Georgetown University
Economics 101 - Microeconomic Theory
Final Exam - May 6, 2008 12:30-2:30 PM

Instructions. The exam period is two hours. Answer all questions.

No calculators, cell phones, notes, books, or other reference materials may be used during the exam.

Part A (35 points)

Give short answers to the following questions.

1. (5 points) How can a meet-the-competition clause help to sustain collusion?

Answer: It could help because it gives customers the incentive to report low price offers from other firms. In other words, it employs customers to monitor whether other firms are cutting price. An alternative answer is that it provides a commitment by firms to sell at a lower price if a competitor ever lowers price. This reduces the competitor's incentive to lower price.

2. (5 points) Suppose that one individual imposes a negative externality on another. The individuals have quasi-linear preferences. Explain why the equilibrium level of the externality-producing activity is independent of the allocation of property rights.

Answer: With quasi-linear preferences, the contract curve in the Edgeworth box is horizontal presuming the externality-producing activity is measured on the vertical axis. The allocation of property rights only affects how much of the other good (money) people have.

3. (5 points) Consider an exchange economy with two individuals, Ann and Bill. There are two goods, X and Y. Ann's endowment is $(4, 6)$ and Bill's is $(6, 6)$. Ann has the utility function

$$u(x, y) = \min\{x, y\},$$

while Bill has the utility function

$$u(x, y) = x + y.$$

Describe the contract curve (verbally, graphically, or with an equation).

Answer: All Pareto efficient points have $x_A = y_A$, so the contract curve has the equation $y_A = x_A$ until it reaches (10,10).

4. (5 points) A market has the demand curve $p = 10 - q$. The market is a duopoly. Each firm has average cost equal to 2. Write down firm 1's profit as a function of the firms' quantities and determine firm 1's best-response function under Cournot competition.

Answer: Firm 1's profit is $\Pi_1(q_1, q_2) = [10 - (q_1 + q_2) - 2]q_1$. The derivative with respect to q_1 is $8 - 2q_1 - q_2$. Setting it equal to zero gives $q_1 = 4 - \frac{1}{2}q_2$.

5. Suppose that a real estate agent is paid a 6% commission for selling a \$1 million house. Describe in general terms an alternative scheme that is better for the seller and generates a higher incentive for effort.

Answer: One example has the agent receiving no compensation for prices below a certain level, but a commission above 6% for amounts above that level.

6. (5 points) George produces and consumes two goods: hamburgers and fries. His production possibility frontier has the equation $h = \sqrt{100 - f^2}$, where h is the number of hamburgers and f is the numbers of servings of fries. George has the utility function $u(h, f) = h^{\frac{2}{3}} f^{\frac{1}{3}}$. If the prices of hamburgers and fries are $p_H = p_F = \$2$, what is the value of George's marginal rate of substitution at the optimal consumption bundle? (George can buy or sell hamburgers and fries.)

Answer: The marginal rate of substitution equals 1. Since $p_H = p_F$, the budget line has slope -1.

7. (5 points) Explain how moral hazard in an insurance market might lead to individuals being induced to purchase incomplete insurance.

Answer: If there is moral hazard, individuals might not exercise the right amount of care. Incomplete insurance will give an incentive to exercise more care.

Part B (65 points)

Provide solutions to the following longer questions.

1. Jill and Jack start a business and agree to work 50 hours per week each. They make cotton candy and donuts. Each hour, Jill is able to make 8 sticks of cotton candy or 8 donuts. Jack is able to make 16 sticks of cotton candy or 8 donuts in an hour.

(i) (5 points) On a single diagram, draw Jill's and Jack's production possibility frontiers, with donuts on the vertical axis. Be sure to label the axes and intercepts.

Answer: Jack's PPF has intercepts of 800 on the horizontal axis and 400 on the vertical. Jill's PPF has intercepts of 400 on both axes.

(ii) (5 points) On a separate diagram, draw the business's aggregate production possibility frontier, again labeling axes and intercepts.

Answer: The aggregate PPF has intercepts of 1,200 on the horizontal axis and 800 on the vertical. It has a kink at the point (800, 400).

(iii) (5 points) If the price of donuts is $1.50 and the price of cotton candy is $1, what quantity of each good will the business produce to maximize its weekly revenue?

Answer: The two segments of the aggregate PPF have slopes of $-\frac{1}{2}$ and -1. The iso-revenue lines have a slope of $-\frac{2}{3}$. Thus, they will produce at the kink, with Jack making 800 sticks of cotton candy and Jill making 400 donuts.

2. Consider the market for used computers. There are 100 computers for sale; half of them are "duds," while the others are "gems." The owner of a dud is willing to sell it for any price above $400, but the owner of a gem is only willing to sell it for at least $1000. Duds are worth $600 to buyers, and gems are worth $1200 to them.

(i) (5 points) At a Pareto efficient allocation, which computers will be sold?

Answer: All will be sold, since the value to a buyer of any given computer is greater than the value to a seller.

(ii) (5 points) If buyers cannot observe the quality of the computer for sale, how much would they be willing to pay for a computer if they believed that all of the computers were being sold?

Answer: If all are sold, the expected value of a computer to a buyer is $\frac{1}{2} \times 600 + \frac{1}{2} \times 1200 = 300 + 600 = 900$.

