Georgetown University
Economics 101 - Microeconomic Theory
Final Exam - May 11, 2011

Instructions. The exam period is two hours. Answer all questions.

No phones, calculators, notes, books or other reference materials may be used during the exam.

Part A (35 points)

Give short answers to the following questions.

1. (5 points) Robinson Crusoe makes apples and bananas according to the production possibility frontier $a^2 + b^2 = 100$. He has preferences for apples and bananas given by the utility function $u(a, b) = a^{\frac{1}{2}} + b^{\frac{1}{2}}$. Crusoe can buy or sell apples and bananas. If the prices are $p_A = p_B = 2$, what is the value of Crusoe's marginal rate of substitution at the optimal consumption bundle?

Answer: The marginal rate of substitution equals 1. Since $p_A = p_B$, the budget line has slope -1.

2. (5 points) Explain why moral hazard and adverse selection in an insurance market might lead some individuals to purchase a contract with a deductible.

Answer: When there is moral hazard, the insured may have insufficient incentive to exercise care. When there is adverse selection, offering the Pareto efficient contracts would lead high-risk types to buy the policy intended for the low-risk types. To stop high-risk types from making such a purchase, the policy offered to low-risk types must involve a deductible.

3. (5 points) Two developers plan to put up condominium apartment buildings and then sell the units at the market-clearing price. If firm 1 builds first, how does its profit compare to its Cournot equilibrium profit? Explain.

Answer: Firm 1 could choose its Cournot equilibrium output, thereby inducing firm 2 to choose its Cournot equilibrium output. If firm 1 chooses something different, its profit from doing so must be higher.

4. (5 points) Two individuals have the utility function $u(x, m) = \sqrt{x} + m$. Good X produces an externality. What does the Coase Theorem say about the impact of property rights in this case?

Answer: The Coase Theorem says that the level of X does not depend on how property rights are assigned. The assignment of property rights only affects how much money (m) people have.

5. (5 points) A retailer bargains with a supplier as follows. First, the retailer offers a payment, x. Next, the supplier accepts or rejects the offer. If the supplier accepts, the retailer gets a payoff of $12 - x$ and the supplier gets $x - 6$. If the supplier rejects, the retailer and the supplier both get a payoff of zero. What offer does the retailer make in a subgame-perfect Nash equilibrium and what are the equilibrium payoffs?

Answer: Solve the game backwards, beginning with the supplier's turn. The supplier will accept any $x \geq 6$. Now consider the retailer. Any $x > 6$ leaves the retailer worse off than $x = 6$; any $x < 6$ is also worse as it gives her a payoff of zero. Thus, the retailer will offer $x = 6$ and the supplier will accept. The retailer gets a payoff of $12 - 6 = 6$ while the supplier gets 0.

6. (5 points) Good X and good Y are perfect complements sold by independent monopolists. Will the sum of the prices, $p_X + p_Y$, change if the two monopolists begin to collude and charge prices that maximize their combined profit? Explain.

Answer: The sum of the prices will fall. The independent monopolists charge prices that are higher since each firm ignores the negative externality that it imposes on the other.

7. (5 points) One-half of consumers are willing to pay \$16 for an entree and \$4 for dessert; the others are willing to pay \$12 for an entree and \$8 for dessert. If the marginal cost of an entree is \$5 and the marginal cost of a dessert is \$2, what is the optimal pricing scheme for a monopolist? Explain.

Answer: The monopolist will maximize profit by selling the goods as a bundle, for a price of \$20. This realizes all gains from trade, and the monopolist extracts all surplus.

Part B (65 points)

Provide solutions to the following longer questions.

1. A monopolist sells prescription drugs in the U.S. and Canada. In the U.S., the quantity demanded is $Q = 20 - 2P$. In Canada, it is $Q = 12 - 2P$. Marginal cost is $MC = 2$. There are no fixed costs.

(i) (5 points) Find the profit-maximizing quantities under first-degree price discrimination.

Answer: It is optimal to sell the Pareto efficient quantities. Setting $P = MC$, we get $Q = 20 - 2(2) = 16$ in the U.S. and $Q = 12 - 2(2) = 8$ in Canada.

(ii) (5 points) Find the profit-maximizing prices under third-degree price discrimination.

Answer: The demand curve in each country is of the form $P = A - BQ$. Letting marginal cost (= average cost) be C, profit is

$$\Pi(Q) = (A - BQ - C)\,Q.$$

It is maximized when

$$\Pi'(Q) = A - 2BQ - C = 0 \Rightarrow Q = \frac{A - C}{2B}.$$

Price is then

$$P = A - B\frac{A - C}{2B} = \frac{A + C}{2}.$$

Since $Q = 20 - 2P$ in the U.S., the demand curve is $P = 10 - \frac{Q}{2}$. We therefore have $P = 6$ in the U.S. Likewise, $Q = 12 - 2P$ in Canada so the demand curve is $P = 6 - \frac{Q}{2}$. We therefore have $P = 4$ in Canada.

(iii) (5 points) Now suppose that a regulation requires that the monopolist charge the same price in each country. Find the profit-maximizing uniform price.

Answer: For $P > 6$, only the U.S. consumes. Setting $P > 6$ is not optimal since profit from the U.S. alone is higher when $P = 6$.

Now consider $P \leq 6$. We need to determine aggregate demand. The aggregate demand is $Q = (20 - 2P) + (12 - 2P) = 32 - 4P$. This means the demand curve is $P = 8 - \frac{Q}{4}$. From above, the optimal uniform price is $P = \frac{8+2}{2} = 5$.

