Notes on Models with Calvo Price and Wage Setting[1]

M. Canzoneri, R. Cumby and B. Diba

Georgetown University, March 28, 2005

This evolving set of notes documents some models we have been using recently, and the way we solve them using dynare. All of the models have NNS preferences, and Calvo wage and/or price setting. Model 1 has multiple sectors, and no capital; it comes in three versions: "nominal", "real", and "wage & price inflation". As explained below, we moved from one version to another as problems with dynare seemed to dictate. Model 2 has only one sector, but it adds capital and habit, and shows how to add "rule of thumb" agents if desired. Model 3 is a two country model of a monetary union.

Model 1: the Multi Sector Model.

The Basic Multi-sector Framework:

Sectors are characterized by their productivity shock and by the type (wage/price) and degree of their nominal inertia. There is no labor mobility across sectors. Each household is allocated to a sector, and works at all of the firms in that sector. In the flexible wage version of the model, all households in a given sector the identical (despite the fact that firms may charge different prices); so, we can easily aggregate utility in the end. In the sticky wage version of the model, the households in a given sector are different (since they require different wages), and aggregation is more difficult. Workers are allocated across sectors in such a way that sectoral wages equalize in the steady state; there is no incentive for labor to move across sectors in the steady state, and our "no labor mobility" assumption does not create steady state welfare losses. More specifically –

1. There are S sectors indexed by $s \in [1, \dots, S]$. Households are indexed by $h \in [0, 1]$; they are monopolistic suppliers of labor within their sector. H_s – with measure n_s – is the set of households working in sector s; $\sum_{s=1}^{S} n_s = 1$.

---

[1] These notes compliment Canzoneri, Cumby and Diba (2003), and the accompanying "Notes on: Monopolistic Competition and Nominal Inertia", which discuss fundamental aspects of this literature within the simpler framework of one period nominal wage/price "contracts".

2. Firms are indexed by $f \in [0, 1]$; they are monopolistic suppliers of consumption goods. F_s – with measure f_s – is the set of firms in sector s; $\sum_{s=1}^{S} f_s = 1$.
3. Each firm's production is linear in a composite labor input of all the households working in the sector. Sectoral goods are a composite of the output of firms in the sector. And, the final consumption good is a composite of the S sectoral goods.
4. Here, we use Chari, Kehoe and McGrattan's (2000) artifice of a "bundler" to describe the algebra of composite goods. For a more detailed discussion of composite goods, see Canzoneri, Cumby, and Diba, "Notes on Monopolistic Competition and Nominal Inertia", which are available on Canzoneri's webpage.

The composite labor input, and production, in sector s:

(1) $N_{s,\tau}(f) \equiv [n_s^{-1/\phi} \int_{Hs} L_{s,\tau}(h,f)^{(\phi-1)/\phi} dh]^{\phi/(\phi-1)}$, $\phi > 1$ (think of a bundler for each firm f)

$W_{s,\tau} = [n_s^{-1} \int_{Hs} W_{s,\tau}(h)^{1-\phi} dh]^{1/(1-\phi)}$ (household wage does not depend on firm)

$Y_{s,\tau}(f) = Z_{s,\tau} N_{s,\tau}(f)$ (firm's production function)

$L_{s,\tau}(h) = \int_{Fs} L_{s,\tau}(h,f) df$ and $N_{s,\tau} = \int_{Fs} N_{s,\tau}(f) df$ (aggregate hours worked and labor input)

$L^d_{s,\tau}(h,f) = (W_{s,\tau}/W_{s,\tau}(h))^{\phi}(N_{s,\tau}(f)/n_s)$ (demand of bundler for firm f)

$L^d_{s,\tau}(h) = (W_{s,\tau}/W_{s,\tau}(h))^{\phi}(N_{s,\tau}/n_s)$ (integrating demands over f)

where, $W_{s,\tau}(h)$ is the household's wage, $L_{s,\tau}(h,f)$ is hours worked at firm f, $L_{s,\tau}(h)$ is total hours worked, $L^d_{s,\tau}(h) = \int_{Fs} L^d_{s,\tau}(h,f) df$ is total demand for the hours of household h, similar definitions hold for composite inputs (the N's), and $Z_{s,\tau}$ is a stochastic sectoral productivity.

The composite sectoral good, $Y_{s,\tau}$, is given by:

(2) $Y_{s,\tau} \equiv [f_s^{-1/\sigma} \int_{Fs} Y_{s,\tau}(f)^{(\sigma-1)/\sigma} df]^{\sigma/(\sigma-1)}$, $\sigma > 1$

$P_{s,\tau} = [f_s^{-1} \int_{Fs} P_{s,\tau}(f)^{1-\sigma} df]^{1/(1-\sigma)}$

$Y^d_{s,\tau}(f) = (P_{s,\tau} / P_{s,\tau}(f))^{\sigma}(Y_{s,\tau}/f_s)$

The composite final good, Y_τ, is given by:

(3) $Y_\tau \equiv [\sum_{s=1}^{S} \gamma_s Y_{s,\tau}^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$, $\eta > 1$

$P_\tau = [\sum_{s=1}^{S} \gamma_s^{\eta} P_{s,\tau}^{1-\eta}]^{1/(1-\eta)}$,

$Y^d_{s,\tau} = (P_\tau / P_{s,\tau})^{\eta}(Y_\tau/\gamma_s^{-\eta})$ [or equivalently $Y^d_{s,\tau} = (P_\tau / P_{s,\tau})^{\eta}(Y_\tau/\gamma_s^{-\eta})$]

Remarks:

1. For the Cobb-Douglass case, set η very close to one. Dynare has no difficulty with this.
2. Our notation makes a distinction between hours worked by the household (L) and the composite labor input (N). We have not found it necessary to made similar distinctions for the composite consumption goods.
3. The algebra of "bundlers" – leading to the results asserted above – is discussed next.

The algebra of competitive bundlers:

The "bundler" for (say) the sector s composite good is a competitive (zero profit) agent who buys the firm's $Y_{s,t}(f)$ at the price $P_{s,t}(f)$, bundles them into the composite good $Y_{s,\tau} \equiv [f_s^{-1/\sigma}\int_{Fs} Y_{s,\tau}(f)^{(\sigma-1)/\sigma}df]^{\sigma/(\sigma-1)}$, and sells it at the price $P_{s,t}$.

The bundler's cost minimization problem for CES aggregators:

$\min_{Ys,t(f)} \int_{Fs} P_{s,t}(f)Y_{s,t}(f)df \quad \text{s.t.} \quad \bar{Y}_{s,t} = [f_s^{-1/\sigma}\int_{Fs} Y_{s,t}(f)^{(\sigma-1)/\sigma}df]^{\sigma/(\sigma-1)}$

$\mathcal{L} = \int_{Fs} P_{s,t}(f)Y_{s,t}(f)df + \mu\{\bar{Y}_{s,t} - [f_s^{-1/\sigma}\int_{Fs} Y_{s,t}(f)^{(\sigma-1)/\sigma}df]^{\sigma/(\sigma-1)}\}$

Note: Lagrangian multiplier $\mu = MC = P_{s,t}$, since bundler is competitive

First Order condition –

$P_{s,t}(f) = P_{s,t}[f_s^{-1/\sigma}\int_{Fs} Y_{s,t}(f)^{(\sigma-1)/\sigma}df]^{[\sigma/(\sigma-1)]-1} f_s^{-1/\sigma} Y_{s,t}(f)^{[(\sigma-1)/\sigma]-1} = P_{s,t}Y_{s,t}^{1/\sigma} f_s^{-1/\sigma} Y_{s,t}(f)^{-1/\sigma}$

$= P_{s,t}[(Y_{s,t}/f_s)/Y_{s,t}(f)]^{1/\sigma}$

$\Rightarrow \; Y_{s,t}(f) = (P_{s,t}(f)/P_{s,t})^{-\sigma}(Y_{s,t}/f_s)$, the bundler's demand for firm-f's product