(iii) (5 points) Which computers will be sold in equilibrium in this market?

Answer: If a price of $900 is offered, sellers of gems will not sell, so the only computers that could be sold are duds.

(iv) (5 points) Now suppose that the fraction g of the computers in the market are gems. If buyers still cannot observe the quality, how large must g be for all computers to be sold in equilibrium?

Answer: Now a buyer's expected value of a computer, given that all are sold, is $(1-g)\times 600 + g\times 1200 = 600 + g\times 600$. Sellers of both types of computers are willing to sell at such a price if it exceeds 1,000; meaning $600 + g\times 600 > 1{,}000 \Rightarrow g > \frac{2}{3}$.

3. A monopolist produces output with constant marginal cost equal to 2. There are 100 consumers of type A and 100 of type B. Consumers of type A have the inverse demand function

$$p_A(x) = 12 - \frac{x}{2},$$

and consumers of type B have the inverse demand function

$$p_B(x) = 10 - x.$$

(i) (5 points) What are the Pareto efficient levels of consumption for types A and B? Call these x_A^* and x_B^*, respectively.

Answer: The Pareto efficient output levels are 20 for A types and 8 for B types. At these points, marginal benefits are equated to marginal costs (i.e., the height of the demand curve equals marginal cost), so $12 - \frac{x_A^*}{2} = 2$ and $10 - x_B^* = 2$.

(ii) (5 points) The monopolist can identify which consumer is which. Suppose that the monopolist charges a price p_A for type As and p_B for type Bs. What are the profit-maximizing prices?

Answer: The prices equate marginal revenue to marginal cost. For group A, the marginal revenue is

$$\begin{aligned} MR_A &= \frac{d}{dx}\left[\left(12 - \frac{x}{2}\right)x\right] \\ &= 12 - x. \end{aligned}$$

The quantity it sells to A types x_A satisfies $12 - x_A = 2$, or $x_A = 10$. The price it charges is $p_A = 12 - \frac{x_A}{2} = 7$.

For B types, the marginal revenue is $MR_B = 10 - 2x$, so the quantity sold is $x_B = 4$, and the price is $p_B = 10 - x_B = 6$.

(iii) (5 points) Now suppose that the monopolist cannot tell which consumer is which, so she offers a pair of purchase options, (x'_A, R'_A) and (x'_B, R'_B). That is, individuals can either purchase a quantity x'_A and pay an amount $\$R'_A$, or a quantity x'_B and pay an amount $\$R'_B$. How do the profit-maximizing quantities compare to the Pareto efficient quantities?

Answer: The monopolist will sell the Pareto efficient quantity to type-A consumers, so $x'_A = 20$. It will need to reduce the amount it sells to type-B consumers below their Pareto efficient level. In particular, it will reduce x'_B to the point where the gain from the type As just matches the loss from type Bs.

(iv) (5 points) Now suppose that the monopolist knows each individual consumer's demand. The monopolist will charge an entry fee plus a per-unit price. That is, it will charge consumer A an entry fee of F_A and a per-unit price $\widehat{p}_A$, and consumer B an entry fee of F_B and a unit price $\widehat{p}_B$. How big is the deadweight loss now?

Answer: There is no deadweight loss as this is first-degree price discrimination. The monopolist charges the same per-unit price to each type of consumer, $\widehat{p}_A = \widehat{p}_B = MC = 2$. It appropriates the total consumer surplus through entry fees, so $F_A = \frac{1}{2} \times (16 \times 8) = 64$, and $F_B = \frac{1}{2} \times (6 \times 6) = 18$.

4. This question consists of two parts.

(i) (5 points) Find all of the Nash equilibria of the following game.

		Player 2	
		Left	Right
Player 1	Up	(5,4)	(1,3)
	Down	(4,1)	(2,2)

Answer: There are two pure-strategy Nash equilibria: (Up, Left) and (Down, Right). The mixed-strategy equilibrium has each player playing both strategies with probability $\frac{1}{2}$. Then, player 1 has an expected payoff of 3 from either strategy while player 2 gets $2\frac{1}{2}$ from either strategy.

(ii) (5 points) Suppose that the following game is played for an infinite number of periods.

		Player B	
		Low	High
Player A	Low	1,1	4,0
	High	0,4	2,2

The players discount the future at the rates δ_A and δ_B, respectively. Find conditions under which the players can sustain the outcome $(2, 2)$ in every period.

Answer: Suppose that B follows the strategy of playing High unless someone ever played Low in the past. If A plays High every period, he gets a discounted payoff of

$$2 + 2\delta_A + 2\delta_A^2 + ... = \frac{2}{1 - \delta_A}.$$

If A plays Low this period (and Low thereafter), he gets

$$4 + \delta_A + \delta_A^2 + ... = 4 + \frac{\delta_A}{1 - \delta_A}.$$

The former exceeds the latter if

$$\frac{2}{1 - \delta_A} > 4 + \frac{\delta_A}{1 - \delta_A} \Rightarrow 2 > 4(1 - \delta_A) + \delta_A \Rightarrow \delta_A > \frac{2}{3}.$$

Using the symmetric argument for B, we see that the outcome (2,2) can be sustained if $\delta_A > \frac{2}{3}$ and $\delta_B > \frac{2}{3}$.