# Revealed Political Power

**Jinhui H. Bai*  and  Roger Lagunoff†**

February 9, 2010
Preliminary‡

Abstract

This paper adopts a "revealed preference" approach to the question of what can be inferred about bias in a political system. We examine a dynamic environment in which individuals differ by income each period. Long run preference profiles are unobserved to an outside observer but are known to belong to a well behaved class in which individual preferences are ordered by income in each state. Policy data is summarized by a Markov policy rule. The observer makes inferences about the underlying distribution of political power as if political power were derived from a wealth-weighted voting system with weights that can vary across states. The weights determine the nature and magnitude of the wealth bias. Positive weights on relative income in any period indicate an "elitist" bias in the political system whereas negative weights indicate a "populist" one.

We ask: what class of weighted systems can rationalize a given policy rule as a weighted-majority outcome each period? Our first result shows that without further knowledge, all forms of bias are possible: *any* Markov policy rule can be shown to be rationalized by *any* system of wealth-weighted voting. An additional single crossing restriction on preferences can, however, rule out certain weighting systems. We then augment policy data with polling data and show that the set of rationalizing wealth-weights are bounded above and below. In some cases, polls can provide information about the change in political inequality across time.

JEL Codes: C73, D63, D72, D74, H11

Key Words and Phrases: wealth-bias, elitist bias, populist bias, weighted majority winner, rationalizing weights, "Anything Goes Theorems".

---

*Department of Economics, Georgetown University, Washington, DC 20057, USA. E-mail: jb543@georgetown.edu, Phone: +1-202-687-0935, Fax: +1-202-687-6102. USA.

†Department of Economics, Georgetown University, Washington, DC 20057, USA. 202 687-1510. `lagunofr@georgetown.edu`. `www.georgetown.edu/lagunoff/lagunoff.htm` .

‡We thank seminar participants at Maryland, Northwestern, Towson, and the 2009 Summer Econometric Society Meetings for helpful comments and suggestions.

# 1 Introduction

The principle of *political equality* is widely accepted as a governing philosophy in most democracies. According to the principle, all individuals, regardless of income or background are to be endowed with the same political power or influence. On paper, electoral processes in most democracies satisfy some rough form of it, often taking the form of "one-man-one-vote" electoral systems. Examples include Winner-take-all Presidential elections (in the U.S. and Latin America) and Proportional Representation in Parliamentary elections (e.g., Western Europe).[1]

It is unlikely, however, that the de facto distribution of power in these countries is equal. There is anecdotal evidence, and some systematic evidence, that wealth matters in the political process. For instance, Benabou (2000) presents patterns of the political participation, including rates of voter turnout, contribution, and influence activities among various groups in the income distribution. He finds that the propensity to participate in every reported form of political activities rises with income.

Campante (2008) examines campaign contribution data in the 2000 US presidential election. He shows that income inequality increases the share of contributions coming from relatively wealthy individuals.

Bartels (2008) offers perhaps the most sweeping look at the relation between economic and political inequality. Chapter 9 in his book specifically examines whether economic inequality creates political inequality in the *policy process*. Using data from the Senate Election Study, he finds that Senators' voting records are unresponsive to preferences of those in the lower third of the income distribution.[2] By contrast, Senator's responsiveness to middle and upper thirds is virtually linear to income.

These studies all document some form of wealth-bias in the political system. Their findings suggest that the *de facto* allocation of power is such that richer individuals have a disproportionate influence in the policy process. The result is that policies enacted appear to favor wealthier rather than poorer individuals. Consequently, economic inequality apparently produces political inequality to some degree.

The present paper takes a step back by asking whether and how bias can be identified directly from policy data. When, for instance, can the egalitarian distribution of power based on "one-man-one-vote" be ruled out? To address these issues we view a dynamic economy and its political system from the point of view of an "outside observer." This outside observer either knows or observes the tangible features of the economy (e.g., the income distribution)

[1] Clearly, there are well known exceptions. In the U.S. representation in the Senate is equal across states, so that voters in small states have disproportionate political power in that governing body.

[2] The Senate Election Study consists of survey data conducted after the November elections of 1988, 1990, 1992.

and the observable policies generated by a political system over a finite span of time. However, he does not observe the preferences profile of the citizenry. Nor does he observe certain latent characteristics of the political system. The outsider's job is therefore to infer something about the underlying distribution of political power that generated these policies.

Our concern with consistency of observed policy data with political fundamentals draws an obvious parallel to Revealed Preference Theory (RPT) which typically examines consistency of consumption data with budget-constrained utility maximization.[3] In particular, our interest in *finite* policy data is in the spirit of Afriat (1967) who examines how individual utility function can be constructed from finite consumption and price data.[4]

One difference is that our consistency check involves aggregation of choice: we check whether observed policies could have arisen as an equilibrium of a (possibly biased) voting process. This is reminiscent of the well known Sonnenshein-Mantel-Debreu result checking whether an aggregate excess demand function is consistent with economy-wide aggregation of optimizing choices.[5] Another difference is that the policy outcomes studied here consist of a time series produced by the same underlying polity. For this reason, the present model is dynamic. This puts the present work closer to that of Boldrin and Montrucchio (1986) who examine whether a given policy rule could have been rationalized by a single dynamically-consistent decision maker in a capital accumulation model.

The present work can be loosely viewed as the political economy analogue of Boldrin and Montrucchio. We posit a simple dynamic policy environment consisting of a continuum of citizens or citizen-types, each type differentiated by income each period. The income distribution in period $t$ is summarized by a state variable $\omega_t$ (e.g., public capital), and next period's state evolves from the current state and policy choice $a_t$. Each type's income grows as the state increases, but overall inequality may increase or decrease.

All these tangible attributes of the economy are observed by or known to the outside observer. However, the observer does not observe the preference profile. Instead, he knows only that long run preferences over policies each period belong to the class of single peaked preferences satisfying a single crossing condition. Each preference from this class is well ordered so that an individual's preferred policy is increasing in income.

Given this structure, the observer is a witness to the time series of states and policies. While the time horizon is infinite, we restrict attention to data consistent with a Markov rule of the form $\Psi(\omega_t) = a_t$. The Markov assumption clearly rules out some interesting data series.

[3]See Richter (1966) and more recently Varian (2006) for summaries and surveys of Samuelson's iconic contributions and others' as well.

[4]See also Varian (1982) and Chiappori and Rochet (1987).

[5]References for this result are Sonnenshein (1973), Mantel (1974), Debreu (1974). The consistency check here imposes extra degree of difficulty since we assume that elements of both the political system and the underlying preferences of voters are latent. See also recent applications by Brown and Matzkin (1996) to general equilibrium theory.

Nevertheless, it seems reasonable for large, anonymous polities in which reputation and other history-dependent enforcement mechanisms do not arise.

We then ask whether there exist sequences of distributions of political power that rationalize the observed Markov data as majority-winning outcomes of a voting process. Specifically, consider a weighted voting rule where the weights are attached to one's income and can vary with the state, independent of the income-generating process. The weights can be positive, indicating a pro-wealth bias. In this case, a wealthy individual's vote is worth more than a poorer one. We refer to this case as an *elitist bias*.

There are a number of theories put forth in the literature for why an elitist might exist. One prominent theory links bias to differential participation rates among the rich and poor. Examples include Benabou (2000) and Bourguignon and Verdier (2000). In these models, the poor vote less frequently, the effect being that wealthier voters have a disproportionate influence on policy. A second type theory concerns the effect of campaign contributions, for instance, Austen-Smith (1987), Grossman and Helpman (1996), Prat (2002), Coate (2004), Campante (2008), etc. In these models, the money either "buys" influence directly or it affects policy indirectly by changing the electoral odds toward candidates ideologically predisposed toward the rich. Because contributions skew toward the wealthy, policies are biased in their favor. Finally, a third type of theory centers on disenfranchising investments, e.g., Acemoglu and Robinson (2008), made by a wealthy elite in order to disinherit the poor from the political process.

In the present, "detail free" approach, one need not take a stand on which, if any, of these theories is the "right one." In fact, we do not rule out the possibility of an anti-wealth bias *a priori*. We refer to this case, where weights are negative, as a *populist bias*. Under a populist bias, the poorer individual's vote is worth more than a richer one.

More generally, an increase in the wealth-weight works in favor of the wealthy, while a decrease works in favor of the poor. The case where the weights are exactly zero corresponds to the standard system of "one-man-one vote" or equal representation. We refer to this as the *unbiased* system.[6]

We emphasize that voting is not, or need not be, explicit. Policies are chosen *as if* they came from wealth-weighted voting. In this sense, the weights correspond to an implied distribution of political power. Given a set of weights, a "Political Lorenz Curve" can be calculated to express the implied vote share (hence, "political power") of the poorest $j$th percent of the population, for each possible $j$. A policy rule $\Psi$ is then a *Weighted majority winner (WMW)* of the wealth-weighted voting system if in each state, $\Psi(\omega)$ wins in a weighted majority vote against any alternative. The system of wealth-weighted voting is then said to *rationalize the policy rule* $\Psi$ *in a given class of payoff profiles* if there exists a profile in the class under which $\Psi$ is a weighted majority winner (WMW).

[6]Though there may be other reasonable notions of unbiased, the present definition seems natural to us and is very much in line with that of the literature, including Benabou (2000) and Campante (2008).

The main results address two types of questions. First, what type of wealth bias (if any) can rationalize a given policy rule? Second, what type of wealth bias (if any) *cannot* rationalize a given policy rule? Our first result addresses both questions. We show that *any* Markov policy rule $\Psi$ can be rationalized by *any* wealth bias. That is, without further structure on preferences or additional data, the policy data alone is not very discerning; the data is consistent with every type of income-weighted bias. This is reminiscent of the "Anything Goes Theorems" of Sonnenshein-Mantel-Debreu and Boldrin-Montrucchio.[7] The proof, in fact, adapts and augments a construction by Boldrin and Montrucchio to our dynamic, heterogeneous-agent voting environment.

To make a meaningful inference of bias, two routes are taken. First, we further restrict the class of preferences to those that necessarily generate *monotone* policy rules. In certain types of policy rules, this restriction is shown to narrow the range of rationalizing bias weights. Any decreasing policy rule, for instance, could not have been rationalized by an unbiased polity.