To find $P_{s,t}$, use FOC to eliminate $Y_{s,t}(f)$ in $Y_{s,t} = [f_s^{-1/\sigma}\int_{Fs} Y_{s,t}(f)^{(\sigma-1)/\sigma}df]^{\sigma/(\sigma-1)}$

$Y_{s,t} = [f_s^{-1/\sigma}\int_{Fs} Y_{s,t}(f)^{(\sigma-1)/\sigma}df]^{\sigma/(\sigma-1)} = \{f_s^{-1/\sigma}\int_{Fs}[(P_{s,t}(f)/P_{s,t})^{-\sigma}(Y_{s,t}/f_s)]^{(\sigma-1)/\sigma}df\}^{\sigma/(\sigma-1)}$

$= P_{s,t}^{\sigma}(Y_{s,t}/f_s)\{f_s^{-1/\sigma}\int_{Fs} P_{s,t}(f)^{-(\sigma-1)}df\}^{\sigma/(\sigma-1)}$

$P_{s,t}^{-\sigma} = f_s^{-1}\{f_s^{-1/\sigma}\int_{Fs} P_{s,t}(f)^{-(\sigma-1)}df\}^{\sigma/(\sigma-1)}$

$\Rightarrow P_{s,t} = f_s^{1/\sigma}\{f_s^{-1/\sigma}\int_{Fs} P_{s,t}(f)^{(1-\sigma)}df\}^{1/(1-\sigma)} = \{f_s^{(1-\sigma)/\sigma} f_s^{-1/\sigma}\int_{Fs} P_{s,t}(f)^{(1-\sigma)}df\}^{1/(1-\sigma)} = \{f_s^{-1}\int_{Fs} P_{s,t}(f)^{1-\sigma}df\}^{1/(1-\sigma)}$

Collecting results:

$Y_{s,t} = [f_s^{-1/\sigma}\int_{Fs} Y_{s,t}(f)^{(\sigma-1)/\sigma}df]^{\sigma/(\sigma-1)}$	CES aggregator for the sectoral good s
$P_{s,t} = [f_s^{-1}\int_{Fs} P_{s,t}(f)^{1-\sigma}df]^{1/(1-\sigma)}$	Price of sectoral good $Y_{s,t}$
$Y^d_{s,t}(f) = (P_{s,t}/P_{s,t}(f))^{\sigma}(Y_{s,t}/f_s)$	Demand for product of firm f

Remarks:

1. Similar algebra applies to the labor aggregator and the final goods aggregator.
2. In equilibrium, all households in sector s will look alike in the flexible wage version of the model, but since $n_s \neq 1$, we can not make both sectoral wages and sectoral work efforts equal to their household values. We define units of the composite labor input so that $W_{s,t} = W_{s,t}(h)$ in equilibrium, but then $N_{s,t} = n_s L_{s,t}(h)$ in equilibrium.

The Household's Intertemporal Maximization Problem:

Utility of household h working in sector s:

(4) $U_t(h) = E_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}[(1-\rho)^{-1}C_\tau(h)^{1-\rho} - \kappa(1+\chi)^{-1}L_{s,\tau}(h)^{1+\chi} + \nu\cdot v((M_\tau(h)/P_\tau)]$

Budget constraint of household h working in sector_s s:

(5) $M_\tau(h) + E_\tau[\delta_{\tau,\tau+1}B_{\tau+1}(h)] + P_\tau C_\tau(h) + P_\tau T_\tau = S_w W_{s,\tau}(h)L^d_{s,\tau}(h) + S_m M_{\tau-1}(h) + B_\tau(h) + D_\tau(h)$

where $B_{\tau+1}(h)$ is a state contingent claim, $\delta_{\tau,\tau+1}$ is the stochastic discount factor; $D_\tau(h)$ are dividends, T_τ is a lump sum tax (used to balance it's budget each period), S_w is a wage subsidy, and S_m is a subsidy on money holdings.

Remarks:

1. Monopolistic wage setting implies an inefficiently low level of work effort; fiscal policy can eliminate this inefficiency with a wage subsidy. See Canzoneri, Cumby & Diba (2002).
2. We will generally be considering interest rate rules; in that case, the way money enters U_t and the determination of real balances is not of much interest to us.
3. The parsimonious notation for contingent claims in comes from Woodford (1997). Cochrane (2001,Ch. 3) introduces contingent claims in the following way: let $p(B) = \sum_\xi pc(\xi)B(\xi)$ be the price of a portfolio B of contingent claims; the ξ's denote states of nature, $pc(\xi)$ is the price of a claim on one dollar received in $\tau+1$ contingent on the state ξ occurring, and $B(\xi)$ is the number of such claims in portfolio B. Letting $\pi(\xi)$ be the probability of state ξ, $p(B) = \sum_\xi \pi(\xi)[pc(\xi)/\pi(\xi)]B(\xi) = E[\delta(\xi)B(\xi)]$, where $\delta(\xi) \equiv pc(\xi)/\pi(\xi)$ is the "stochastic discount factor". $B_{\tau+1}(h,f)$ and $\delta_{\tau,\tau+1}$ in (5) correspond to $B(\xi)$ and $\delta(\xi)$. All households face the same asset prices and have the same subjective probabilities; so, all households face the same discount factor, $\delta_{\tau,\tau+1}$, in (5). This means that the λ_t defined below will

equalize across households.

4. The “risk free” rate of return: Consider a bond that costs 1 dollar in t and pays I dollars in all states in t+1. From remark 2: $1 = E_t[\delta_{t,t+1}I_t] \Rightarrow I_t^{-1} = E_t[\delta_{t,t+1}]$

5. We may assume that each household owns a representative share in all of the firms. We have suppressed the buying and selling of shares since, as explained below, state contingent claims make the distribution of dividends irrelevant in this model.

Household's problem: choose $B_{t+1}(h)$, $C_t(h)$, $M_t(h)$, and $W_{s,t}(h)$ to maximize (4) subject to (3) and (5). FOC include:

(6a) $C_t(h)$: $\lambda_t P_t = C_t(h)^{-\rho}$

(7a) $B_{t+1}(h)$: $\delta_{t,t+1} = \beta\lambda_{t+1}/\lambda_t \Rightarrow I_t^{-1} = E_t[\delta_{t,t+1}] = \beta E_t[\lambda_{t+1}/\lambda_t]$

(7b) $M_t(h)$: $\nu v(\cdot)'/P_t = \lambda_t - \beta S_m E_t(\lambda_{t+1}) = \lambda_t[1 - \beta S_m E_t(\lambda_{t+1}/\lambda_t)] \Rightarrow \nu v(\cdot)' = C_t(h)^{-\rho}(1 - S_m I_t^{-1})$

(8) $W_{s,t}(h)$: $W_{s,t}(h) = \kappa(M_\tau(h)/P_\tau)(\mu_w/S_w)L_{s,t}(h)^\chi/\lambda_t$ [$= \kappa(\mu_w/S_w)(N_{s,t}/n_s)^\chi/\lambda_t$ in equilibrium],

where $\mu_w \equiv \phi/(\phi-1) > 1$, and λ_t is the marginal utility of nominal wealth.

