4. Utility

Varian, Chapter 4

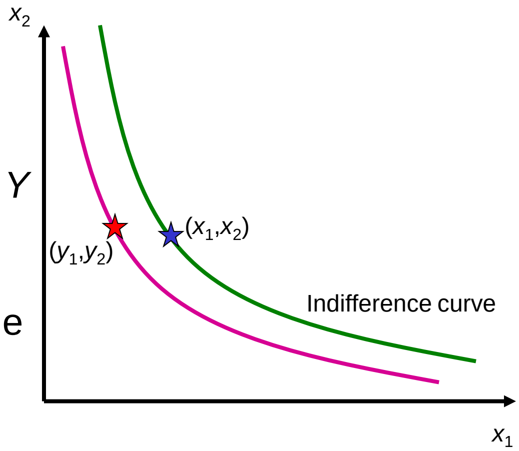
Utility functions

- *Continuity* of the preference relation means that if $X \succ Y$, and Z is sufficiently close to X, then $Z \succ Y$ as well

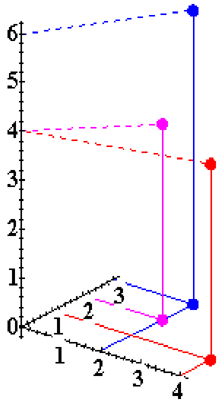
- A preference relation that is complete, reflexive, transitive, continuous and monotonic can be represented by a continuous *utility function*

Utility and Indifference Curves

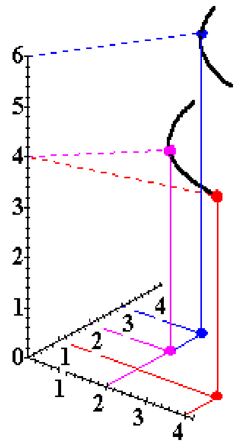
- Preferences can be described by a utility function with $u(X) \geq u(Y)$ if and only if $X \succcurlyeq Y$
- An indifference curve (IC) is a level set of the utility function