Second, the observer is allowed access to external information in the form of polling data. Polls provide data on specific aggregate binary orderings between benchmark policies — typically those that are being considered in the political process. Two simple polls are analyzed in each state. The wealth-weighted majority winner $\Psi(\omega)$ is compared two alternatives, a policy located to its right, and one located to its left.

We characterize both sufficient and necessary conditions on the wealth weights in each state. Regarding the latter, it turns out that fairly minimal amounts of polling data can nevertheless provide clear restrictions on the bias. Upper and lower bounds on the bias are characterized state-by-state. An upper bound represents a maximal degree of positive wealth-bias — the largest possible bias in favor of wealthy individuals. The lower bound represents the lowest possible bias. Together, these bounds define an admissible band of bias weights that can rationalize the policy data.

Holding fixed the Political Lorenz curve in a given state, the larger is income inequality in a given state, the narrower is this band. This is not surprising when bias band implies an elitist system. In that case, the pro-wealth bias must be lower to have off-set the greater income inequality. However, the result holds even when the band implies a populist system. There, the bias may be more elitist despite the greater income inequality that already benefits richer individuals. The reason is that with a populist system, political inequality is a weighted mirror image of income inequality. Hence, an increase in relative income of the top 10% translates into an weighted decrease in their political power. Hence, the degree of elitism in the bias must increase to offset this income change.

Finally, we later use information specific to the dynamics of the system. Period-by-period

[7] Of the two, Boldrin's and Montrucchio's result is arguably closer to ours. The S-M-D results show that fairly weak conditions are required for consistency between aggregate demand and individual optimization. Boldrin and Montrucchio show that any capital accumulation rule is consistent with an individual's dynamic optimization.

changes in poll data are used to examine whether political power to the wealthy increases or decreases over time.

The paper is organized as follows. Section 2 lays out the economic side of the model. Section 3 then describes the political side: an implied voting process with latent, exponential weights. The first result shows that any weighted system can rationalize any policy rule. We also show, however, that a simple supermodularity requirement on preferences can imply some restriction on the bias. Section 4 examines the implications of polling data. When coupled with policy data, polls imply significant restrictions on the bias. Section 5 gives an extended discussion of the literature on wealth-bias and of extensions that could enrich the framework for future work.

# 2 The Economic Side

This section models the "economic side" of the environment. As the Introduction emphasized, the model is specified from the point of view of an outside observer who knows or observes the tangible attributes of the economy such as the feasible choice sets, the income generating process, and so forth. The observer also observes the dynamic paths of states and policies. However, the observer does not see the actual, parametric preferences from which the policies are distilled. Both the observed and unobserved attributes are laid out in the following subsections. The political side is taken up in Section 3.

## 2.1 Income Inequality

An infinite horizon economy is populated by a continuum of $I = [0, 1]$ of *citizen-types*. A citizen-types is an index that orders individuals by income, with higher types accorded higher income. A citizen of type $i \in I$ holds income $y(i, \omega_t)$ in period $t$ that depends on the value of an aggregate state variable $\omega_t$. For concreteness, this state can be interpreted as an economy-wide public capital stock, such as public infrastructure. The set of possible states is denoted by $\Omega$ with $\Omega$ assumed to be a finite subset of $\mathbb{R}$. The function $y$ is assumed to be continuous in $i$, strictly increasing in each of its arguments, and $\log y$ has monotone differences in the pair $(i, \omega)$.[8] Finally, assume that $y(0, \omega) > 0$.

These assumptions imply the following. Higher citizen types are wealthier; everyone's income is increasing in the state; and by the Monotone differences property, Lorenz income inequality can be ranked unambiguously across any two states. Formally, the standard Lorenz

[8] $\log y$ has monotone differences if for any pair of states, $\omega$ and $\hat{\omega}$, the difference $\log y(i, \omega) - \log y(i, \hat{\omega})$ is either strictly increasing or strictly decreasing in $i$.

curve, defined by

$$L(j,\omega) \;=\; \frac{\int_0^j y(i,\omega)di}{\int_0^1 y(i,\omega)di}, \tag{1}$$

describes the proportion of income held by the lowest $j\%$ in state $\omega$. Monotone differences then implies either $L(j,\omega) \geq L(j,\hat{\omega})$ for all $j$, or $L(j,\omega) \leq L(j,\hat{\omega})$ for all $j$.

Each period the state (e.g., public infrastructure) can be augmented by a public investment denoted by $a_t$. We refer to $a_t$ as the *policy choice*. Assume $a_t \in A$ with $A$ a compact interval in $\mathbb{R}$. The state evolves according to a simple Markov transition technology whereby next period's state is determined by $\omega_{t+1} = Q(\omega_t, a_t)$. The transition function $Q$ is strictly increasing in both arguments.

Putting these attributes together, the physical environment is summarized by following list $(I, \Omega, A, Q, y)$. The outside observer sees/knows all attributes of the physical environment which remains fixed for the rest of the analysis.

## 2.2 Policy Data

To the outside observer (as well the participants themselves), the state $\omega_t$ and policy $a_t$ are observed in each period $t$. We limit the analysis to data that satisfy a Markov data property. Namely, the sequences $\{\omega_0, \omega_1, \ldots\}$ and $\{a_0, a_2, \ldots\}$ are such that every state occurs at least once in the state path, i.e., for all $\omega \in \Omega$, there exists $t$ such that $\omega = \omega_t$, and for any two dates $t$ and $s$, if $\omega_t = \omega_s$, then $a_t = a_s$. Data that satisfy this requirement consist of states and policies that policies vary only in the state variable, and all states (but not necessarily all policies) occur on the realized path.

The Markov data property comes at the price of some loss of generality. However, it has the benefit of admitting a tractable characterization of the data. Policy data that satisfy the Markov data property are fully characterized by an initial state $\omega_0$ and a Markov policy function, $\Psi : \Omega \to A$. Given a state $\omega_t$, the chosen policy is therefore given by

$$\Psi(\omega_t) \;=\; a_t.$$

Note also that if some states were never observed then it would be easier to show that the observations are consistent with any given type of political bias because an extra degree of freedom exists in the way that "off-path" behavior can be specified.

## 2.3 Preferences

In revealed preference theory, the standard interpretation is that of an economy from the viewpoint of an outside observer. The outside observer knows something about the nonparametric structure of citizens' preference profiles, but would not know their precise, parametric form. That view is adopted here as well.

Specifically, preferences are assumed to be represented by a function, $U(i, \omega_t, a_t; \Psi)$ denoting the long run payoff to a citizen-type $i$ of policy $a_t$ in state $\omega_t$ under a policy rule $\Psi$ there determines future policies. The precise form of function $U$ is not known to the outside observer. However, $U$ is known to belong to a set $\mathcal{U}$ of payoff functions satisfying:

(A1) $U$ is continuous in the index $i$, and strictly concave in its $a_t$th argument.

(A2) (Single Crossing) $U$ is satisfies the single crossing property in $(i, a)$: for all $a > \hat{a}$,

$$U(i, \omega_t, a; \Psi) - U(i, \omega_t, \hat{a}; \Psi) \ > \ 0 \ \text{ implies } \ U(j, \omega_t, a; \Psi) - U(j, \omega_t, \hat{a}; \Psi) \ > \ 0 \ \ \forall j > i.$$

(A3) (Recursive Consistency). There exist flow payoff function $u$ and discount factor $\delta$ such that

$$U(i, \omega_t, a_t; \Psi) \ = \ u(\omega_t, y(i, \omega_t), a_t) \ + \ \delta \, U(i, Q(\omega_t, a_t), \Psi(Q(\omega_t, a_t)); \Psi).$$

Note that the single crossing property implies that in every state, wealthier citizens always prefer larger policies than poorer citizens.[9] Note also that flow payoffs do not depend directly on one's type; all types have the same underlying preferences, and consequently heterogeneity comes exclusively from differences in income.

Though these assumptions are fairly standard in political economy, they may appear restrictive at first. In fact, because we are interested in working backward from the observed equilibrium policy rule $\Psi$, the restrictiveness in the class of preferences actually *strengthens* rather than weakens certain of our results. The reason is that the larger the set of feasible preference orderings, the easier it is to find one that "works" in the sense that a voting system can produce $\Psi$ under such preferences. The narrower the class of preferences the more difficult it is for a particular voting system to have generated the data. Hence, possibility results (i.e., assertions that $\Psi$ *can* be rationalized by a voting system) are stronger under narrower classes of preferences, while impossibility results (i.e., assertions that $\Psi$ *cannot* be rationalized) are weaker, all else equal.

Clearly, we do require, and later verify, that the class of $U$ satisfying (A1)-(A3) is nonempty. The theory would be vacuous if this were not the case. Let $\mathcal{U}$ denotes the collection of all payoff functions $U$ satisfying (A1)-(A3). Henceforth, we refer to $\mathcal{U}$ as the class of *feasible preferences.*

[9]Because policies have no specific interpretation, notions of "large" and "small" arbitrary. Hence, one could just as easily have assumed decreasing differences between $i$ and $a$.

### 2.4 An Example

For illustrative purposes, the following example adapted from Bai and Lagunoff (2009) fits the framework.[10] Let $\omega_t$ denote a stock of public capital such as infrastructure, and $a_t$ the investment in infrastructure at date $t$. The transition rule is

$$\omega_{t+1} \;=\; Q(\omega_t, a_t) \;=\; \omega_t(1-d) \;+\; a_t, \quad 0 < d \leq 1.$$

determining next period's stock as the sum of current investment and depreciated capital (with depreciation rate $d$).

A type $i$ individual's income is determined by his inelastic labor and by the stock of infrastructure, with $g(i)$ and $f(i)$ as the marginal productivities of labor and infrastructure, resp. Hence, $i$'s income process is given by

$$y\,(i, \omega_t) = g\,(i) + f(i)\omega_t$$

The investment level $a_t$ is funded by lump sum revenue where $\mathcal{T}_t = \frac{1}{2}a_t^2 =$ is the revenue required to provide investment $a_t$. The flow payoff to an individual in this case is his income net of tax payments, expressed as

$$u \;=\; y\,(i, \omega_t) - \mathcal{T}_t \;=\; g\,(i) + f(i)\omega_t - \frac{a^2}{2}$$

# 3 The Political Side

In this section the payoff function $U$ is fixed and notationally suppressed where convenient. We attempt to identify the de facto distribution of political power from a class of parameterized distributions that all resemble, and have similar properties to, a standard Lorenz curve. Each "political Lorenz curve" in this class describes a proportion of political power held by the poorest $j\%$ of the population in each state. The interpretation is that of an implied, weighted vote. Power is measured by whether and how much of a weight would one have to give to income or wealth so that the observed policy is consistent with voting.