Remarks:

1. Derivation of the FOC for $W_{s,t}(h)$:

$$\mathcal{L} = E_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}\{... - \kappa(1+\chi)^{-1}[(W_{s,\tau}(h)/W_{s,\tau})^{-\phi}N_{s,\tau}/n_s]^{1+\chi} + ...] + \lambda_\tau[S_w W_{s,\tau}(h)[(W_{s,\tau}(h)/W_{s,\tau})^{-\phi}N_{s,\tau}/n_s + ...]\}$$

$$= E_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}\{... - \kappa(1+\chi)^{-1}[(W_{s,\tau}(h)/W_{s,\tau})^{-\phi}N_{s,\tau}/n_s]^{1+\chi} + ...] + \lambda_\tau[S_w W_{s,\tau}(h)^{1-\phi}(1/W_{s,\tau})^{-\phi}N_{s,\tau}/n_s + ...]\}$$

$$W_{s,t}(h): -\kappa L_{s,t}(h)^\chi(-\phi)(W_{s,t}(h)/W_{s,t})^{-\phi}(N_{s,t}/n_s)W_{s,t}(h)^{-1} + \lambda_t S_w(1-\phi)W_{s,t}(h)^{-\phi}(1/W_{s,t})^{-\phi}N_{s,t}/n_s = 0$$

$$\kappa L_{s,t}(h)^\chi\phi L_{s,t}(h)W_{s,t}(h)^{-1} + \lambda_t S_w(1-\phi)L_{s,t}(h) = 0 \text{ and dividing by } L_{s,t}(h)$$

$$\kappa L_{s,t}(h)^\chi\phi W_{s,t}(h)^{-1} + \lambda_t S_w(1-\phi) = 0 \Rightarrow W_{s,t}(h) = \kappa(\mu_w/S_w)L_{s,t}(h)^\chi/\lambda_t$$

2. All households face the same $\delta_{t,t+1}$, so λ’s and C’s equalize across households.

3. In equilibrium, $C_t \equiv \int_0^1 C_t(h)dh = C_t(h)$.

4. Setting $S_w = \mu_w$ eliminates the monopolistic distortion; see Canzoneri, Cumby & Diba (2002).

Note (from (3)) that: $C_{s,t} = (P_t / P_{s,t})^{\eta}(C_t/\gamma_s^{-\eta}) \Rightarrow P_t / P_{s,t} = (C_{s,t}/C_t)^{1/\eta}/\gamma_s \Rightarrow C^{-\rho} = \lambda_t P_t = \lambda_t P_{s,t}(C_{s,t}/C_t)^{1/\eta}/\gamma_s$

So, in equilibrium the Euler equation (6a) can be written as a sectoral Euler Equation:

(6b) $\lambda_t P_{s,t} = \gamma_s C_t^{(1/\eta)-\rho} C_{s,t}^{-1/\eta}$

The Firm's Calvo Pricing Behavior:

1. Firm-f in F_s gets to set a new price with probability $1-\alpha_s$.
2. The expected length of the "contract" is: $(1-\alpha_s)\cdot 1 + (1-\alpha_s)\cdot\alpha_s\cdot 2 + ... + (1-\alpha_s)\cdot\alpha_s^{n-1}\cdot n + ... = (1-\alpha_s)^{-1}$.

   For example, if $1-\alpha_s = ¼$, then a quarter of the firms adjust each quarter, and the average length of "contracts" is a year; this is the benchmark value in King and Wolman (1996).
3. The fraction of firms with "contracts" set j periods ago is: $(1-\alpha_s)\alpha_s^{\,j}$.

In this section, we lighten the notation by dropping the sector subscripts "s" where possible.

Optimal price setting in period t –

Firm-f seeks to maximize it's market value:

$MV_t = E_t\sum_{j=t}^{\infty}\beta^{j-t}\lambda_j[S_p P_j(f)Y_j(f) - TC_j(Y_j(f))]$, where TC is total cost and S_p is a price subsidy.

With probability α^{j-t}, the new price $P_t^*(f)$ will be in effect in period j; so, firm-f sets $P_t^*(f)$ to maximize:

$MV_t = E_t\sum_{j=t}^{\infty}(\alpha\beta)^{j-t}\lambda_j[S_p P_t^*(f)Y_j(f) - TC_j(Y_j(f))]$, where $Y_j(f) = (P_t^*(f)/P_j)^{-\sigma}(Y_j/f_s)$

$= E_t\sum_{j=t}^{\infty}(\alpha\beta)^{j-t}\lambda_j[S_p P_t^*(f)^{1-\sigma}(1/P_j)^{-\sigma}(Y_j/f_s) - TC_j(P_t^*(f)^{-\sigma}(1/P_j)^{-\sigma}(Y_j/f_s)]$

FOC:

$0 = E_t\sum_{j=t}^{\infty}(\alpha\beta)^{j-t}\lambda_j[S_p(1-\sigma)P_t^*(f))^{-\sigma}(1/P_j)^{-\sigma}(Y_j/f_s) + \sigma MC_j(\cdot)P_t^*(f)^{-\sigma-1}(1/P_j)^{-\sigma}(Y_j/f_s)]$

(dividing by $P_t^*(f)^{-\sigma}/f_s$, which is a constant, and noting that MC = W/Z)

$= E_t\sum_{j=t}^{\infty}(\alpha\beta)^{j-t}\lambda_j[S_p(1-\sigma)P_j^{\sigma}Y_j + \sigma(W_j/Z_j)P_j^{\sigma}Y_j/P_t^*(f)]$

So,

$P_t^*(f)E_t\sum_{j=t}^{\infty}(\alpha\beta)^{j-t}\lambda_j P_j^{\sigma}Y_j = (\mu_p/S_p)E_t\sum_{j=t}^{\infty}(\alpha\beta)^{j-t}\lambda_j(W_j/Z_j)P_j^{\sigma}Y_j$ where $\mu_p \equiv \sigma/(\sigma-1) > 1$

Remark: Setting the subsidy $S_p = \mu_p$ eliminates the monopolistic distortion, see CC&D (2002).

Reintroducing the sectoral subscripts, we have:

(9) $P_{s,t}^{*} = (\mu_p/S_p)(PB_{s,t}/PA_{s,t})$

where

$$PB_{s,t} = E_t\sum_{\tau=t}^{\infty}(\alpha_s\beta)^{\tau-t}\lambda_\tau(W_{s,\tau}/Z_{s,\tau})P_{s,\tau}{}^{\sigma}Y_{s,\tau} = \alpha_s\beta E_t PB_{s,t+1} + \lambda_t(W_{s,t}/Z_{s,t})P_{s,t}{}^{\sigma}Y_{s,t}$$

$$PA_{s,t} \equiv E_t\sum_{\tau=t}^{\infty}(\alpha_s\beta)^{\tau-t}\lambda_\tau P_{s,\tau}{}^{\sigma}Y_{s,\tau} = \alpha_s\beta E_t PA_{s,t+1} + \lambda_t P_{s,t}{}^{\sigma}Y_{s,t}$$

The aggregate sectoral price level –

$$P_{s,t} = [\int_{Fs}P_{s,t}(f)^{1-\sigma}df]^{1/(1-\sigma)} = [\sum_{j=0}^{\infty}(1-\alpha_s)\alpha_s^{\,j}(P_{s,t-j}^{*}(f))^{1-\sigma}]^{1/(1-\sigma)}$$

Lagging $P_{s,t}$ in the equation above, it is straightforward to show that:

(10) $P_{s,t}^{1-\sigma} = (1-\alpha_s)P_{s,t}^{*1-\sigma} + \alpha_s(P_{s,t-1})^{1-\sigma}$

Remarks: (9) and (10) give new and aggregate prices in "nominal" terms. This seems to give us problems in dynare with some interest rate policies. So, we converted to "real" prices in some programs.

Converting to the "real" model – Here we normalize nominal variables on P_t (= 1).

Let $\partial U_t/\partial C_{s,t} = P_{s,t}\lambda_t = p_{s,t}\Lambda_t$

where $p_{s,t} = P_{s,t}/P_t$ and $\Lambda_t = P_t\lambda_t$ is the marginal utility of the final consumption good, C_t.

