

Homework 2 - Neoclassical Growth Theory

1. Suppose that the function $Y = F(K, L) = K^{.5}L^{.5}$ describes the production technology for an economy with fixed population. Can this economy obtain an arbitrarily high income level over time simply by building up the capital stock, given that capital depreciates? Explain.

2. Suppose that there is an economy that follows the logic of the Solow growth model. This economy is initially in steady state with an unchanging population and an unchanging technology. If this economy experiences a one-time increase in population, then what happens over time to (a) total output and (b) output per capita?

3. Suppose that three countries have the same technology and follow the logic of the Solow growth model. Country 1 always saves 10 percent of output (i.e $s = .1$), country 2 always saves 20 percent of output (i.e $s = .2$) and country 3 always saves 30 percent of output. What is GDP per unit of labor input in each country in steady state? What is the marginal product of capital in each country in steady state?

Additional Assumptions:

$$Y_t = F(K_t, L_t) = (K_t)^{.3}(L_t)^{.7}, K_{t+1} = K_t(1 - \delta) + I_t$$

$$I_t = sF(K_t, L_t), L_{t+1} = L_t(1 + n)$$

$$\delta = .06, n = 0, g = .0 \text{ and } s = .1, .2, .3$$

Hint: Find the capital-labor ratio k that makes investment $i = sF(k, 1)$ equal to depreciation δk . It is easy to see that $y = F(k, 1) = k^{.3}$ for the production function above.

4. Calculate the Golden Rule capital-labor ratio. Do this for the model economy considered in problem 3 above. To answer this question it is helpful to know that the marginal product of capital is $F_k(k, 1) = .3k^{-.7}$.

5. Theory tells us that the real interest rate equals the marginal product of capital less the depreciation rate. Theory also tells us that the marginal product of capital decreases when the capital-labor ratio increases. We know from observation that the capital-labor ratio has been rising steadily for over a century, but that the real interest rate and the depreciation rate have remained relatively constant. How can all of these statements be correct? Explain.

Hint: One possible answer uses technological change in an important way. Graphing the production function for different technology levels may be helpful.

BONUS QUESTION: How Fast Will Mexico Converge to the US?

6. Suppose that two countries both share a common technology and that both countries have the same saving rate. Suppose further that one economy (US) is in steady state, whereas the other (Mexico) has a per capita output level that is one fourth the US level.

Under the additional assumptions listed below, how many model periods will it take Mexico to have a per capita output level that is one half the US level?

Additional Assumptions:

$$F(K_t, L_t A_t) = (K_t)^{.3} (L_t A_t)^{.7}, A_{t+1} = A_t(1 + g)$$
$$K_{t+1} = K_t(1 - \delta) + I_t, I_t = sF(K_t, L_t A_t), L_{t+1} = L_t(1 + n)$$

$\delta = .06$ - depreciation rate

$n = 0$ - population growth rate

$g = .02$ - technology growth rate

$s = .2$ - savings rate