We proceed in two steps. First, we fix given state and define political bias as a weighting parameter that measures the departure from equal representation (political equality). Next, we allow this parameter to vary across states. In a polity with significant inertia, the weights on votes (though not necessarily the de facto distribution of power) will change very little across time.

[10]The Reader can skip this section without losing the main strand of the theory.

### 3.1 Elitist versus Populist Bias

Time subscripts may be dropped for now, with $\omega$ representing a generic state. Define the share of political power allocated to citizen-type $i$ in state $\omega$ by

$$\lambda(i,\omega,\alpha) \;=\; \frac{y(i,\omega)^{\alpha(\omega)}}{\int_0^1 y(j,\omega)^{\alpha(\omega)}dj} \tag{2}$$

Equation (2) describes a weighting system that determines the effective political power of each citizen-type. Because higher $i$-types have higher incomes, political power is increasing in income if $\alpha(\omega) > 0$, decreasing in income if $\alpha(\omega) < 0$, and invariant to income if $\alpha(\omega) = 0$. We allow that $\alpha(\omega)$ can take values in the entire real line.

This specification has a very simple interpretation. Consider policies are determined by some unspecified pairwise voting process. Each time a vote is taken, $\lambda(i,\omega,\alpha)$ is $i$'s endowment of *vote share* in state $\omega$. The exponent $\alpha(\omega)$ may then be thought of as the geometric weight attached to one's relative wealth, and $1-\alpha(\omega)$ is the weight attached to equal vote share or *equal representation* in voting. To see this more transparently, write (2) as

$$\lambda(i,\omega,\alpha) \;=\; \frac{y(i,\omega)^{\alpha(\omega)}1^{1-\alpha(\omega)}}{\int_0^1 y(j,\omega)^{\alpha(\omega)}1^{1-\alpha(\omega)}dj}$$

When $\alpha(\omega) = 0$, the polity may be said to be *unbiased* in the sense that each person's vote, hence their political weight in the distribution, is the same. We will refer to the $\alpha(\omega) > 0$ case as an *elitist bias* since power accrues to the wealthy elite. For instance when $\alpha(\omega) = 1$ then an individual who possesses twice as much wealth as another has twice as many votes, hence twice as much political power.

The case of $\alpha(\omega) < 0$ is referred to as a *populist bias* since political power is redistributed away from wealth. In either case, the absolute value $|\alpha(\omega)|$ measures the intensity of the bias. The cases where $|\alpha(\omega)| > 1$ are particularly stark since this indicates a distribution of power that disproportionately rewards the fringes of the distribution. Extreme inequality occurs in the limit as $|\alpha(\omega)| \to \infty$.

As with income inequality, political inequality can be similarly by measured by a "Political Lorenz curve,"

$$L^P(j,\omega,\alpha) \;=\; \int_0^j \lambda(i,\omega,\alpha)di \tag{3}$$

which gives the proportion of *political power* held by the lowest $j\%$ of types in state $\omega$.

Figure 1 displays the Political Lorenz curve, i.e., the graph of $L^P$, when the bias is elitist. Specifically the case of $0 < \alpha(\omega) < 1$ is displayed. Wealthier individuals have greater political weight than do poorer individuals, however, their increased weight is smaller than

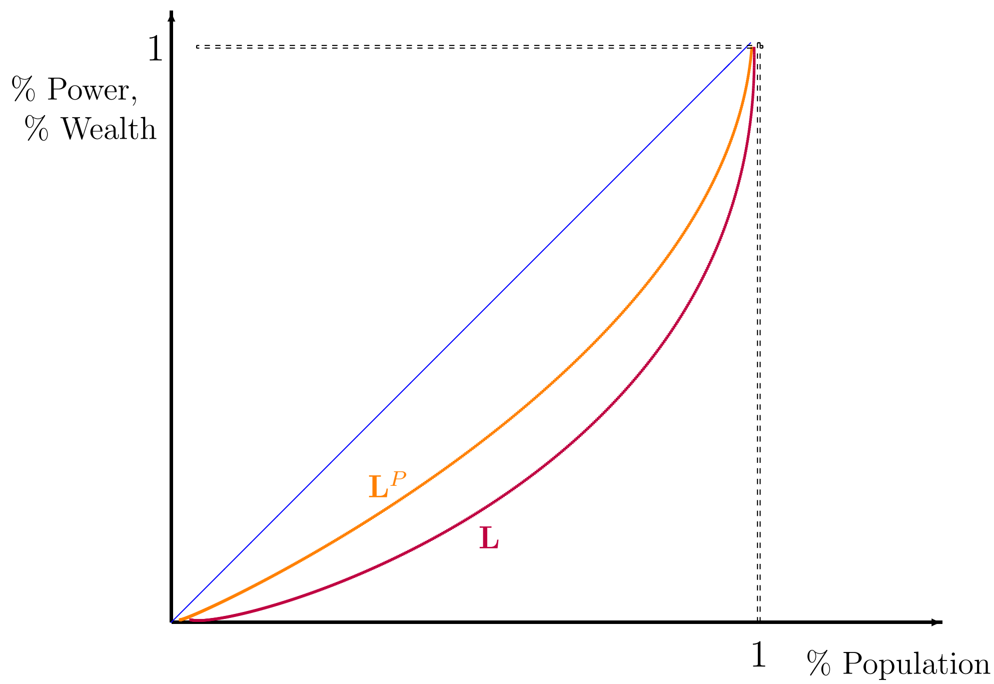



Figure 1: Political Lorenz Curve with an Elitist Bias

their weight in the income distribution. Political inequality therefore lies somewhere between income inequality and full equality. From the assumptions on $y(\cdot)$, it is not difficult to show that, changes in the state imply unambiguous changes in political inequality just as with income inequality. Graphically this means that the Political Lorenz curves corresponding to two different states $\omega$ and $\omega'$ do not cross.

Figure 2 displays the Political Lorenz curve when $\alpha(\omega) > 1$. In that case the elitist bias is more pronounced, with political inequality that exceeds income inequality in the degree that the wealthy are accorded power.

Figure 3 illustrates the case of a populist bias. When $\alpha(\omega) < 0$ the Political Lorenz curve is a "mirror image" of one with bias $-\alpha(\omega) > 0$. Most theories we are aware of predict an elitist bias if any. Nevertheless, it does not seem sensible to rule out the $\alpha(\omega) < 0$ case, a priori.

Notice that for any fixed state, the Political Lorenz curve is monotone increasing in the geometric weight $\alpha(\omega)$. For this reason, there exists a "pivotal" type $i = \mu(\omega, \alpha)$ in the income distribution that implicitly solves

$$L^P(\mu(\omega, \alpha), \omega, \alpha) \;=\; \frac{1}{2} \tag{4}$$

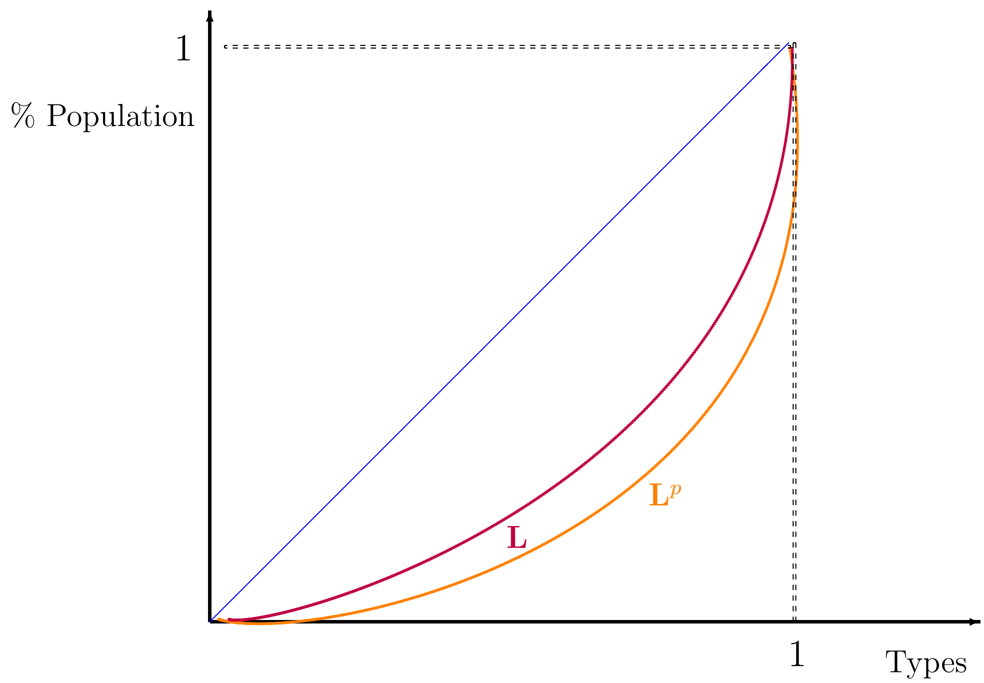



Figure 2: Political Lorenz Curve with Pronounced Elitist Bias

That is, $\mu(\omega, \alpha)$ is the weighted median type according to a distribution of power defined by wealth-weight $\alpha(\omega)$. It is easy to show that this pivotal type is increasing in $\alpha(\omega)$, meaning that the pivotal voter is wealthier, the more elitist the bias weights. For instance, the determination of $\mu(\omega, \alpha)$ is shown in Figure 4 for a particular $\alpha(\omega) > 0$. Clearly, since $\alpha$ is latent, the pivotal type is as well. However, it will sometimes prove more convenient to consider inference over $\mu(\omega, \alpha)$ rather than over $\alpha$ directly.

Notice finally that political inequality, as measured by $L^P$, can change over time. It can change for two distinct reasons. First, changes in the state directly affect income inequality, and political inequality is a function of income inequality. Second, even if income inequality does not change, political inequality can change because the wealth weight $\alpha(\omega)$ itself varies with changes in the state.