In equilibrium, the Euler equation (7) becomes:

$(7)_{real}$ $I_t^{-1} = \beta E_t[\lambda_{t+1}/\lambda_t] = \beta E_t[(\Lambda_{t+1}/\Lambda_t)(P_t/P_{t+1})]$

In equilibrium, the flexible wage setting equation (8) becomes:

$(8)_{real}$ $w_{s,t} = \kappa(\mu_w/S_w)(L_{s,t}/n_s)^{\chi}/\Lambda_t$ where $w_{s,t} = W_{s,t}/P_t$

The new price setting equation (9) becomes:

$(9)_{real}\ p_{s,t}^{*} = (1/P_t)(\mu_p/S_p)(PB_{s,t}/PA_{s,t}) \equiv (\mu_p/S_p)(pb_{s,t}/pa_{s,t})$

where $p_{s,t}^{*} = P_{s,t}^{*}/P_t$ and

$pb_{s,t} \equiv PA_{s,t}/P_t^{\sigma} = P_t^{-\sigma}E_t\sum_{\tau=t}^{\infty}(\alpha_s\beta)^{\tau-t}\lambda_\tau(W_{s,\tau}/Z_{s,\tau})P_{s,\tau}^{\sigma}Y_{s,\tau} = E_t\sum_{\tau=t}^{\infty}(\alpha_s\beta)^{\tau-t}\Lambda_\tau(w_{s,\tau}/Z_{s,\tau})(P_{s,\tau}/P_t)^{\sigma}Y_{s,\tau}$

$= \alpha_s\beta E_t[(P_{t+1}/P_t)^{\sigma}pb_{s,t+1}] + \Lambda_t(w_{s,t}/Z_{s,t})p_{s,t}^{\sigma}Y_{s,t}$

$pa_{s,t} \equiv (P_t)PA_{s,t}/P_t^{\sigma} = (P_t/P_t^{\sigma})E_t\sum_{\tau=t}^{\infty}(\alpha_s\beta)^{\tau-t}\lambda_\tau P_{s,\tau}^{\sigma}Y_{s,\tau} = E_t\sum_{\tau=t}^{\infty}(\alpha_s\beta)^{\tau-t}\Lambda_\tau(P_t/P_\tau)(P_{s,\tau}/P_t)^{\sigma}Y_{s,\tau}$

$= \alpha_s\beta E_t[(P_{t+1}/P_t)^{\sigma-1}pa_{s,t+1}] + \Lambda_t p_{s,t}^{\sigma}Y_{s,t}$

And finally, the aggregate sectoral price (10) becomes:

$(10)_{real}\ p_{s,t}^{1-\sigma} = (1-\alpha_s)p_{s,t}^{*1-\sigma} + \alpha_s p_{s,t-1}^{1-\sigma}(P_{t-1}/P_t)^{1-\sigma}$

A computable expression for aggregate sectoral output (or employment) –

Recall:

(11a) $L_{s,t}(h) = \int_{Fs}L_{s,t}(h,f)df$

(11b) $Y_{s,t}(f) = Z_{s,t}N_{s,t}(f)$ (11c) $N_{s,t}(f) \equiv [n_s^{-1/\phi}\int_{Hs}L_{s,t}(h,f)^{(\phi-1)/\phi}dh]^{\phi/(\phi-1)}$

(11d) $Y_{s,t} \equiv [f_s^{-1/\sigma}\int_{Fs}Y_{s,t}(f)^{(\sigma-1)/\sigma}df]^{\sigma/(\sigma-1)}$ (11e) $Y^d_{s,t}(f) = (P(f)_{s,t}/P_{s,t})^{-\sigma}(Y_{s,t}/f_s)$

So, the aggregate sectoral demand for the composite labor input is:

(12a) $N_{s,t} = \int_{Fs}N_{s,t}(f)df = (1/Z_{s,t})\int_{Fs}Y_{s,t}(f)df = (1/Z_{s,t})\int_{Fs}(P_{s,t}(f)/P_{s,t})^{-\sigma}(Y_{s,t}/f_s)df = (1/Z_{s,t})Y_{s,t}DP_{s,t}$

where $DP_{s,t} \equiv (1/f_s)\int_{Fs}(P_{s,t}(f)/P_{s,t})^{-\sigma}df$

Or equivalently, the aggregate sectoral output is:

(12b) $Y_{s,t} = Z_{s,t}N_{s,t}/DP_{s,t}$

Recall: $\int_{Fs}P_{s,t}(f)^x df = f_s\sum_{j=0}^{\infty}(1-\alpha_s)\alpha_s^j P_{s,t-j}^{*}(h)^x$ for any power x.

So, $DP_{s,t} \equiv (1/f_s)\int_{Fs}(P_{s,t}(f)/P_{s,t})^{-\sigma}df = (1/f_s)P_{s,t}^{\sigma}\int_{Fs}(P_{s,t}(f)^{-\sigma}df = (1/f_s)P_{s,t}^{\sigma}[f_s\sum_{j=0}^{\infty}(1-\alpha_s)\alpha_s^j P_{s,t-j}^{*}(h)^{-\sigma}]$

$= P_{s,t}^{\sigma}(1-\alpha_s)\sum_{j=0}^{\infty}\alpha_s^j P_{s,t-j}^{*}(f)^{-\sigma}]$

$= P_{s,t}^{\sigma}(1-\alpha_s)P_{s,t}^{*}(f)^{-\sigma} + P_{s,t}^{\sigma}(1-\alpha_s)\sum_{j=1}^{\infty}\alpha_s^j P_{s,t-j}^{*}(f)^{-\sigma}$

$= P_{s,t}^{\sigma}(1-\alpha_s)P_{s,t}^{*}(f)^{-\sigma} + (P_{s,t}/P_{s,t-1})^{\sigma}[P_{s,t-1}^{\sigma}(1-\alpha_s)\sum_{j=0}^{\infty}\alpha_s^{j+1}P_{s,t-j-1}^{*}(f)^{-\sigma}]$

$= P_{s,t}^{\sigma}(1-\alpha_s)P_{s,t}^{*}(f)^{-\sigma} + (P_{s,t}/P_{s,t-1})^{\sigma}\alpha_s[P_{s,t-1}^{\sigma}(1-\alpha_s)\sum_{j=0}^{\infty}\alpha_s^j P_{s,t-j-1}^{*}(f)^{-\sigma}]$

$= P_{s,t}^{\sigma}(1-\alpha_s)P_{s,t}^{*}(f)^{-\sigma} + (P_{s,t}/P_{s,t-1})^{\sigma}\alpha_s DP_{s,t-1}$

And finally,

$(13)_{nominal}$ $DP_{s,t} = (1-\alpha_s)(P_{s,t}/P_{s,t}^*(f))^\sigma + (P_{s,t}/P_{s,t-1})^\sigma \alpha_s DP_{s,t-1}$

Converting to the "real" model – Here we normalize nominal variables on P_t.

$(13)_{real}$ $DP_{s,t} = (1-\alpha_s)(p_{s,t}/p_{s,t}^*(f))^\sigma + (p_{s,t}/p_{s,t-1})^\sigma(P_t/P_{t-1})^\sigma \alpha_s DP_{s,t-1}$

where $p_{s,t} = P_{s,t}/P_t$.

Remarks:

1. Total hours worked by household h are divided across all firms in the sector (equation (11a)); each firm f uses a composite labor input (equations (11b) and (11c)); the sectoral good is a composite of the firms outputs (equation (11d)). Since firms charge different prices, outputs and work efforts may differ across firms; this dispersion is determined by the firms' demand functions (equation (11e), and summarized by $DP_{s,t}$.

2. **Inefficiency of firms' price dispersion in the flexible wage version of the model:** Since $N_{s,t}(f) = n_s L_{s,t}(h,f)$, $Y_{s,t}(f) = Z_{s,t}N_{s,t}(f) = Z_{s,t}n_s L_{s,t}(h,f)$. So, sectoral output becomes $Y_{s,t} \equiv [f_s^{-1/\sigma}\int_{Fs} Y_{s,t}(f)^{(\sigma-1)/\sigma}df]^{\sigma/(\sigma-1)} = Z_{s,t}n_s[f_s^{-1/\sigma}\int_{Fs} L_{s,t}(h,f)^{(\sigma-1)/\sigma}df]^{\sigma/(\sigma-1)}$. The linear resource constraint (11a) says that an extra hour's work at any firm f incurs the same utility cost. To maximize $Y_{s,t}$ for a given amount of $L_{s,t}(h)$, the social planner would demand equal amounts of output and labor from each firm; but our sectoral output bundler will not do that if the firms charge different prices. Output will not be maximized for a given labor utility cost. This inefficiency is measured by the price dispersion term, $DP_{s,t}$, in (12b); $DP_{s,t} = 1$ when there is no price dispersion.

