Homework 5 - Equilibrium Properties of the Life-Cycle Model

We can use the Life-Cycle model to analyze the effect of one-time shocks. A potentially interesting case of a one-time shock is a temporary increase in government spending due to a war. In this homework we will analyze a very stylized increase in government spending.

We make the following assumptions:

1. the economy is in steady state initially at time $t = 0$.
2. at time $t = 1$ government spending increases from it previous level of zero to a positive level to fight the war and at time $t \geq 2$ the war is over and government spending is again zero.
3. private agents do not value this war expenditure as a substitute for private consumption.
4. the old and young agents who are alive at the time of the war pay equally for the costs of the war in that the government charges each young and old agent a lump-sum tax to pay for the war.

(a) What are the effects of the war and the war financing scheme on the time profile of the capital-labor ratio, the output-labor ratio, the wage rate and the real interest rate?

[HINT: The graph of the law of motion for the capital-labor ratio shifts. The shift is related to the fact that young agents at $t = 1$ have a lower present value of labor income after taxes than would have been the case absent the war.]

(b) There is a graph attached to the homework which plots time series of nominal interest rates on long-term bonds in the United Kingdom along with a measure of government military spending as a fraction of GDP. To what degree is the theoretical model's prediction consistent with the patterns in the graph?

This graph comes from Robert Barro (1987), "Government Spending, Interest Rates, Prices and Budget Deficits in the United Kingdom, 1701- 1918", Journal of Monetary Economics, vol. 20, 221- 47.

(c) What does theory say must be true about the connection between (i) the real interest rate and the marginal product of capital on the one hand and (ii) the real interest rate and the marginal rate of substitution of young agents on the other hand? Explain. For item (ii) how does consumption growth for young agents relate to the marginal rate of substitution?

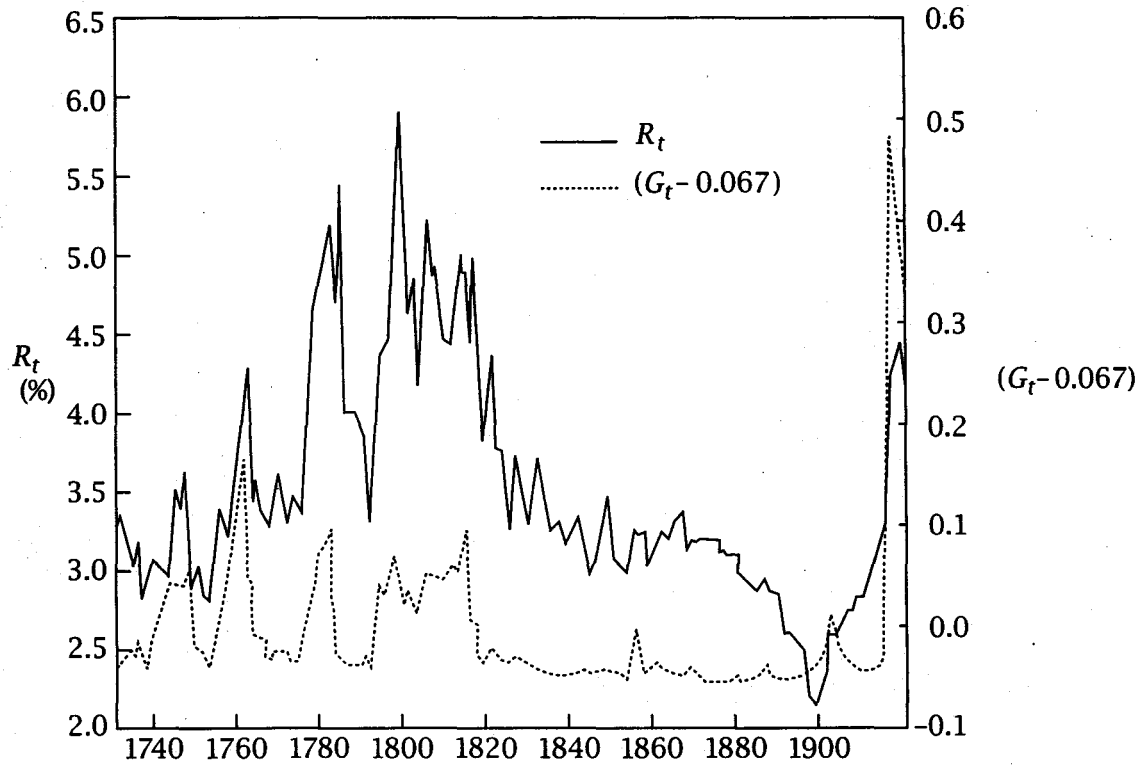

FIGURE 2.10 Temporary military spending and the long-term interest rate in the United Kingdom (from Barro, 1987; used with permission)