## 3.2 Rationalizing a Policy Rule

So far, political inequality has been implicitly associated with a weighted vote share. To make this idea operational, one must also describe how these vote shares are used. The special case of $\alpha(\omega) = 0$ illustrates this well. Each individual is allocated one vote in a simple majority

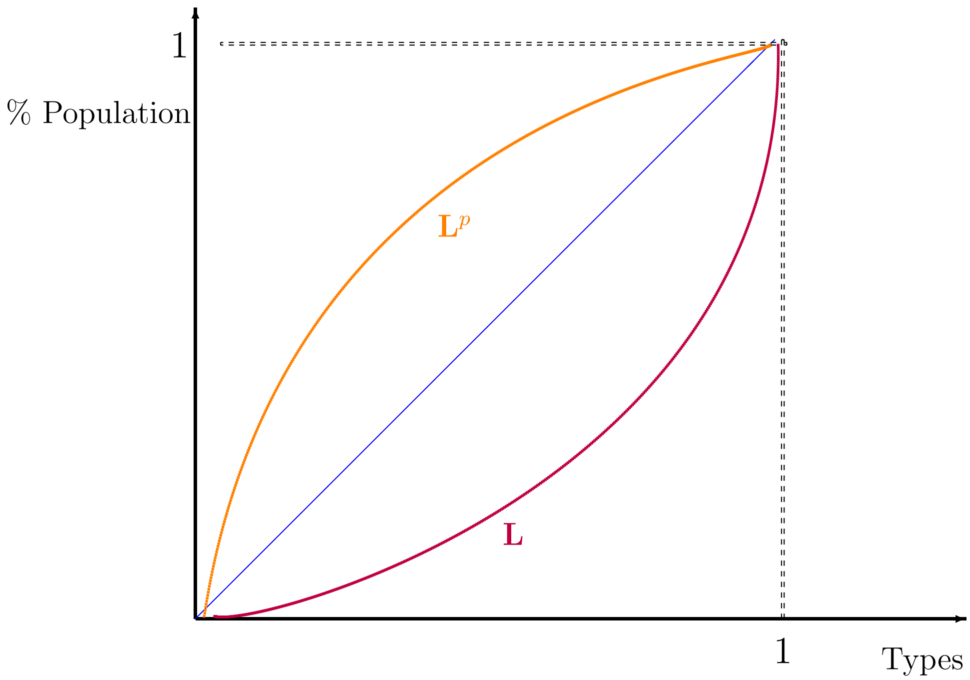



Figure 3: Political Lorenz Curve with Populist Bias

vote between any pair of policies. A Majority Winning (sometimes referred to as Condorcet Winning) policy exists if that policy survives against all others in a simple majority vote. This idea can be extended to any weighted voting rule with $\alpha(\omega) \neq 0$.

We first introduce notation for voters who prefer one policy vis-a-vis another. For any policy rule $\Psi$, any state $\omega$, and any arbitrary pair of policies $a$ and $\hat{a}$, let

$$B(\omega, a, \hat{a};\ U, \Psi) \ \equiv\ \{i:\ U(i, \omega, a; \Psi) > U(i, \omega, \hat{a}; \Psi)\} \tag{5}$$

This notation will be used throughout the paper. Our equilibrium restriction is now stated in the following definition.

**Definition 1** Given a policy rule $\Psi$, a policy $a$ is an $\alpha$-*Weighted Majority Winner (WMW)* in state $\omega$ under payoff function $U$ if, for all policies $\hat{a}$,

$$\int_{B(\omega, a, \hat{a};\ U, \Psi)} \lambda(i, \omega, \alpha)\ di \ \ \geq \ \ 1/2$$

In other words, an $\alpha$-weighted majority winner, or $\alpha$-WMW, is a policy that survives against all others in a majority vote when each type $i$ is allocated $\lambda(i, \omega, \alpha)$ votes and long run payoffs are given by $U$.

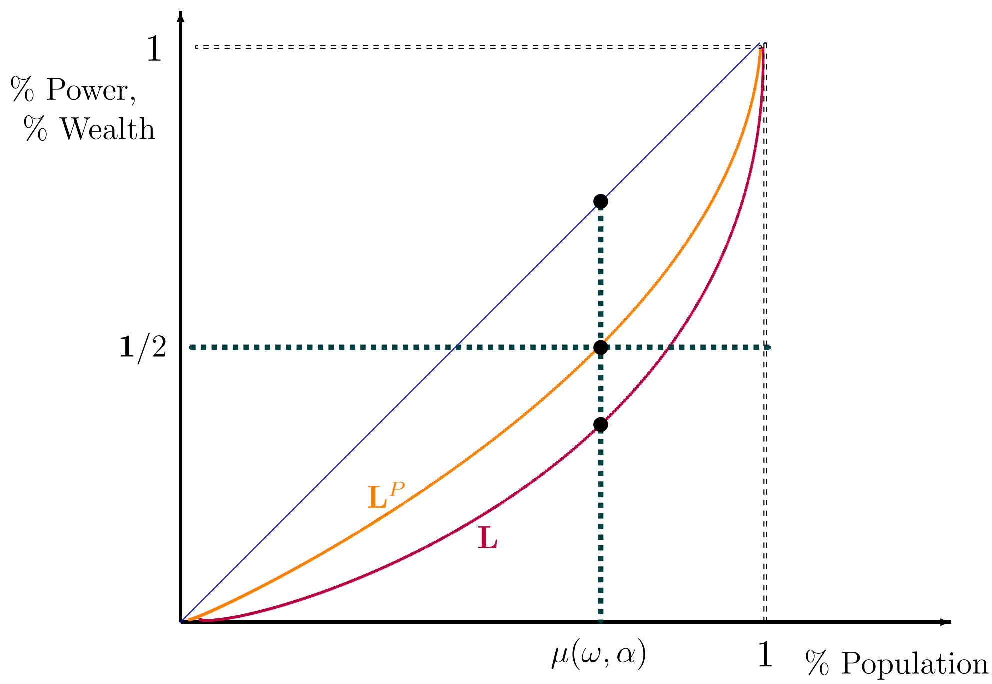



Figure 4: Identifying a Pivotal Voter under an Elitist Bias

If the payoff function $U$ were known precisely to the outside observer, then the political distribution of power that rationalizes $\Psi$ is indeed be pinned down by $\Psi$. But because $U$ is not known, it natural to ask whether a policy rule $\Psi$ might be "rationalized" by the weighting function $\alpha$ for at least some $U$ in a class preferences. Formally,

**Definition 2** A weighting function $\alpha$ *rationalizes the policy rule* $\Psi$ *in the feasible preference class* $\mathcal{U}$ if there exists a payoff function $U \in \mathcal{U}$ such that for all $\omega$, $\Psi(\omega)$ is an $\alpha$-weighted majority winner under $U$.

## 3.3 An "Anything Goes" Theorem

**Theorem 1** *Every policy rule* $\Psi$ *can be rationalized by every weighting function* $\alpha$ *in the feasible preference class* $\mathcal{U}$.

According to Theorem 1, without specific information about preference orderings, the policy data does not tell us anything about political bias, whether it exists or whether its magnitude is large. Since, among all other $\alpha$, the unbiased weight $\alpha(\omega) = 0 \ \forall\omega$ can also rationalize $\Psi$ we cannot rule out the possibility of a completely unbiased polity.

The Proof of Theorem 1 (given below) makes use of a modified, dynamic version of the standard Median Voter Theorem. We state this without proof in the Lemma below since it follows directly from the single crossing property (A2).

**Lemma** *A weighting function $\alpha$ rationalizes the policy rule $\Psi$ in the feasible preference class $\mathcal{U}$ iff there exists a payoff function $U \in \mathcal{U}$ such that for all $\omega$,*

$$\Psi(\omega) \ \in \ arg\max U(\mu(\omega,\alpha),\omega;\Psi)$$

**Proof of Theorem 1**. First, notice that as direct consequence of the single crossing property (A2), for any policy rule $\Psi$, and any two policies $a$ and $\hat{a}$ such that $a < \hat{a}$. Then the set $\{i : \ U(i,\omega,a;\Psi) \geq U(i,\omega,\hat{a};\Psi)\}$ is either empty or of the form $[0,j]$ for some $0 \leq j \leq 1$. That is, the set of individuals who prefer the smaller policy always constitutes the poorer interval of citizen-types.

Let $U$ be a long run payoff function defined by

$$U(i,\omega,a) = -\ \frac{1}{2}\mu(\omega,\alpha)[a^2 - (\Psi(\omega))^2] \ + \ i\Psi(\omega)[a - \Psi(\omega)]$$

where, recall, $\mu(\omega,\alpha)$ is the pivotal type defined by (4). We now verify that $U \in \mathcal{U}$.

First, observe that $U$ is clearly concave in $a$ and continuous in $i$. The first order condition for citizen $i = \mu(\omega,\alpha)$ gives $\Psi(\omega) = a$ as the solution to max $U$. Moreover, $U$ satisfies single crossing in $(i,a)$, and so the preferences are ordered by citizen-types. Higher types (wealthier individuals) prefer higher policies, as required. Hence, by the Lemma, the fact that $\Psi(\omega)$ is the preferred policy by the weighted median type $i = \mu(\omega,\alpha)$ in each state implies that $\alpha$ rationalizes $\Psi$, provided that the remaining property (A3) on $U$ can be verified.

Given the assumptions on $y$, there is an inverse function $i = h(\omega,y)$ such that $h$ is increasing in $y$ and $\omega$ and has increasing differences in $y$ and $\omega$. To verify (A3), we now find the flow payoff $u$ as the difference:

$$\begin{aligned} u(\omega,y,a) \ &= \ U(h(\omega,y),\omega,a) \ - \ \delta U(h(\omega,y),Q(\omega,a),\Psi(Q(\omega,a))\ ) \\ &= \ -\ \tfrac{1}{2}\mu(\omega,\alpha)[a^2 - (\Psi(\omega))^2] \ + \ h(\omega,y)\Psi(\omega)[a - \Psi(\omega)] \end{aligned} \tag{6}$$

Hence $U \in \mathcal{U}$ and we conclude the proof. ∎

## 3.4 Monotone Comparative Restrictions

At this stage there are two possible ways one could rule out certain bias weights. First, one could add direct information about specific binary rankings. Such information could come, for instance, from polls. We consider this option in the next Section. Before proceeding with that option, however, it is worth exploring a second option: that the class of preferences considered here is "too large." Is there a sensible subset $\mathcal{U}^* \subset \mathcal{U}$ such that there are $\alpha$ that *cannot* rationalize the policy data $\Psi$ in $\mathcal{U}^*$? One possible, though by no means only, requirement is that an individual's bliss rule is monotone, increasing in the value of the state. Formally, consider

(A4) $U$ satisfies single crossing in the pair $(\omega, a)$ for each $i$.