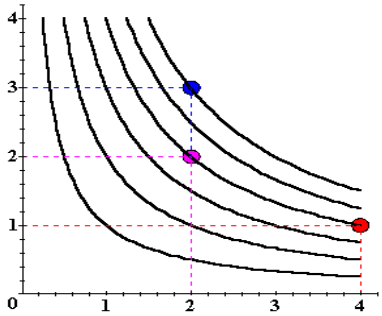
Utility and Indifference Curves



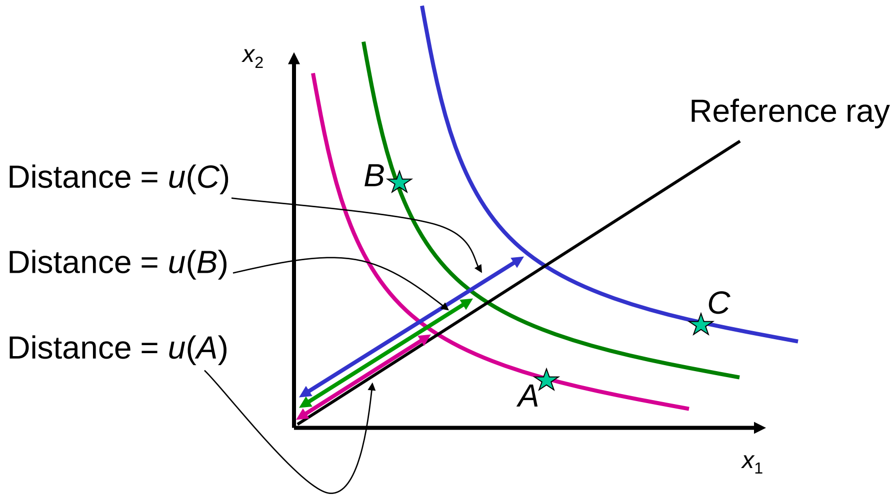
Utility and Indifference Curves



Utility and Indifference Curves

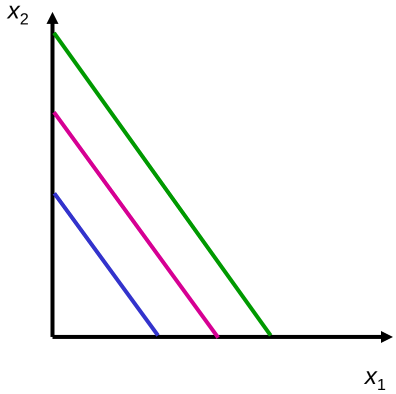


Constructing a utility function



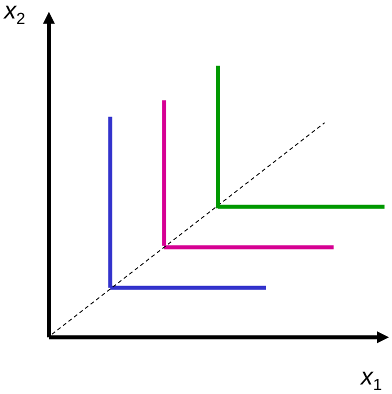
Perfect substitutes

- $u(x_1,x_2) = x_1 + x_2$
- Indifference curve:
  $x_1+x_2 = u_0 \Rightarrow x_2 = u_0-x_1$
- $u^*(x_1,x_2) = x_1+ 5x_2$



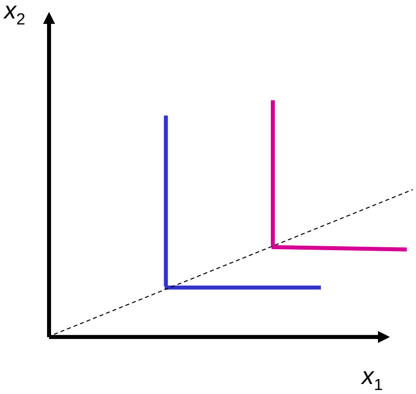
Perfect complements

- $u(x_1,x_2) = \min\{x_1,x_2\}$



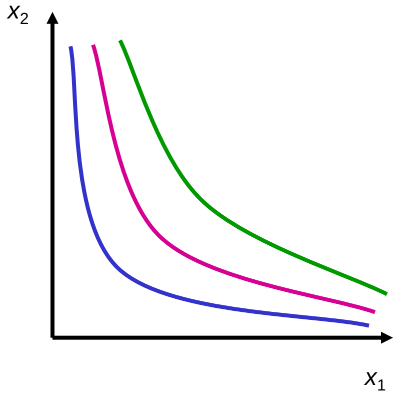
Perfect complements

- $u^*(x_1,x_2) = \min\{x_1,2x_2\}$



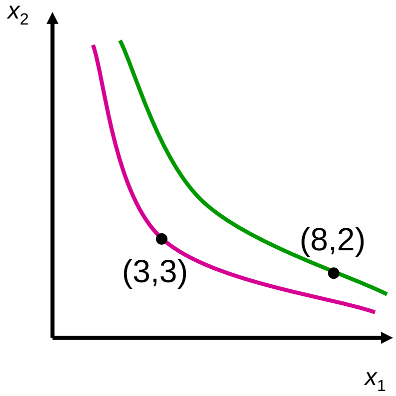
Cobb-Douglas utility

- $u(x_1,x_2) = x_1x_2$
- Indifference curve:
  $x_1x_2 = u_0 \Rightarrow x_2 = u_0/x_1$
- $v(x_1,x_2) = ln(x_1) + ln(x_2)$
- $ln(x_1) + ln(x_2) = v_0$
  $\Rightarrow ln(x_1x_2) = v_0$



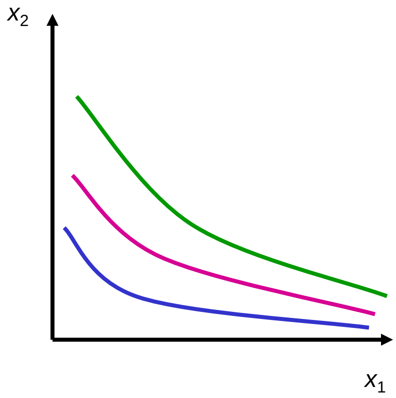
Cobb-Douglas utility (cont.)