3. We can use dynare to compute second order approximations to (12b) and (13). Earlier work (following Woodford (????)) calculated first order approximations by brute force.

Model with Calvo price setting and flexible wages –

"Nominal" model:

(1) $C_t = [\sum_{s=1}^{S} \gamma_s C_{s,t}^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$

(2) $\lambda_t P_{s,t} = \gamma_s C_t^{(1/\eta)-\rho} C_{s,t}^{-1/\eta}$

(3) $P_t = [\sum_{s=1}^{S} \gamma_s^{\eta} P_{s,t}^{1-\eta}]^{1/(1-\eta)}$

(4) $P_{s,t}^{*} = (\mu_p/S_p)(PB_{s,t}/PA_{s,t})$

(5) $PB_{s,t} = \alpha_s \beta E_t PB_{s,t+1} + \lambda_t (W_{s,t}/Z_{s,t}) P_{s,t}^{\sigma} C_{s,t}$

(6) $PA_{s,t} = \alpha_s \beta E_t PA_{s,t+1} + \lambda_t P_{s,t}^{\sigma} C_{s,t}$

(7) $P_{s,t}^{1-\sigma} = (1-\alpha_s) P_{s,t}^{*1-\sigma} + \alpha_s P_{s,t-1}^{1-\sigma}$

(8) $C_{s,t} = Y_{s,t} = Z_{s,t} N_{s,t}/DP_{s,t}$

(9) $DP_{s,t} = (1-\alpha_s)(P_{s,t}/P_{s,t}^{*}(h))^{\sigma} + \alpha_s (P_{s,t}/P_{s,t-1})^{\sigma} DP_{s,t-1}$

(10) $W_{s,t} = \kappa(\mu_w/S_w)(N_{s,t}/n_s)^{\chi}/\lambda_t$

(11) $I_t^{-1} = \beta E_t[\lambda_{t+1}/\lambda_t]$

where $\sum_{s=1}^{S} \gamma_s = 1$, $\mu_p = \sigma/(\sigma-1)$, $\mu_w = \phi/(\phi-1)$.

Steady State Equations:

$C = [\sum_{s=1}^{S} \gamma_s C_s^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$

$\lambda P_s = \gamma_s C^{(1/\eta)-\rho} C_s^{-1/\eta}$

$P = [\sum_{s=1}^{S} \gamma_s^{\eta} P_s^{1-\eta}]^{1/(1-\eta)}$

$P_s^{*} = (\mu_p/S_p)(PB_s/PA_s)$

$PB_s = (1-\beta\alpha_s)^{-1} \lambda (W_s/Z_s) P_s^{\sigma} C_s$

$PA_s = (1-\beta\alpha_s)^{-1} \lambda P_s^{\sigma} C_s$

$P_s = P_s^{*}$

$C_{s,t} = Y_{s,t} = N_{s,t}$

$DP_{s,t} = 1$

$W_s = \kappa(\mu_w/S_w)(N_s/n_s)^{\chi}/\lambda$

$I_t^{-1} = \beta$

"Real" model:

(1) $C_t \equiv [\sum_{s=1}^{S} \gamma_s C_{s,t}^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$

(2) $\Lambda_t p_{s,t} = \gamma_s C_t^{(1/\eta)-\rho} C_{s,t}^{-1/\eta}$

(3) $1 = [\sum_{s=1}^{S} \gamma_s^{\eta} p_{s,t}^{1-\eta}]^{1/(1-\eta)}$

(4) $p_{s,t}^{*} = (\mu_p/S_p)(pb_{s,t}/pa_{s,t})$

(5) $pb_{s,t} = \alpha_s \beta E_t[(P_{t+1}/P_t)^{\sigma} pb_{s,t+1}] + \Lambda_t (w_{s,t}/Z_{s,t}) p_{s,t}^{\sigma} Y_{s,t}$

(6) $pa_{s,t} = \alpha_s \beta E_t[(P_{t+1}/P_t)^{\sigma-1} pa_{s,t+1}] + \Lambda_t p_{s,t}^{\sigma} Y_{s,t}$

(7) $p_{s,t}^{1-\sigma} = (1-\alpha_s) p_{s,t}^{*1-\sigma} + \alpha_s p_{s,t-1}^{1-\sigma} (P_{t-1}/P_t)^{1-\sigma}$

(8) $C_{s,t} = Y_{s,t} = Z_{s,t} N_{s,t}/DP_{s,t}$

(9) $DP_{s,t} = (1-\alpha_s)(p_{s,t}/p_{s,t}^{*}(f))^{\sigma} + (p_{s,t}/p_{s,t-1})^{\sigma} (P_t/P_{t-1})^{\sigma} \alpha_s DP_{s,t-1}$

(10) $w_{s,t} = \kappa(\mu_w/S_w)(N_{s,t}/n_s)^{\chi}/\Lambda_t$

(11) $I_t^{-1} = \beta E_t[(\Lambda_{t+1}/\Lambda_t)(P_t/P_{t+1})]$

where $\sum_{s=1}^{S} \gamma_s = 1$, $\mu_p = \sigma/(\sigma-1)$, $\mu_w = \phi/(\phi-1)$ and $\Lambda_t = P_t \lambda_t$.

Steady State Equations:

$C = [\sum_{s=1}^{S} \gamma_s C_s^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$

$\Lambda p_s = \gamma_s C^{(1/\eta)-\rho} C_s^{-1/\eta}$

$1 = [\sum_{s=1}^{S} \gamma_s^{\eta} p_s^{1-\eta}]^{1/(1-\eta)}$

$p_s^{*} = (\mu_p/S_p)(pb_s/pa_s)$

$pb_{s,t} = (1-\beta\alpha_s)^{-1} \Lambda w_s p_s^{\sigma} Y_s$

$pa_{s,t} = (1-\beta\alpha_s)^{-1} \Lambda p_s^{\sigma} Y_s$

$p_s = p_s{*}$

$C_{s,t} = Y_{s,t} = N_{s,t}$

$DP_{s,t} = 1$

$w_{s,t} = \kappa(\mu_w/S_w)(N_{s,t}/n_s)^{\chi}/\Lambda_t$

$I^{-1} = \beta$

Steady State properties: Choose n_s and γ_s so that employment and wages equalize across sectors. Consider the CES aggregator (Cobb-Douglass is exactly analogous) –

(4), (5) & (6) $\Rightarrow$ $P_s = P_s{}^* = \mu_p W_s$ for all s

From (2b), $\lambda P_s = \gamma_s C^{(1/\eta)-\rho} C_s{}^{-1/\eta}$ $\Rightarrow$ $\gamma_s/\gamma_{s'} = (P_s/P_{s'})(C_s/C_{s'})^{1/\eta} = (W_s/W_{s'})(L_s/L_{s'})^{1/\eta}$

From (10), $W_s = \kappa\mu_w(N_s/n_s)^\chi/\lambda$ $\Rightarrow$ $(N_s/n_s)/(N_{s'}/n_{s'}) = (W_s/W_{s'})^{1/\chi}$

So, $\gamma_s/\gamma_{s'} = (W_s/W_{s'})(N_s/N_{s'})^{1/\eta} = (n_s/n_{s'})^{1/\eta}(W_s/W_{s'})^{1+(1/\chi\eta)}$

So, for any α, $[\gamma_s{}^\eta = \alpha n_s$ and $\gamma_{s'}{}^\eta = \alpha n_{s'}]$ $\Rightarrow$ $W_s = W_{s'}$

What is the proportionality factor α? $\gamma_s{}^\eta = \alpha n_s$ for all s $\Rightarrow$ $\sum_{s=1}^S \gamma_s{}^\eta = \alpha\sum_{s=1}^S n_s = \alpha\cdot 1$

And, $W_s = \kappa\mu_w(N_s/n_s)^\chi/\lambda = \kappa\mu_w(L_s(h))^\chi/\lambda$ $\Rightarrow$ $L_s(h) = L_{s'}(h)$

So, finally, $n_s = \gamma_s{}^\eta/\sum_{s=1}^S \gamma_s{}^\eta$ for all s $\Rightarrow$ wages & employment levels equalize across sectors.