(iv) (5 points) Repeat (iii) for the case of $MC = 4$.

Answer: When $P = 8 - \frac{Q}{4}$ (which happens when selling to both markets), the monopolist would set $P = \frac{8+4}{2} = 6$, but then only the U.S. consumes. The optimal price when selling to the U.S. alone is $P = \frac{10+4}{2} = 7$, so $P = 7$ is optimal here.

2. Larry and Anna make books and DVDs. Each year, Larry is able to make 5 books or 10 DVDs. Anna is able to make 10 books or 10 DVDs in a year.

(i) (5 points) On a single diagram, draw Larry's and Anna's production possibility frontiers, with DVDs on the vertical axis.

Answer: Anna's has intercepts at (10,0) and (0,10). Larry has intercepts at (5,0) and (0,10).

(ii) (5 points) Draw the aggregate production possibility frontier. (Label the intercepts and kinks, if any.)

Answer: It has intercepts at (15,0) and (0,20). The kink is at (10,10).

(iii) (5 points) If the price of a book is 9 and the price of a DVD is 4, what does Larry produce and what does Anna produce?

Answer: Both produce books.

3. Consider a labor market in which 100 individuals are willing to work. One-half of them have high ability; the others have low ability. Low-ability workers are willing to work for \$100 per day, and high-ability workers are willing to work for \$200 per day. Employers are willing to pay low-ability workers \$120 per day and high-ability workers \$220 per day.

(i) (5 points) If the employer can tell who is who, how many people will be employed?

Answer: All 100 workers will be employed.

(ii) (5 points) If the employer cannot tell who is who, and must therefore pay the same daily wage to all, which workers will find a job in equilibrium?

Answer: The employer is willing to pay $\$\left(\frac{1}{2}\times 120+\frac{1}{2}\times 220\right) = \$170$ for a random worker. The high-ability workers would not be willing to work at that wage so only the low-ability workers will work.

(iii) (5 points) Suppose that workers could enhance their productivity through education. Would high-ability workers take the Pareto efficient amount of education?

Answer: High-ability workers might take the Pareto efficient amount of education; however, if low-ability workers would mimic that level in order to get the higher wage, the high-ability types would take just enough additional education so that low-ability types would not mimic them.

4. This question asks you to analyze various games.

(i) (5 points) Find all Nash equilibria and corresponding payoffs of the following game.

		Roger	
		Forehand	Backhand
Rafael	Forehand	50,50	80,20
	Backhand	80,20	50,50

Answer: There cannot be a pure-strategy equilibrium. If Rafael selects Forehand with probability p, Roger gets expected payoffs of $50p + 20(1-p)$ and $20p + 50(1-p)$ from Forehand and Backhand, respectively. Indifference requires $50p + 20(1-p) = 20p + 50(1-p) \Rightarrow 20 + 30p = 50 - 30p \Rightarrow p = \frac{1}{2}$. If Roger selects Forehand with probability q, Rafael gets expected payoffs of $50q + 80(1-q)$ and $80q + 50(1-q)$ from Forehand and Backhand, respectively. Indifference requires $80 - 30q = 50 + 30q \Rightarrow q = \frac{1}{2}$. Thus, each player selects Forehand and Backhand with probability $\frac{1}{2}$.

Rafael has an expected payoff of $80 - 30q = 50 + 30q = 65$ while Roger has an expected payoff of $50p + 20(1-p) = 20p + 50(1-p) = 35$.

(ii) (5 points) Find all Nash equilibria and corresponding payoffs of the following game.

		Bella	
		Left	Right
Alex	Up	1,0	0,1
	Down	2,2	1,2

Answer: Alex has a dominant strategy of playing Down, so he will play Down. Bella is then indifferent between Left and Right. There are two pure-strategy equilibria: (Down,Left) and (Down,Right). Since Bella is indifferent, there are also mixed-strategy equilibria. The equilibria have Alex playing Down and Bella playing Left with probability q, where $0 \leq q \leq 1$. Bella always has a payoff of 2 whereas Alex has an expected payoff of $2q + 1(1-q) = 1 + q$ when Bella plays Left with probability q.

(iii) (5 points) Suppose that the following game is played for an infinite number of periods.

		Bellissimo	
		Low	High
Apex	Low	1,1	4,0
	High	0,4	2,2

The players discount the future at the rates δ_A and δ_B, respectively. Under what conditions can the players sustain the outcome $(2,2)$ in every period?

Answer: Assume that B follows the strategy of playing High unless someone ever played Low in the past. If A plays High every period, it gets a discounted payoff of

$$2 + 2\delta_A + 2\delta_A^2 + ... = \frac{2}{1-\delta_A}.$$

If A plays Low this period (and Low thereafter), it gets

$$4 + \delta_A + \delta_A^2 + ... = 4 + \frac{\delta_A}{1-\delta_A}.$$

The former exceeds the latter if

$$\frac{2}{1-\delta_A} > 4 + \frac{\delta_A}{1-\delta_A} \Rightarrow 2 > 4(1-\delta_A) + \delta_A \Rightarrow \delta_A > \frac{2}{3}.$$

Using the symmetric argument for B, we see that the outcome (2,2) can be sustained if $\delta_A > \frac{2}{3}$ and $\delta_B > \frac{2}{3}$.