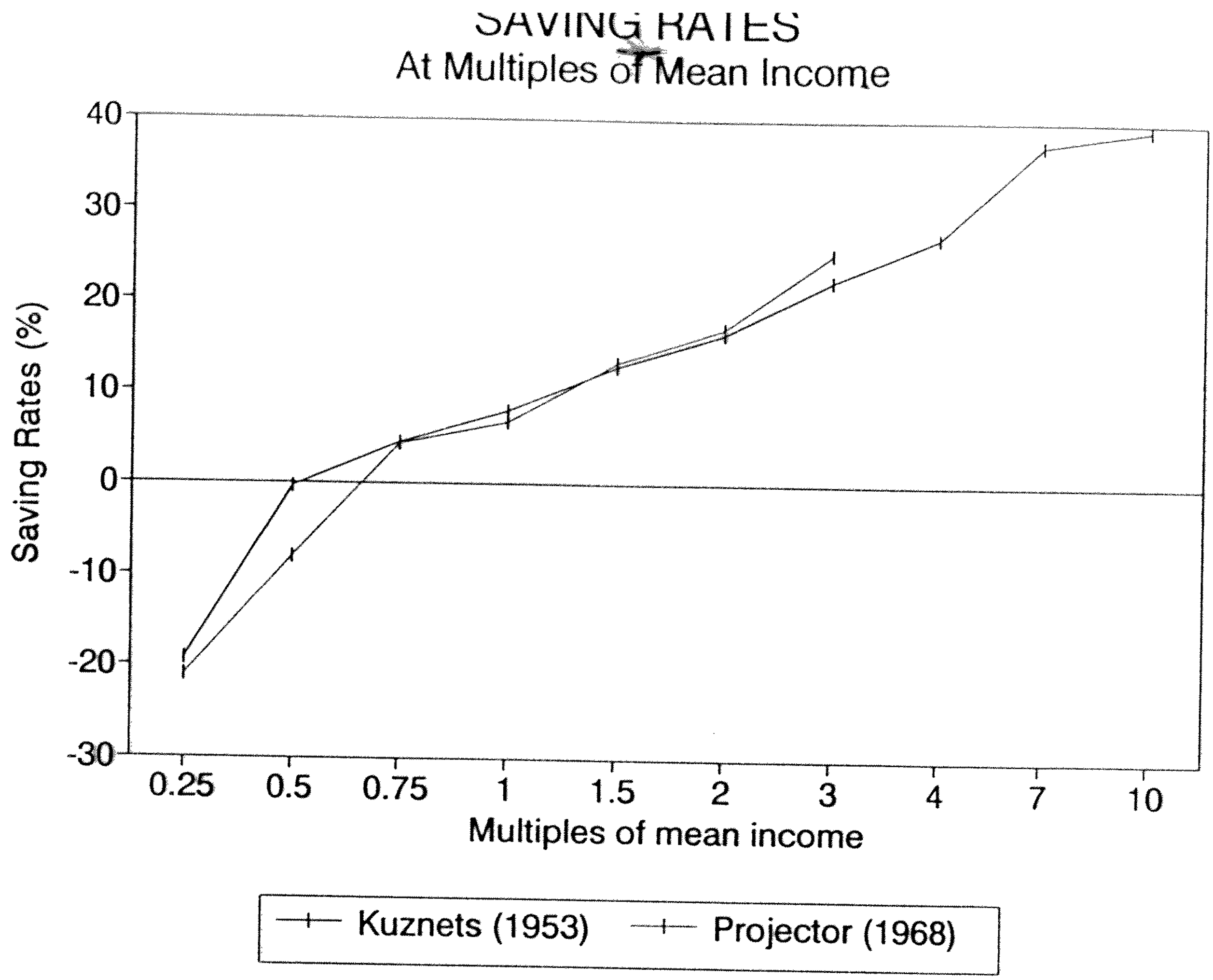


Table 3

Savings Rates at Multiples of Mean Income: US 1929-1950

Income Multiple	1929	1935	1941	1942	1945	1946	1947	1948	1949	1950
.25	-30.4	-32.1	-15.6	-25.1	4.9	-9.3	-14.8	-22.2	-31.1	-15.9
.50	-1.3	-7.4	0.2	-0.1	7.9	1.9	1.4	-1.3	-5.7	-0.8
.75	8.1	-1.5	5.3	8.3	10.7	7.0	4.6	3.2	-0.6	3.9
1.0	11.6	3.5	5.0	10.9	12.9	10.8	7.0	6.4	5.0	7.4
1.5	16.3	9.4	10.7	15.9	15.7	15.9	10.2	10.8	11.2	12.1
2.0	19.5	14.1	13.9	18.2	19.6	19.7	14.0	14.0	15.6	15.4
3.0	23.6	21.9	19.3	22.7	28.6	24.9	21.5	18.5	21.8	20.2
4.0	29.0	27.2	24.8	27.2						
7.0	3[illegible].0	37.5								
10.0	38.5	39.8								
25.0	43.1	49.2								