Note: for $\eta = 1$, this reduces to $\gamma_s = n_s$ (since $\sum_{s=1}^S \gamma_s = 1$).

Social Utility (with Calvo pricing and flexible wages) –

In equilibrium, household utility (for a household working in sector s) is given by:

$$U_t(h) = E_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}[(1-\rho)^{-1}C(h)_\tau{}^{1-\rho} - \kappa(1+\chi)^{-1}L(h)_{s,\tau}{}^{1+\chi} + v(M(h)_\tau/P_\tau)]$$
$$= E_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}[(1-\rho)^{-1}C_\tau{}^{1-\rho} - \kappa(1+\chi)^{-1}(N_{s,\tau}/n_s)^{1+\chi} + v(M(h)_\tau/P_\tau)]$$

Remarks:

1. Recall: each household works at all of the firms in the sector to which it has been allocated. Household work efforts may differ across sectors, but they equalize within a given sector (since the households' flexible wage rates equalize within a given sector). We aggregate by summing over sectors (weighted by n_s). This could be interpreted as ex-ante welfare (before knowing which sector the household is placed in).
2. Following the literature, we generally ignore the $v(\cdot)$ term. Justification: it is thought to be small in developed countries. Attractiveness: it allows us to ignore questions of seigniorage, including the time consistency aspect. But note, the monopolistic markups still imply a temptation to expand.

<u>Social Welfare</u>:

(11) $U_t = E_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}[(1-\rho)^{-1}C_\tau{}^{1-\rho} - \kappa(1+\chi)^{-1}\sum_{s=1}^S n_s(N_{s,\tau}/n_s)^{1+\chi}]$

where $n_s = \gamma_s$ for $\eta = 1$, and $n_s = \gamma_s{}^\eta/\sum_{s=1}^S \gamma_s{}^\eta$ for $\eta > 1$.

Adding "Calvo" wage setting:

1. We continue to assume each household in sector s works at all of the firms in sector s. So, firms continue to use the composite labor input. However, the flexible wages, described by equation (9), are replaced by "Calvo" wage setting.
2. Each household in sector s gets to set a new wage with probability $1-\omega_s$. The basic structure is analogous to the firms' Calvo price setting. The fraction wages set j periods: $(1-\omega_s)\omega_s^{\,j}$.

Optimal wage setting in period t –

In this section, we lighten the notation by dropping the sector subscripts "s" where possible.

With probability ω^{j-t}, the new wage will be in effect in period j; so, household h (which gets to reset it's wage) sets $W^*_t(h)$ to maximize:

$$U(h)_t = E_t\sum_{j=t}^{\infty}\beta^{j-t}[u(\cdot) - \kappa(1+\chi)^{-1}L_j(h)^{1+\chi} + v(\cdot)],\ \text{ where } L_j(h) = (W^*_t(h)/W_j)^{-\phi}N_j/n$$

$$= E_t\sum_{j=t}^{\infty}(\omega\beta)^{j-t}[u(\cdot) - \kappa(1+\chi)^{-1}[W^*_t(h)^{-\phi}W_j^{\phi}L_j/n]^{1+\chi} + v(\cdot)]$$

subject to:

$$M_j(h) + E_j[\delta_{j,j+1}B_{j+1}(h)] + P_jC_j(h) + P_jT_j = S_wW^*_t(h)L_j(h) + M_{j-1}(h) + B_j(h) + D_j(h) = S_wW^*_t(h)^{1-\phi}W_j^{\phi}L_j/n +$$

FOC:

$$0 = E_t\sum_{j=t}^{\infty}(\omega\beta)^{j-t}[-\kappa L_j(h)^{\chi}(-\phi)W^*_t(h)^{-\phi-1}W_j^{\phi}(N_j/n) + \lambda_jS_w(1-\phi)W^*_t(h)^{-\phi}W_j^{\phi}N_j/n]$$

(dividing by $W^{*-\phi}_t(h)$ and replacing $L_j(h) = (W^*_t(h)/W_j)^{-\phi}(N_j/n)$)

$$= E_t\sum_{j=t}^{\infty}(\omega\beta)^{j-t}\{\kappa[(W^*_t(h)/W_j)^{-\phi}(N_j/n)]^{\chi}\phi W^*_t(h)^{-1}W_j^{\phi}N_j/n - \lambda_jS_w(\phi-1)W_j^{\phi}N_j/n\}$$

$$= E_t\sum_{j=t}^{\infty}(\omega\beta)^{j-t}[\kappa W^*_t(h)^{-\phi\chi-1}W_j^{\phi\chi+\phi}(N_j/n)^{1+\chi}\phi - \lambda_jS_w(\phi-1)W_j^{\phi}N_j/n]$$

So, dividing by $(\phi-1)/S_wn$ and rearranging,

$$W^*_t(h)^{1+\phi\chi}E_t\sum_{j=t}^{\infty}(\omega\beta)^{j-t}\lambda_jW_j^{\phi}N_j = \kappa[\phi/(\phi-1)/S_w](1/n)^{\chi}E_t\sum_{j=t}^{\infty}(\omega\beta)^{j-t}N_j^{1+\chi}W_j^{\phi(1+\chi)}$$

Reintroducing the sectoral subscripts, we have:

(12) $W_{s,t}^{*1+\phi\chi} = (\mu_w/S_w)(\kappa/n_s^{\chi})(WB_{s,t}/WA_{s,t})$

where $\mu_w \equiv \phi/(\phi-1)$

$WB_{s,t} \equiv E_t\sum_{j=t}^{\infty}(\omega_s\beta)^{j-t}N_{s,j}^{1+\chi}W_{s,j}^{\phi(1+\chi)} = \omega_s\beta E_tWB_{s,t+1} + N_{s,t}^{1+\chi}W_{s,t}^{\phi(1+\chi)}$

$WA_{s,t} \equiv E_t\sum_{j=t}^{\infty}(\omega_s\beta)^{j-t}\lambda_jN_{s,j}W_{s,j}^{\phi} = \omega_s\beta E_tWA_{s,t+1} + \lambda_tN_{s,t}W_{s,t}^{\phi}$

The aggregate sectoral wage level –

$W_{s,t} = [\int_{Ns} W_{s,t}(h)^{1-\phi}dh]^{1/(1-\phi)} = [\sum_{j=0}^{\infty}(1-\omega_s)\omega_s^j(W_{s,t-j}^*(h))^{1-\phi}]^{1/(1-\phi)}$

Lagging $W_{s,t}$ in the equation above, it is straightforward to show that:

(13) $W_{s,t}^{1-\phi} = (1-\omega_s)W_{s,t}^{*1-\phi} + \omega_s(W_{s,t-1})^{1-\phi}$

Converting to the "real" model – Here we normalize nominal variables on P_t.

As before, let $\partial U_t/\partial C_{s,t} = P_{s,t}\lambda_t = p_{s,t}\Lambda_t$

where $p_{s,t} = P_{s,t}/P_t$ and $\Lambda_t = P_t\lambda_t$ is the marginal utility of the final consumption good, C_t.