Assumption (A4) implies that every type's most preferred policy is (weakly) increasing in the state. This monotonicity restriction is fairly common when the policy is a complementary input in the production process. A standard example is investment in human capital. In growing economies, citizens prefer greater investment.

Let $\mathcal{U}^* \subset \mathcal{U}$ be the set of payoff functions that satisfy (A1)-(A4). If $U \in \mathcal{U}^*$ and $\tilde{\Psi}(i, \omega_t)$ is policy that maximizes $U(i, \omega_t, a_t; \Psi)$, then it straightforward to show that $\tilde{\Psi}(i, \omega_t)$ is increasing in both $i$ and $\omega_t$. The following result shows that Theorem 1 does not hold in the more restricted class $\mathcal{U}^*$.

**Theorem 2** *A weighting function $\alpha$ rationalizes $\Psi$ in the preference class $\mathcal{U}^*$ only if there exist $\bar{\beta}, \underline{\beta}$ with $\bar{\beta} \geq \underline{\beta} \geq 0$ such that $\forall\ \omega' > \omega$,*

$$\begin{aligned} \Psi(\omega') - \Psi(\omega) &\geq \underline{\beta}\,[\mu(\omega', \alpha) - \mu(\omega, \alpha)] \quad \textit{whenever}\ \mu(\omega', \alpha) > \mu(\omega, \alpha) \\ \Psi(\omega') - \Psi(\omega) &\geq \bar{\beta}\,[\mu(\omega', \alpha) - \mu(\omega, \alpha)] \quad \textit{whenever}\ \mu(\omega', \alpha) < \mu(\omega, \alpha) \end{aligned} \tag{7}$$

*Moreover, if (7) holds with $\bar{\beta} = \underline{\beta}$, then the weighting function $\alpha$ rationalizes $\Psi$ in the preference class $\mathcal{U}^*$.*

Notice that if the policy rule is always increasing, i.e., $\Psi(\omega') > \Psi(\omega)$ for all $\omega' > \omega$ then $\Psi$ can be rationalized by any $\alpha$ since in that case $\bar{\beta} = \underline{\beta} = 0$ satisfies (7). However, if $\Psi$ is ever decreasing, there may be some $\alpha$ for which no such $\bar{\beta}$ exists (see Corollaries below).

**Proof**. We first prove the "only if" part. Suppose $\alpha$ rationalizes $\Psi$ in $\mathcal{U}^*$ as hypothesized. In particular, let $U \in \mathcal{U}^*$ denote the preference profile under which $\alpha$ rationalizes $\Psi$. Define

$$\tilde{\Psi}(i, \omega) \in \arg\max U(i, \omega, a;\ \Psi)$$

By (A4), $\tilde{\Psi}(i, \omega)$ is weakly increasing in $\omega$ and in $i$, and by the definition of rationalizing, $\Psi(\omega) = \tilde{\Psi}(\mu(\omega, \alpha), \omega)$ for all $\omega$. Next, define

$$\underline{\beta} = \inf_{\{\omega', \omega:\ \omega' > \omega\}} \frac{\tilde{\Psi}(\mu(\omega', \alpha), \omega') - \tilde{\Psi}(\mu(\omega, \alpha), \omega')}{\mu(\omega', \alpha) - \mu(\omega, \alpha)}$$

and

$$\underline{\beta} \;=\; \sup_{\{\omega',\omega:\, \omega'>\omega\}} \frac{\tilde{\Psi}(\mu(\omega',\alpha),\omega') - \tilde{\Psi}(\mu(\omega,\alpha),\omega')}{\mu(\omega',\alpha) - \mu(\omega,\alpha)}$$

(where we set the term inside sup{} to one if both numerator and denomenator are zero). Clearly, the monotonicity of $\tilde{\Psi}$ implies that $\bar{\beta} \geq \underline{\beta} \geq 0$. Let $\mu(\omega',\alpha) > \mu(\omega,\alpha)$. Then

$$\begin{aligned}
& \Psi(\omega') \;-\; \Psi(\omega) \\
= \;& \tilde{\Psi}(\mu(\omega',\alpha),\omega') \;-\; \tilde{\Psi}(\mu(\omega,\alpha),\omega) \\
= \;& [(\tilde{\Psi}(\mu(\omega',\alpha),\omega') - \tilde{\Psi}(\mu(\omega,\alpha),\omega')\ )] + [\tilde{\Psi}(\mu(\omega,\alpha),\omega') - \tilde{\Psi}(\mu(\omega,\alpha),\omega)\ ] \\
\geq \;& \underline{\beta}[\mu(\omega',\alpha) - \mu(\omega,\alpha)\ ] + [\tilde{\Psi}(\mu(\omega,\alpha),\omega') - \tilde{\Psi}(\mu(\omega,\alpha),\omega)\ ] \\
\geq \;& \underline{\beta}[\mu(\omega',\alpha) - \mu(\omega,\alpha)]
\end{aligned}$$

Similarly, let $\mu(\omega',\alpha) < \mu(\omega,\alpha)$. Then

$$\begin{aligned}
& \Psi(\omega') \;-\; \Psi(\omega) \\
= \;& \tilde{\Psi}(\mu(\omega',\alpha),\omega') \;-\; \tilde{\Psi}(\mu(\omega,\alpha),\omega) \\
= \;& [(\tilde{\Psi}(\mu(\omega',\alpha),\omega') - \tilde{\Psi}(\mu(\omega,\alpha),\omega')\ )] + [\tilde{\Psi}(\mu(\omega,\alpha),\omega') - \tilde{\Psi}(\mu(\omega,\alpha),\omega)\ ] \\
\geq \;& \bar{\beta}[\mu(\omega',\alpha) - \mu(\omega,\alpha)\ ] + [\tilde{\Psi}(\mu(\omega,\alpha),\omega') - \tilde{\Psi}(\mu(\omega,\alpha),\omega)\ ] \\
\geq \;& \bar{\beta}[\mu(\omega',\alpha) - \mu(\omega,\alpha)]
\end{aligned}$$

To show sufficiency, suppose that the expressions in (7) hold for some $\beta = \bar{\beta} = \underline{\beta} \geq 0$. Now construct $U$ as follows. Define

$$U \;=\; [\beta(i - \mu(\omega,\alpha)\ ) + \Psi(\omega)]a \;-\; \frac{1}{2}a^2$$

Notice that F.O.C. implies $a = \beta(i - \mu(\omega,\alpha)\ ) + \Psi(\omega)$ and, in particular, $a = \Psi(\omega)$ whenever $i = \mu(\omega,\alpha)$ as required for $\alpha$ to rationalize $\Psi$. It remains to show that $U$ satisfies (A1)- (A4).

Clearly (A1) is trivial. (A2) is straightforward as well since $\beta \geq 0$. To verify (A4), observe that for $a' > a$,

$$\begin{aligned}
& U(i,\omega,a';\Psi) - U(i,\omega,a;\Psi) \\
= \;& [\beta(i - \mu(\omega,\alpha)\ ) + \Psi(\omega)](a' - a) - \tfrac{1}{2}(a'^2 - a^2)
\end{aligned}$$

But this last expression is increasing in $\omega$ by (7). Finally, (A3) can easily be verified by setting the flow payoff as in (6) in the Proof of Theorem 1. ∎

An important special case of a weighting system satisfying the conditions of the Theorem is the unbiased benchmark, $\alpha(\omega) = 0$ for all $\omega$. The following are a straightforward consequences of the Theorem and demonstrates that monotone comparative statics restrictions have bite.

**Corollary 1** *The unbiased weighting system cannot rationalize a decreasing policy rule* $\Psi$*.*

**Corollary 2** *(Increasing Inequality and Elitism) Suppose that income inequality is increasing in the state. Then any decreasing policy rule* $\Psi$ *(i.e.,* $\Psi(\omega') < \Psi(\omega)$*) cannot be rationalized by any increasingly elitist (i.e.,* $\alpha(\omega') > \alpha(\omega) > 0$*) bias function* $\alpha$*.*

**Corollary 3** *(Decreasing Inequality and Populism) Suppose that income inequality is decreasing. Then any decreasing policy rule* $\Psi$ *cannot be rationalized by any decreasingly populist (i.e.,* $\alpha(\omega') < \alpha(\omega) < 0$*) bias function* $\alpha$*.*

Significantly, (7) minimally requires that the policy be evaluated at two different points in time. In other words, a purely static model would reveal nothing about $\alpha$. Restrictions, when they exist, apply to the dynamic policy path.

# 4 Revealed Political Power and the Power of Polls

External information about policy preferences often exists in the form of polls. This section examines how simple aggregate data from polls might reveal information about political bias.

Consider the following scenario. Each period $t$, a poll is taken in which citizens are asked to compare the actual policy choice $\Psi(\omega_t)$ to some small collection of fixed alternatives in the feasible policy set $A$. Typically, these alternatives are some much discussed policy alternatives, always on the table but not necessarily adopted.

We examine the case of two anonymous binary polls that ask individuals to rank $\Psi(\omega)$ against each of two alternatives, $\bar{a}$ and $\underline{a}$ such that $\underline{a} < \Psi(\omega) < \bar{a}$. Policy alternative $\underline{a}$ can be thought of as the "left wing" alternative to the chosen policy, $\bar{a}$ the "right-wing" alternative. For tractability, these polls are assumed to be accurate in the sense that the sampling error is ignored.

Since preferences are fully summarized by the state, the poll data may be summarized by a pair $(p, q)$ of Markov functions that give fractions $p(\omega)$ and $q(\omega)$ of the population that weakly prefer the weighted-majority winner $\Psi(\omega)$ to the alternatives $\bar{a}$ and $\underline{a}$, respectively, in each state $\omega$.