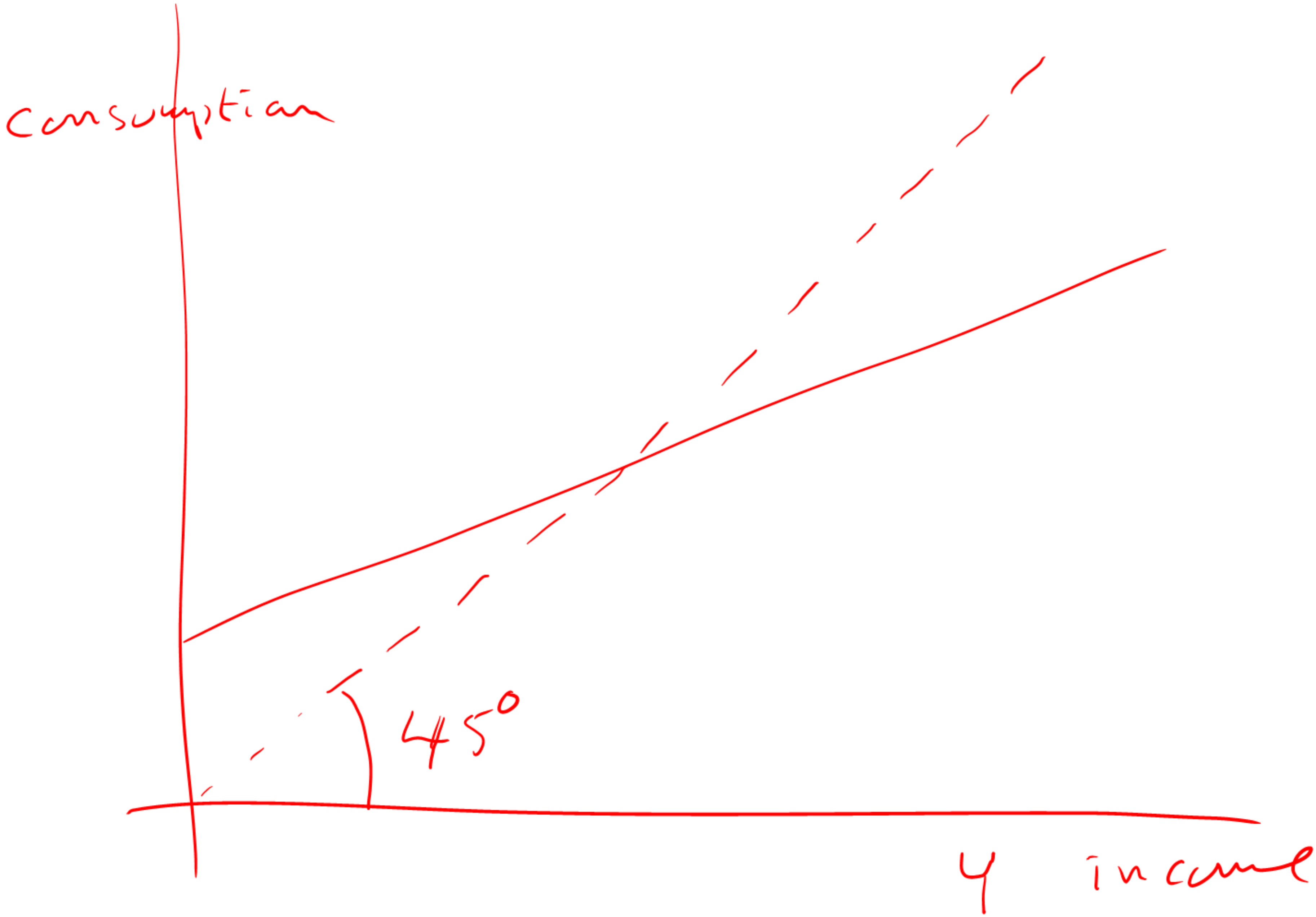
Source: Kuznets (1953, Table 48)

Keynesian Story:

Households follow a simple rule of thumb:

"The fundamental psychological law, upon which we are entitled to depend with great confidence both a priori from our knowledge of human nature and from the detailed facts of experience, is that men are disposed, as a rule and on average, to increase their consumption as their income increases, but not by as much as the increase in their income." - JM Keynes

Keynes: Cnspt. Fn.



Modigliani-Brumberg Story:

1. Labor Income is Hump Shaped

2. Preferences lead to “consumption smoothing” motivated by retirement

3. Low income are largely young + old, whereas high income are largely middle age

Responses to Permanent and Temporary Income Changes

1. Keynesian Story: response should be the same!

2. Modigliani-Brumberg Story: response should depend on how it impacts the present value of lifetime labor income

[numerical example ... connection to 'tax cuts and stimulus' debate]

Theory: $c_j = \frac{\alpha_j PV}{(1+r)^{j-1}}, \forall j$ and $J = 60$

Temporary Shock: compare two situations

1. $w_j = 10$ in all periods

2. $w_1 = 70$ and $w_j = 10$ in all other periods

Consumption change: $\Delta c_j = \frac{\alpha_j \Delta PV}{(1+r)^{j-1}}, \forall j$

Theory: $c_j = \frac{\alpha_j PV}{(1+r)^{j-1}}, \forall j$ and $J = 60$

Permanent Shock: compare two situations

1. $w_j = 10$ in all periods

2. $w_j = 70$ in all periods

Consumption change: $\Delta c_j = \frac{\alpha_j \Delta PV}{(1+r)^{j-1}}, \forall j$