The new wage setting equation (12) becomes:

$(12)_{real}$ $w_{s,t}^{*1+\phi\chi} = (1/P_t^{1+\phi\chi})(\mu_w/S_w)(\kappa/n_s^{\chi})(WB_{s,t}/WA_{s,t}) = (\mu_w/S_w)(\kappa/n_s^{\chi})(wb_{s,t}/wa_{s,t})$

where $w_{s,t}^* = W_{s,t}^*/P_t$ and

$wb_{s,t} \equiv WB_{s,t}/P_t^{\phi(1+\chi)} = [E_t\sum_{j=t}^{\infty}(\omega_s\beta)^{j-t}N_{s,j}^{1+\chi}W_{s,j}^{\phi(1+\chi)}]/P_t^{\phi(1+\chi)}$

$= [E_t\sum_{j=t}^{\infty}(\omega_s\beta)^{j-t}N_{s,j}^{1+\chi}w_{s,j}^{\phi(1+\chi)}P_j^{\phi(1+\chi)}]/P_t^{\phi(1+\chi)} = E_t\sum_{j=t}^{\infty}(\omega_s\beta)^{j-t}N_{s,j}^{1+\chi}w_{s,j}^{\phi(1+\chi)}(P_j/P_t)^{\phi(1+\chi)}$

$= \omega_s\beta E_t[(P_{t+1}/P_t)^{\phi(1+\chi)}wb_{s,t+1}] + N_{s,j}^{1+\chi}w_{s,t}^{\phi(1+\chi)}$

$wa_{s,t} \equiv P_t^{1+\phi\chi}WA_{s,t}/P_t^{\phi(1+\chi)} = P_t^{1+\phi\chi}[E_t\sum_{j=t}^{\infty}(\omega_s\beta)^{j-t}\lambda_jN_{s,j}W_{s,j}^{\phi}]/P_t^{\phi(1+\chi)}$

$= P_t^{1+\phi\chi}[E_t\sum_{j=t}^{\infty}(\omega_s\beta)^{j-t}\Lambda_jN_{s,j}w_{s,j}^{\phi}P_j^{\phi-1}]/P_t^{\phi(1+\chi)} = E_t\sum_{j=t}^{\infty}(\omega_s\beta)^{j-t}\Lambda_jN_{s,j}w_{s,j}^{\phi}(P_j/P_t)^{\phi-1}$

$= \omega_s\beta E_t[(P_{t+1}/P_t)^{\phi-1}wa_{s,t+1}] + \Lambda_tN_{s,t}w_{s,t}^{\phi}$

Finally, the aggregate "real" wage is given by:

$(13)_{real}$ $w_{s,t}^{1-\phi} = (1-\omega_s)w_{s,t}^{*1-\phi} + \omega_s(w_{s,t-1})^{1-\phi}(P_{t-1}/P_t)^{1-\phi}$

Remarks:

1. Here again, setting the wage subsidy $S_w = \mu_w$ eliminates the monopolistic distortion.

Model with Calvo price and wage setting –

“Nominal” model:	**Steady State Equations:**
(1) $C_t = [\sum_{s=1}^{S} \gamma_s C_{s,t}^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$	$C = [\sum_{s=1}^{S} \gamma_s C_s^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$
(2) $\lambda_t P_{s,t} = \gamma_s C_t^{(1/\eta)-\rho} C_{s,t}^{-1/\eta}$	$\lambda P_s = \gamma_s C^{(1/\eta)-\rho} C_s^{-1/\eta}$
(3) $P_t = [\sum_{s=1}^{S} \gamma_s^{\eta} P_{s,t}^{1-\eta}]^{1/(1-\eta)}$	$P = [\sum_{s=1}^{S} \gamma_s^{\eta} P_s^{1-\eta}]^{1/(1-\eta)}$
(4) $P_{s,t}^{*} = (\mu_p/S_p)(PB_{s,t}/PA_{s,t})$	$P_s^{*} = (\mu_p/S_p)(PB_s/PA_s)$
(5) $PB_{s,t} = \alpha_s \beta E_t PB_{s,t+1} + \lambda_t (W_{s,t}/Z_{s,t}) P_{s,t}^{\sigma} C_{s,t}$	$PB_s = (1-\beta\alpha_s)^{-1} \lambda (W_s/Z_s) P_s^{\sigma} C_s$
(6) $PA_{s,t} = \alpha_s \beta E_t PA_{s,t+1} + \lambda_t P_{s,t}^{\sigma} C_{s,t}$	$PA_s = (1-\beta\alpha_s)^{-1} \lambda P_s^{\sigma} C_s$
(7) $P_{s,t}^{1-\sigma} = (1-\alpha_s) P_{s,t}^{*1-\sigma} + \alpha_s (P_{s,t-1})^{1-\sigma}$	$P_s = P_s^{*}$
(8) $C_{s,t} = Y_{s,t} = Z_{s,t} N_{s,t}/DP_{s,t}$	$C_{s,t} = Y_{s,t} = N_{s,t}$
(9) $DP_{s,t} = (1-\alpha_s)(P_{s,t}/P_{s,t}^{*}(h))^{\sigma} + \alpha_s (P_{s,t}/P_{s,t-1})^{\sigma} DP_{s,t-1}$	$DP_{s,t} = 1$
(10) $W_{s,t}^{*1+\phi\chi} = (\mu_w/S_w)(\kappa/n_s^{\chi})(WB_{s,t}/WA_{s,t})$	$W_s^{*1+\phi\chi} = (\mu_w/S_w)(\kappa/n_s^{\chi})(WB_s/WA_s)$
(11) $WB_{s,t} = \omega_s \beta E_t WB_{s,t+1} + N_{s,t}^{1+\chi} W_{s,t}^{\phi(1+\chi)}$	$WB_s = (1-\beta\omega_s) N_s^{1+\chi} W_s^{\phi(1+\chi)}$
(12) $WA_{s,t} = \omega_s \beta E_t WA_{s,t+1} + \lambda_t N_{s,t} W_{s,t}^{\phi}$	$WA_s = (1-\beta\omega_s)^{-1} \lambda N_s W_s^{\phi}$
(13) $W_{s,t}^{1-\phi} = (1-\omega_s) W_{s,t}^{*1-\phi} + \omega_s (W_{s,t-1})^{1-\phi}$	$W_s = W_s^{*}$
(14) $I_t^{-1} = \beta E_t[(\Lambda_{t+1}/\Lambda_t)(P_t/P_{t+1})]$	$I_t^{-1} = \beta$

“Real” model:	**Steady State Equations:**
(1) $C_t \equiv [\sum_{s=1}^{S} \gamma_s C_s^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$	$C = [\sum_{s=1}^{S} \gamma_s C_s^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$
(2) $\Lambda_t p_{s,t} = \gamma_s C_t^{(1/\eta)-\rho} C_{s,t}^{-1/\eta}$	$\Lambda p_s = \gamma_s C^{(1/\eta)-\rho} C_s^{-1/\eta}$
(3) $1 = [\sum_{s=1}^{S} \gamma_s^{\eta} p_{s,t}^{1-\eta}]^{1/(1-\eta)}$	$1 = [\sum_{s=1}^{S} \gamma_s^{\eta} p_s^{1-\eta}]^{1/(1-\eta)}$
(4) $p_{s,t}^{*} = (\mu_p/S_p)(pb_{s,t}/pa_{s,t})$	$p_s^{*} = (\mu_p/S_p)(pb_s/pa_s)$
(5) $pb_{s,t} = \alpha_s \beta E_t[(P_{t+1}/P_t)^{\sigma} pb_{s,t+1}] + \Lambda_t (w_{s,t}/Z_{s,t}) p_{s,t}^{\sigma} Y_{s,t}$	$pb_{s,t} = (1-\beta\alpha_s)^{-1} \Lambda w_s p_s^{\sigma} Y_s$
(6) $pa_{s,t} = \alpha_s \beta E_t[(P_{t+1}/P_t)^{\sigma-1} pa_{s,t+1}] + \Lambda_t p_{s,t}^{\sigma} Y_{s,t}$	$pa_{s,t} = (1-\beta\alpha_s)^{-1} \Lambda p_s^{\sigma} Y_s$
(7) $p_{s,t}^{1-\sigma} = (1-\alpha_s) p_{s,t}^{*1-\sigma} + \alpha_s p_{s,t-1}^{1-\sigma} (P_{t-1}/P_t)^{1-\sigma}$	$p_s = p_s*$
(8) $C_{s,t} = Y_{s,t} = Z_{s,t} N_{s,t}/DP_{s,t}$	$C_{s,t} = Y_{s,t} = N_{s,t}$
(9) $DP_{s,t} = (1-\alpha_s)(p_{s,t}/p_{s,t}^{*}(f))^{\sigma} + (p_{s,t}/p_{s,t-1})^{\sigma} (P_t/P_{t-1})^{\sigma} \alpha_s DP_{s,t-1}$	$DP_{s,t} = 1$
(10) $w_{s,t}^{*1+\phi\chi} = (\mu_w/S_w)(\kappa/n_s^{\chi})(wb_{s,t}/wa_{s,t})$	$w_{s,t}^{*1+\phi\chi} = (\mu_w/S_w)(\kappa/n_s^{\chi})(wb_{s,t}/wa_{s,t})$
(11) $wb_{s,t} = \omega_s \beta E_t[(P_{t+1}/P_t)^{\phi(1+\chi)} wb_{s,t+1}] + N_{s,t}^{1+\chi} w_{s,t}^{\phi(1+\chi)}$	$wb_s = (1-\omega_s\beta)^{-1} N_s^{1+\chi} w_s^{\phi(1+\chi)}$
(12) $wa_{s,t} = \omega_s \beta E_t[(P_{t+1}/P_t)^{\phi-1} wa_{s,t+1}] + \Lambda_t N_{s,t} w_{s,t}^{\phi}$	$wa_s = (1-\omega_s\beta)^{-1} \Lambda N_s w_s^{\phi}$
(13) $w_{s,t}^{1-\phi} = (1-\omega_s) w_{s,t}^{*1-\phi} + \omega_s (w_{s,t-1})^{1-\phi} (P_{t-1}/P_t)^{1-\phi}$	$w_s = w_s*$
(14) $I_t^{-1} = \beta E_t[(\Lambda_{t+1}/\Lambda_t)(P_t/P_{t+1})]$	$I^{-1} = \beta$