- $u(8,2) = 16$
- $x_1x_2 = 16 \Rightarrow x_2 = 16/x_1$
- $u(3,3) = 9$
- $x_1x_2 = 9 \Rightarrow x_2 = 9/x_1$
- $ln(8) + ln(2) = ln(16) >$
  $ln(9) = ln(3) + ln(3)$



General Cobb-Douglas utility

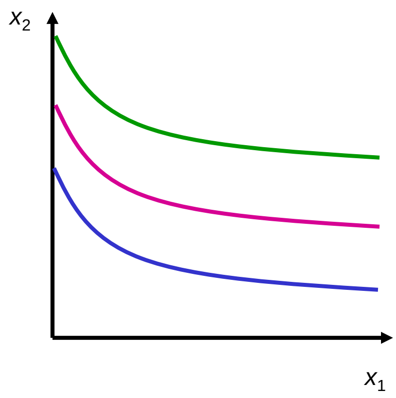
- $u(x_1,x_2) = (x_1)^c\,(x_2)^d$
- $v(x_1,x_2) = c\ln(x_1) + d\ln(x_2)$



Quasi-linear utility

- $u(x_1,x_2) = f(x_1) + x_2$
- Indifference curve:

$$f(x_1) + x_2 = u_0$$
$$\Rightarrow x_2 = u_0 - f(x_1)$$
$$\Rightarrow dx_2/dx_1 = - f'(x_1)$$



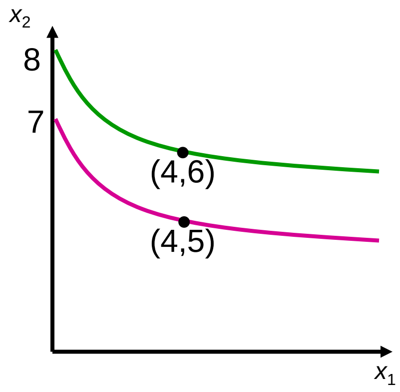
Quasi-linear utility example

- $u(x_1,x_2) = (x_1)^{½} + x_2$
- $u(4,5) = 2 + 5 = 7$
- Indifference curve:

  $(x_1)^{½} + x_2 = 7$

  $\Rightarrow x_2 = 7 - (x_1)^{½}$

  $\Rightarrow dx_2/dx_1 = -½(x_1)^{-½}$



Marginal Utility (*MU*)

- The marginal utility of good 1 at (x_1,x_2) is the rate at which utility increases as the amount of good 1 increases:

$$MU_1 = \partial u(x_1,x_2)/\partial x_1$$

- The marginal utility of good 2 at (x_1,x_2) is:

$$MU_2 = \partial u(x_1,x_2)/\partial x_2$$

MRS and *MU*

- The slope of a level set is the negative of the ratio of the partial derivatives, so the slope of the indifference curve is

$$-\partial u(x_1,x_2)/\partial x_1 \,/\, \partial u(x_1,x_2)/\partial x_2 = -MU_1/MU_2$$

- The marginal rate of substitution is

$$MRS = MU_1/MU_2$$

MRS and *MU* (cont.)

- Heuristic proof:

$$0 = MU_1\Delta x_1 + MU_2\Delta x_2$$

$$\Rightarrow \Delta x_2/\Delta x_1 = -MU_1/MU_2$$

- $MRS = -\Delta x_2/\Delta x_1 = MU_1/MU_2$

Choice of utility functions

- Suppose that *u* is a utility function representing a consumer's preferences
- If *f* is a strictly increasing function (a *monotonic transformation*), preferences are also represented by *v* such that $v(X) = f(u(X))$

Do *MRS* and *MU* depend on the utility function?

- Suppose that $u(x_1,x_2) = x_1^{1/2} x_2^{1/2}$
- $MU_1 = ½x_1^{-1/2} x_2^{1/2}$
- $MU_2 = ½x_1^{1/2} x_2^{-1/2}$
- $MRS = MU_1 / MU_2$

  $= ½x_1^{-1/2} x_2^{1/2}/[½x_1^{1/2} x_2^{-1/2}] = x_2/x_1$

MRS is unaffected by a monotonic transformation

- Suppose that $v(x_1,x_2) = x_1x_2$
- $MU_1 = x_2$
- $MU_2 = x_1$
- $MRS = MU_1 / MU_2 = x_2/x_1$

Quasi-linear utility

- Let $u(x_1,x_2) = f(x_1) + x_2$
- $MRS = f'(x_1)$
- *MRS* does not depend on x_2 since the slope of every indifference curve is the same, for a given value of x_1