Since underlying payoff function $U$ that generates the poll data is itself unobservable, the poll data must be consistent with both $U$ and the observable policy data $\Psi$. The following definition makes use of the notation defined in Equation (5).

**Definition 3** A weighting function $\alpha$ *jointly rationalizes a policy rule* $\Psi$ *and poll data* $(p, q)$ *in the feasible preference class* $\mathcal{U}$ if there exists a long run payoff function $U \in \mathcal{U}$ such that

(i) the policy rule $\Psi(\omega)$ is an $\alpha$-weighted majority winner under payoff function $U$, and

(ii) $U$ satisfies

$$\begin{aligned} p(\omega) &= |B(\omega, \Psi(\omega), \bar{a};\ U, \Psi)|, \quad \textit{and} \\ q(\omega) &= |B(\omega, \Psi(\omega), \underline{a};\ U, \Psi)| \end{aligned} \tag{8}$$

Part (i) is the equilibrium requirement as before. Part (ii) is a consistency requirement. It requires that the long run payoff function $U$ must reflect preferences that can generate the observed poll data $(p, q)$ given $\Psi$.

In the next subsection, we examine necessary conditions for political weights rationalizing policy rules and poll data.

## 4.1 Necessary and Sufficient Conditions

From the single crossing property (A2), it follows that the poorest $p(\omega)\%$ prefer $\Psi(\omega)$ to alternative $\bar{a}$, and the the richest $q(\omega)\%$ prefer $\Psi(\omega)$ to alternative $\underline{a}$. Formally, from the definition in Equation (5),

$$\begin{aligned} B(\omega, \Psi(\omega), \bar{a};\ U, \Psi) &= [0, p(\omega)], \quad \textit{and} \\ B(\omega, \Psi(\omega), \underline{a};\ U, \Psi) &= [1 - q(\omega), 1] \end{aligned}$$

whenever these sets are nonempty. In fact, the assumption that $\Psi$ must be a weighted-majority winner implies certain restrictions on $p(\omega)$ and $q(\omega)$.

First, observe that

$$1 - q(\omega) \leq p(\omega) \tag{9}$$

If this were not the case then a nonempty interval $[p(\omega), 1 - q(\omega)]$ exists, consisting of types that weakly prefer both the smallest policy, $\underline{a}$, and the largest, $\bar{a}$, to $\Psi(\omega)$. This, in turn, is a contradiction of the strict concavity assumption (A1) on $U$.

Second, notice that for any $0 < p(\omega) < 1$, the bias weight $\alpha(\omega)$ cannot be too large since otherwise the richest $(1 - p(\omega))$ fraction could have used its political power to veto $\Psi(\omega)$, in which case $\Psi(\omega)$ could not have been a WMW. This puts an upper bound on $\alpha(\omega)$. To find the upper bound, consider the "worst case" against $\Psi(\omega)$ in which the fraction $(p(\omega), 1]$ who prefer the right-wing alternative $\bar{a}$ is largest. For that to happen, $p(\omega)$ must be pivotal: $p(\omega) = \mu(\omega, \alpha)$.

Third and finally, notice that for any $0 < q(\omega) < 1$, the bias cannot be too small. Otherwise, the poor could have vetoed the WMW, $\Psi(\omega)$, by opting for the left wing alternative $\underline{a}$. This then implies a lower bound on $\alpha(\omega)$. Using the same logic as before, the lower bound can be found by assuming that $1 - q(\omega)$ is pivotal: $1 - q(\omega) = \mu(\omega, \alpha)$.

Together, these three facts establish upper and lower bounds for the set of rationalizing weights $\alpha(\omega)$. To find these bounds, notice from the implicit definition of $\mu$ in (4), that $L^P$ is strictly decreasing in the weight $\alpha(\omega)$. This means that the equation $\mu(\omega, \alpha)$ for the pivotal type is invertible. Therefore, let $M(j, \omega)$ denote the inverse of $\mu$ (fixing the state $\omega$) so that $\alpha(\omega) = M(\mu(\omega, \alpha), \omega)$ holds by definition. The arguments above then imply that only the types in the interval $[1 - q(\omega), p(\omega)]$ can be pivotal. Consequently, $M(1 - q(\omega), \omega) \leq \alpha(\omega) \leq M(p(\omega), \omega)$. As it turns out, these inequalities provide a full characterization for rationalization of a policy rule in the presence of polling data.

**Theorem 3** *Let $\Psi$ be any policy rule for which $\underline{a} < \Psi(\omega) < \bar{a}$ for all $\omega$ and let $(p, q)$ be the corresponding poll data relating $\Psi(\omega)$ to $\bar{a}$ and $\underline{a}$, resp. Then a weighting function $\alpha$ jointly rationalizes policy rule $\Psi$ and poll data $(p, q)$ in $\mathcal{U}$ if and only if*

$$M(1 - q(\omega), \omega) \ \leq \ \alpha(\omega) \ \leq \ M(p(\omega), \omega), \ \ \forall \ \omega \tag{10}$$

*with strict inequality in one of the two inequalities above.*

The "sufficiency" part of the proof, i.e., the construction of a $U$ under which $\Psi$ and $(p, q)$ are jointly rationalized, appears in the Appendix.

It is easily verified that $M$ is strictly increasing in $j$, and so the condition $1 - q(\omega) \leq p(\omega)$ also implies that the interval of biases that can rationalize $\Psi$ is nonempty. We refer to this interval $[M(1 - q(\omega), \omega), M(p(\omega), \omega)]$ as the *bias band.* Figure 4 expresses a graph of $M$ and the bias band. The bounds of the band are are displayed on the vertical axis. In the graph, the range of bias band includes 0, the unbiased weight. It also includes a subinterval of elitist biases, as well as a subinterval of populist ones. Given properties of $M$, some simple comparative statics facts can easily be discerned.

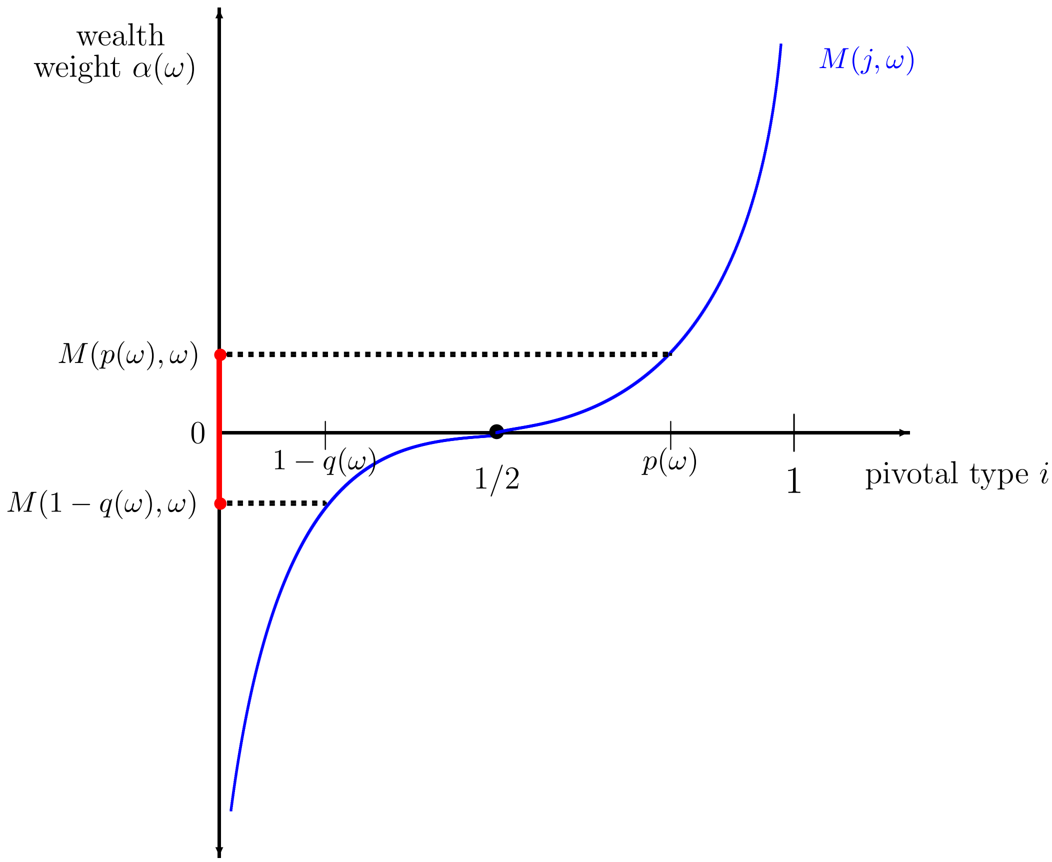



Figure 5: Bias Band and Bounding Function

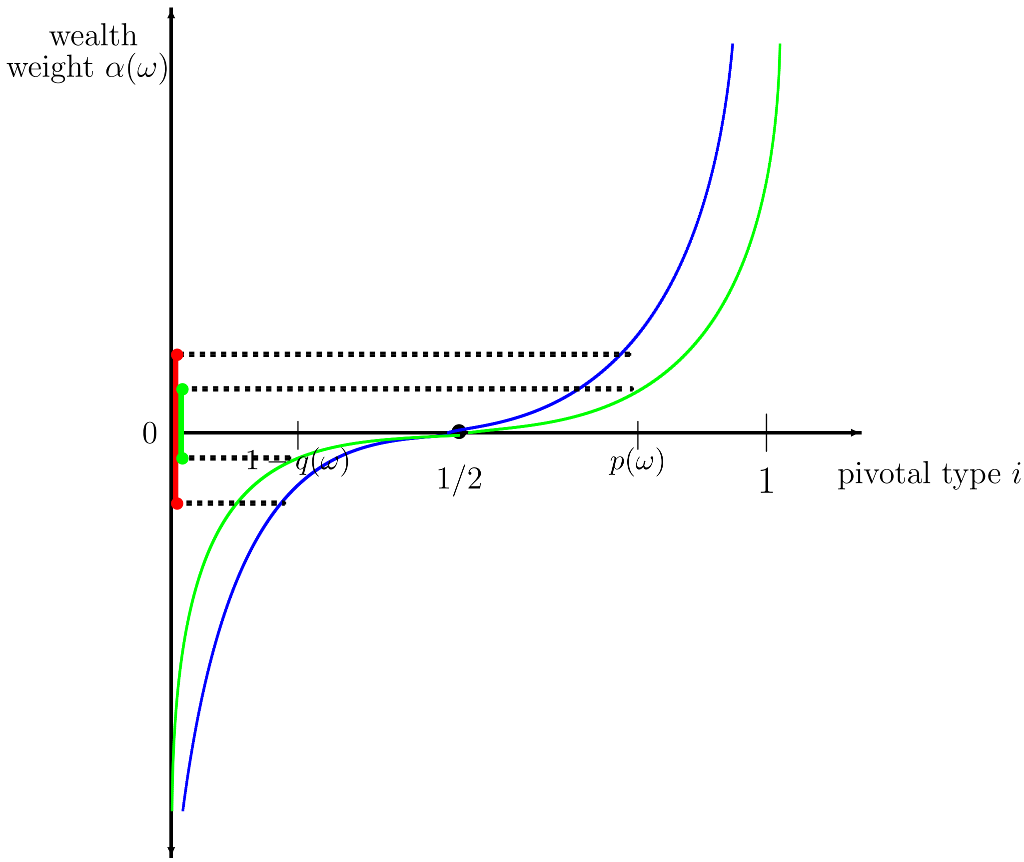
**Theorem 4** *Fix $\omega$, $\Psi$ and poll data $(p,q)$. Then*

*(i)* $M(1-q(\omega),\omega) > 0$ *whenever* $q(\omega) < 1/2$ *in which case the polity is known to express an elitist bias.*

*(ii)* $M(p(\omega),\omega) < 0$ *whenever* $p(\omega) < 1/2$ *in which case the polity is known to express a populist bias.*

*(iii) The larger is income inequality then the smaller is* $|M(j,\omega)|$ *for any fixed* $j$*. In particular, if* 0 *(the unbiased weight) belongs to the band, then larger income inequality reduces the size of the band around 0.*

Other things equal, increases in income inequality have an equalizing effect politically. When 0 is an admissible weight, then the band shrinks around it. If the band is entirely above 0 (elitism) then it moves closer to 0. This is not surprising since in that case, the pro-wealth bias must be lower to have off-set the greater income inequality. Somewhat more surprising is the fact that when the band is entirely below 0 (populism), greater inequality moves the band closer to 0 as well. In other words the band becomes less populist implying that wealthier individuals receive increased political weight from the bias in addition to increased weight from income alone. Why? Because with a populist system, political inequality is a weighted mirror image of income inequality. Hence, holding the bias weight constant, an increase in relative income of the top 10% translates into an weighted decrease in this group's political power. The bias weight must therefore increase to offset this fall in political power due to income change. This dual effect of greater income inequality is displayed in Figure 5.




In addition to state-by-state bounds implied by polling data, the polls may impose some dynamic restrictions as well. Observer that the Political Lorenz curve can change over time for two reasons. First, changes in the state directly affect income inequality, and power is wealth-weighted. Hence, political inequality changes with income inequality. Second, Political Lorenz curve changes as the bias $\alpha(\omega_t)$ changes.

The inferential problem is complicated in either case by the fact that the distribution of policy preferences changes as the state varies over time. To illustrate this suppose the popularity of the policy choice has increased relative to $\bar{a}$, i.e., $p(\omega_{t+1}) > p(\omega_t)$. Popularity increased as the state changed from $\omega_t$ to $\omega_{t+1}$. On the one hand, it could be due to the fact that the taste distribution on the policy set $A$ has shifted away from $\bar{a}$. On the other hand, it could be due to a change in the distribution of political power. For instance, suppose that bias is fixed such that $\alpha(\omega_{t+1}) = \alpha(\omega_t) \neq 0$. Suppose income inequality decreased, i.e., $L(j, \omega_{t+1}) < L(j, \omega_t)\ \forall j$. Clearly, political inequality must have decreased as well. Holding tastes constant, the fact that $p(\omega_{t+1}) > p(\omega_t)$ is therefore explained by the fact that the new chosen policy in $t+1$ is more widely supported than the old policy at date $t$.

Generally, the two explanations: a change in tastes versus a change in political inequality are hard to decouple. Sometimes, spatial information can be used sort them out. One simple extreme case occurs when $1 - q(\omega_{t+1}) > p(\omega_t)$. That is, the individuals who prefer $\underline{a}$ to $\Psi(\omega_{t+1})$ in state $\omega_{t+1}$ is more numerous than those who prefer $\Psi(\omega_t)$ to $\bar{a}$ in state $\omega_t$. Though it sounds unintuitive, this condition implies that the distribution of ideal points has, roughly speaking, shifted to the left. It is straightforward to show

**Theorem 5** *Suppose that income inequality remains stable. Then:*

*(i) the bias in $t+1$ is unambiguously more elitist than in $t$ if there are individuals who supported both $\bar{a}$ in date $t$ and $\underline{a}$ in $t+1$ (i.e., if there are right-wing dissidents in $t$ who become left-wing dissidents in $t+1$);*

*(ii) the bias in $t+1$ is unambiguously more populist than in $t$ if there are individuals who supported both $\underline{a}$ in date $t$ and $\bar{a}$ in $t+1$. (i.e., if there are left-wing dissidents in $t$ who become right-wing dissidents in $t+1$).*

**Proof**. Consider (i). A type $i$ who supports $\bar{a}$ in date $t$ means that $i > p(\omega_t)$. If that type same supports $\underline{a}$ in $t+1$ then $i < 1 - q(\omega_{t+1})$. Hence, $i \in [p(\omega_t), 1 - q(\omega_{t+1})]$. Consequently, the date $t$ bias band, given by $[M(1-q(\omega_t), \omega_t), M(p(\omega_t), \omega_t)]$ lies entirely below the date $t+1$ bias band $[M(1 - q(\omega_{t+1}), \omega_{t+1}), M(p(\omega_{t+1}), \omega_{t+1})]$. By Theorem 3, $\alpha(\omega_{t+1}) > \alpha(\omega_t)$. In other words, the bias becomes more elitist in $t+1$.

Now consider (ii). A type $i$ who supports both $\underline{a}$ in date $t$ and $\bar{a}$ in $t+1$ means that $i \in [p(\omega_{t+1}), 1 - q(\omega_t)]$. Using the same reasoning as in (i), this implies that the date $t$ bias band lies entirely *above* the date $t+1$ bias band. Thus by Theorem 3, $\alpha(\omega_{t+1}) < \alpha(\omega_t)$ so that the bias becomes more populist in $t+1$. ∎

## 4.2 A Parametric Example

Let an income process with a Pareto Distribution be

$$y(i, \omega) = \left(\frac{1}{1-i}\right)^{\omega} = (1-i)^{-\omega}.$$

Here $\left(\frac{1}{1-i}\right) \in [1,\infty]$ is the capital unit of type $i$, while $\omega$ is the capital intensity of production determined by the available technology level $\omega$. It is easy to see that $y(i,\omega)$ is increasing in $(i,\omega)$. In addition, it is straightforward to check that $\log y$ has increasing differences and so income inequality increases in $\omega$.

The parametric form for $y$ gives rise to a power share for type $i$ given by

$$\lambda(i,\omega,\alpha) \;=\; y(i,\omega)^{\alpha(\omega)} \;=\; (1-i)^{-\alpha(\omega)\omega}.$$

To make $\lambda(i,\omega,\alpha)$ integrable, we need to have $\alpha(\omega) < \frac{1}{\omega}$. This implies

$$\int_0^j \lambda(i,\omega,\alpha)\,di = \int_0^j (1-i)^{-\alpha(\omega)\omega}\,di = \frac{1-(1-j)^{1-\alpha(\omega)\omega}}{1-\alpha(\omega)\omega}.$$

Hence the Political Lorenz curve is

$$L^p(j,\omega,\alpha) = \frac{\int_0^j \lambda(i,\omega,\alpha)\,di}{\int_0^1 \lambda(i,\omega,\alpha)\,di} = 1-(1-j)^{1-\alpha(\omega)\omega}.$$

The pivotal voter rule $\mu(\omega,\alpha)$ is given by

$$\mu(\omega,\alpha) \;=\; 1-2^{\frac{1}{\alpha(\omega)\omega-1}}$$

The inverse function, used to determine the bias from a pivotal voter is given by

$$M(i,\omega) \;=\; \frac{1}{\omega}\left(1+\frac{\log 2}{\log(1-i)}\right).$$

Notice that $M(i,\omega)=0$ if $i=\frac{1}{2}$, and $M(i,\omega)\to\frac{1}{\omega}$ if $i\to 1$, and $M(i,\omega)\to-\infty$ if $i\to 0$. From this, the bias band is given by

$$\begin{aligned} & [M(1-q(\omega),\omega),\; M(p(\omega),\omega)] \\ = & \left[\frac{1}{\omega}\left(1+\frac{\log 2}{\log q(\omega)}\right),\; \frac{1}{\omega}\left(1+\frac{\log 2}{\log(1-p(\omega))}\right)\right] \end{aligned}$$

# 5 Extended Discussion

This paper adapts ideas from revealed preference theory to understand political bias. At present, there are (to us) two clear limitations in the present setup.

The first is the generous use of the single crossing restriction (Assumption (A2)) in order to generate preferences that are well ordered by wealth/income. Although the single crossing assumption is standard in voting theory, in our case preferences are single peaked by (A1), and so WMWs would exist even in the absence of single crossing.[11] Hence in principle, the theory can be articulated without it. The interpretation, however, loses most of its force since there would be no systematic way in that case to relate the income/wealth distribution to the political power distribution.

The second limitation is the restriction to a specific functional form — the geometric mean — to determine the vote/power allocation. As with the single crossing restriction, the basic theory as well as some of the results, including Theorem 1, can be readily articulated without the restriction. To see this, consider:

$$\lambda\left(i, \omega, \alpha\right) = g\left(y\left(i, \omega\right), \alpha\right),$$

where, as before, $\alpha$ is a map from $\omega$ to the reals measuring the extent of wealth bias in each state. As a power allocation rule, the following desirable properties of the function are required:

(a) **non-negativity:** $\lambda\left(i, \omega, \alpha\right) \geq 0$ for every $y\left(i, \omega\right)$ and $\alpha$, i.e., everybody can only have a non-negative vote shares.

(b) **monotonicity in $y\left(i, \omega\right)$:** when viewed as a function $g : y \to \lambda$, $g\left(y\left(i, \omega\right); \alpha\right)$ is increasing if $\alpha(\omega) > 0$; decreasing if $\alpha(\omega) < 0$; constant if $\alpha(\omega) = 0$. This captures the idea that a positive $\alpha(\omega)$ corresponds to an elitist bias, a negative $\alpha(\omega)$ corresponds to a populist bias, and $\alpha(\omega) = 0$ captures an unbiased polity.

The inequality under $\lambda\left(i, \omega\right)$ is naturally measured by a Political Lorenz curve

$$L^p\left(j; \omega, \alpha\right) = \frac{\int_0^j \lambda\left(i, \omega\right) di}{\int_0^1 \lambda\left(i, \omega\right) di}.$$

Following the literature, the order of function $L^p\left(j; \omega, \alpha\right)$ is naturally defined as the Lorenz order. As an inequality measure of vote distribution, we require the following desirable properties of $L^p\left(j; \omega, \alpha\right)$:

(c) **increasing property in $\alpha$:** for each fixed $\omega$, when viewed as a function $L^p\left(j; \omega, \alpha\right) : \alpha(\omega) \to L^p$, $L^p\left(j; \omega, \alpha\right)$ is increasing in $\alpha(\omega)$. This captures the idea that a higher positive $\alpha(\omega)$ corresponds to a larger elitist bias, and a more negative $\alpha(\omega)$ corresponds to a higher populist bias.

[11] We remark that even in the absence of single peaked preferences and/or multi-dimensional, it is possible to articulate a well defined theory. In such as case, a weaker equilibrium notion, the *weighted minmax maority winner* always exists. Formally, a policy $a$ is a *Weighted-Minmax Majority Winner (WMMW)* in state $\omega$ if

$$\int_{\{i:\, U(i,\omega,a;\Psi) \geq U(i,\omega,\hat{a};\Psi)\}} \lambda(i, \omega, \alpha)\ di \quad \geq \int_{\{i:\, U(i,\omega,a';\Psi) \geq U(i,\omega,\hat{a}';\Psi)\}} \lambda(i, \omega, \alpha)\ di$$

for all $\hat{a}$, $a'$ and $\hat{a}'$. It's straightforward to show that WMMW coincides with WMW whenever the latter exists.

(d) **monotonicity in $\omega$:** for each fixed $\hat{\alpha} = \alpha(\omega)$, when viewed as a function $L^p(j;\omega,\alpha)$ : $\omega \to L^p$, $L^p(j;\omega,\alpha)$ is monotone (increasing/decreasing) in $\omega$. This is desirable since it gives a way to relate income/wealth inequality to political inequality for a fixed political bias weight.

Properties (a)-(d) determine a general class of Political Lorenz functions, each function representing a one-parameter family of Lorenz curves. The results then go through for this general model.

# 6 Appendix

**Proof**. Given the structure of $M$, (10) implies

$$1 - q(\omega) \;\leq\; \mu(\omega,\alpha) \;\leq\; p(\omega)$$

with strict inequality somewhere in the string and where $\mu(\omega,\alpha)$ is defined, as before, $L^P(\mu(\omega,\alpha),\omega;\alpha) = 1/2$. We now construct a long run payoff function $U \in \mathcal{U}$ as follows. Let

$$U(i,\omega,a;\Psi) = -\frac{1}{2}\left(a^2 - \Psi^2(\omega)\right) + f(i;\omega)\left(a - \Psi(\omega)\right), \tag{11}$$

where $f(i;\omega)$ is a bivariate function in $(i,\omega)$. Notice that this $U$ easily satisfies (A1). It may also be shown to satisfy (A3) using the same argument as in Proof of Theorem 1. Hence it remains to show that $f(i;\omega)$ can be constructed in a way that satisfies Assumption (A2) and the equilibrium requirement that $\Psi$ is a $\alpha$-WMW. These conditions imply that $U$ must satisfy the following four restrictions:

First, $U$ needs to satisfy the equilibrium requirement. From the First Order conditions

$$f(i;\omega) = a \tag{12}$$
$$\Longrightarrow \quad f(\mu(\omega,\alpha);\omega) = \Psi(\omega). \tag{13}$$

Second, $U$ must satisfy the polling equation for $\overline{a}$. If the support rate for $\Psi(\omega)$ against $\overline{a}$ is $p(\omega)$. Under increasing difference property, it implies that the pivotal decision maker is $p(\omega)$. Then we have

$$U(p(\omega),\omega,\overline{a}) = U(p(\omega),\omega,\Psi(\omega)) = 0 \tag{14}$$
$$\Longrightarrow \quad -\frac{1}{2}\left(\overline{a}^2 - \Psi^2(\omega)\right) + f(p(\omega);\omega)\left(\overline{a} - \Psi(\omega)\right) = 0 \tag{15}$$
$$\Longrightarrow \quad f(p(\omega);\omega) = \frac{1}{2}\left(\overline{a} + \Psi(\omega)\right). \tag{16}$$

Third, $U$ must satisfy the polling equation for $\underline{a}$. Let the polling support rate for $\Psi(\omega)$ against $\underline{a}$ equal to $q(\omega)$. Under increasing difference property, it implies that pivotal decision maker is $(1 - q(\omega))$. We have

$$U(1 - q(\omega), \omega, \underline{a}) = U(1 - q(\omega), \omega, \Psi(\omega)) = 0 \tag{17}$$

$$\Longrightarrow \quad -\frac{1}{2}\left(\underline{a}^2 - \Psi^2(\omega)\right) + f(1 - q(\omega); \omega)(\underline{a} - \Psi(\omega)) = 0 \tag{18}$$

$$\Longrightarrow \quad f(1 - q(\omega); \omega) = \frac{1}{2}(\underline{a} + \Psi(\omega)). \tag{19}$$

Fourth, $U(i, \omega, a)$ needs to satisfy increasing difference property (A2), i.e., $f(i; \omega)$ is increasing in $i$ for each $\omega$.

To summarize, the four restrictions lead to the following equation system

$$\begin{pmatrix} f(1 - q(\omega); \omega) \\ f(\mu(\omega, \alpha); \omega) \\ f(p(\omega); \omega) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(\underline{a} + \Psi(\omega)) \\ \Psi(\omega) \\ \frac{1}{2}(\overline{a} + \Psi(\omega)) \end{pmatrix}, \tag{20}$$

such that $f(i; \omega)$ is increasing. In other words, it is a standard interpolation problem at three data points $\left(1 - q(\omega), \frac{1}{2}(\underline{a} + \Psi(\omega))\right)$, $(\mu(\omega, \alpha), \Psi(\omega))$ and $\left(p(\omega), \frac{1}{2}(\overline{a} + \Psi(\omega))\right)$, with the class of interpolants as increasing functions. Notice that $\frac{1}{2}(\underline{a} + \Psi(\omega)) < \Psi(\omega) < \frac{1}{2}(\overline{a} + \Psi(\omega))$.

There are numerous ways to construct such an $f$. For instance a piecewise linear spline easily works. We omit the remaining details and conclude the proof. ∎

# References


[1] Acemoglu, D. and J. Robinson (2008), "Persistence of Powers, Elites and Institutions," *American Economic Review*, 98: 267-293.

[2] Afriat, S. (1967), "The Construction of a Utility Function from Expenditure Data," *International Economic Review*, 8: 67-77.

[3] Austen-Smith, D. (1987), "Interest Groups, Campaign Contributions, and Probabilistic Voting", *Public Choice* 54: 123-139.

[4] Bartels, L. (2005), "Partisan Politics and the U.S. Income Distribution," mimeo, Princeton University.

[5] Benabou, R. (2000), "Unequal Societies: Income Distribution and the Social Contract," *American Economic Review*, 90(1): 96-129.

[6] Boldrin, M. and L. Montruchio (1986), "On the Indeterminacy of Capital Accumulation Paths," *Journal of Economic Theory*, 40: 26-39.

[7] Bourguignon, F., and T. Verdier (2000), "Oligarchy, Democracy, Inequality and Growth," *Journal of Development Economics*, 62: 287-313.

[8] Brown, D. and R. Matzkin (1996), "Testable Restrictions on the Equilibrium Manifold", *Econometrica*, 64, 1249-62.

[9] Campante, F. (2008), "Redistribution in a Model of Voting and Campaign Contributions," mimeo, Harvard University.

[10] Chiappori, P.-A. and J.-C. Rochet (1987), "Revealed Preference and Differentiable Demand, *Econometrica*, 55: 687-91.

[11] Coate, S. (2004), "Political Competition with Campaign Contributions and Informative Advertising", *Journal of the European Economic Association*, 2: 772-804.

[12] Debreu, G. (1974). "Excess Demand Functions," *Journal of Mathematical Economics*, 1: 1521.

[13] Gans, J. and M. Smart (1996), "Majority voting with single-crossing preferences," *Journal of Public Economics*, 59: 219-237.

[14] Grandmont, J.-M. (1978): "Intermediate Preferences and the Majority Rule," *Econometrica*, 46(2): 317-330.

[15] Grossman, G. and E. Helpman (1996), "Electoral Competition and Special Interest Policies", *Review of Economic Studies*, 63: 265-286.

[16] Grossman, G. and E. Helpman (2001), *Special Interest Politics*, Cambridge, MA: MIT Press.

[17] Mantel, R. (1974), "On the characterization of aggregate excess demand," *Journal of Economic Theory* 7: 348353.

[18] Prat, A. (2002), "Campaign Spending with Office-Seeking Politicians, Rational Voters, and Multiple Lobbies", *Journal of Economic Theory*, 103: 162-189.

[19] Richter, M. (1966)...

[20] Rothstein, P. (1990), "Order Restricted Preferences and Majority Rule," *Social Choice and Welfare*, 7: 331-42.

[21] Sonnenschein, H. (1973), "Do Walras' Identity and Continuity Characterize the Class of Community Excess Demand Functions?", *Journal of Economic Theory* 6: 345354.

[22] Varian, H. (1982), "The Non-Parametric Approach to Demand Analysis," *Econometrica*, 50: 945-974.