where $\sum_{s=1}^{S} \gamma_s = 1$, $\mu_p = \sigma/(\sigma-1)$, $\mu_w = \phi/(\phi-1)$, and $\Lambda_t = P_t \lambda_t$

Social Utility (with Calvo wage and price setting) –

In equilibrium, household utility is given by:

$$U_t(h) = E_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}[(1-\rho)^{-1}C(h)_\tau^{1-\rho} - \kappa(1+\chi)^{-1}L(h)_{s,\tau}^{1+\chi} + v(M_\tau(h)/P_\tau)]$$

Remarks:

1. Recall: each household works at all of the firms in the sector to which it has been allocated. As before, household work efforts differ across sectors. But now, they may also differ within a given sector (since households may have different wage rates). As before, we will aggregate by summing over households and sectors.

2. As before, we generally ignore the $v(\cdot)$ term.

3. Inefficiencies due to nominal inertia –

   A. Intertemporal inefficiencies:

   Fluctuations in C and L decrease welfare. These losses presumably increase in ρ and χ. In the fixed wage case, we have different $L_{s,t}(h)$ within a given sector, and calculating the social disutility of work is more difficult. We return to this below.

   B. Intratemporal inefficiencies:

   i. As discussed earlier, price dispersion implies different levels of production (and work) across firms in a given sector, and therefore an inefficient allocation of labor across firms. Less output is gotten from a given amount of work. This inefficiency is captured in our DP term (see (12b) on page 8). Loss should decrease with substitutability parameter, θ.

   ii. With sticky wages, wage dispersion implies different levels of work for households in the labor bundle for firm f, and therefore less a smaller $N_{s,t}(f)$ than could be had for the same total work effort. I think this inefficiency is captured by the appearance of $w_{s,t}$ in the price setting equations. **(Bob and Behzad: is this right?)** This loss should decrease with substitutability parameter, ϕ.

<u>Social Welfare</u>:

(15) $U_t = E_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}[(1-\rho)^{-1}C_\tau^{1-\rho} - \kappa(1+\chi)^{-1}\sum_{s=1}^{S}DW_{s,\tau}]$

where $DW_{s,\tau} \equiv \int_{Ns}L_{s,\tau}(h)^{1+\chi}dh$, $n_s = \gamma_s$ for $\eta = 1$, and $n_s = \gamma_s^{\eta}/\sum_{s=1}^{S}\gamma_s^{\eta}$ for $\eta > 1$.

How to calculate $DW_{s,t} = \int_{Ns}L_{s,t}(h)^{1+\chi}dh$ –

Recall: 1. $L_{s,t}^{d}(h) = (W_{s,t}(h)/W_{s,t})^{-\phi}(N_{s,t}/n_s)$

2. $\int_{Ns}W_{s,t}(h)^x dh = n_s\sum_{j=0}^{\infty}(1-\omega_s)\omega_s^j W_{s,t-j}^*(h)^x$ for any power x

$DW_{s,t} = \int_{Ns}L_{s,t}(h)^{1+\chi}dh = (N_{s,t}/n_s)^{1+\chi}\int_{Ns}(W_{s,t}(h)/W_{s,t})^{-\phi(1+\chi)}dh$

$= (N_{s,t}/n_s)^{1+\chi}\{n_s\sum_{j=0}^{\infty}(1-\omega_s)\omega_s^j(W_{s,t-j}^*(h)/W_{s,t})^{-\phi(1+\chi)}\} = \theta_t\sum_{j=0}^{\infty}\omega_s^j(W_{s,t-j}^*(h)/W_{s,t})^{-\phi(1+\chi)}$

where $\theta_t \equiv (1-\omega_s)N_{s,t}^{1+\chi}n_s^{-\chi}$ and small w's will represent real wages

$D_{s,t} \equiv \sum_{j=0}^{\infty}\omega_s^j(W_{s,t-j}^*(h)/W_{s,t})^{-\phi(1+\chi)} = (w_{s,t}^*(h)/w_{s,t})^{-\phi(1+\chi)} + \sum_{j=1}^{\infty}\omega_s^j(W_{s,t-j}^*(h)/W_{s,t})^{-\phi(1+\chi)}$

$= (w_{s,t}^*(h)/w_{s,t})^{-\phi(1+\chi)} + (W_{s,t-1}/W_{s,t})^{-\phi(1+\chi)}\sum_{j=1}^{\infty}\omega_s^j(W_{s,t-j}^*(h)/W_{s,t-1})^{-\phi(1+\chi)}$

$= (w_{s,t}^*(h)/w_{s,t})^{-\phi(1+\chi)} + [(w_{s,t-1}/w_{s,t})(P_{t-1}/P_t)]^{-\phi(1+\chi)}\sum_{j=0}^{\infty}\omega_s^{j+1}(W_{s,t-1-j}^*(h)/W_{s,t-1})^{-\phi(1+\chi)}$

$= (w_{s,t}^*(h)/w_{s,t})^{-\phi(1+\chi)} + \omega_s[(w_{s,t-1}/w_{s,t})(P_{t-1}/P_t)]^{-\phi(1+\chi)}D_{s,t-1}$

So finally:

(16) $DW_{s,t} = (1-\omega_s)N_{s,t}^{1+\chi}n_s^{-\chi}D_{s,t}$

where $D_{s,t} = (w_{s,t}^*(h)/w_{s,t})^{-\phi(1+\chi)} + \omega_s[(w_{s,t-1}/w_{s,t})(P_{t-1}/P_t)]^{-\phi(1+\chi)}D_{s,t-1}$

Note: as $\omega_s \rightarrow 0$ (that is, as wages become flexible), (16) is consistent with (11).

Remarks:

1. In the 'nominal' model, dynare seemed to have trouble pinning down the price level for interest rules. So, on the advice of Martin Uribe and others, we went to the 'real' model.
2. In the 'real' model, dynare had some problems solving for a steady state. The problem seemed to reside in the exponential terms in the real wage nexus. So, we went to the 'wage and price inflation' model, in which $w_{s,t}^*$ is measured relative to the aggregate wage rate.

"Wage & Price Inflation" model: | **Steady State Equations:**

(1), ... , (9) are the same as in the "real" model.

(10) $w_{s,t}^{*1+\phi\chi} = (\mu_w/S_w)(\kappa/n_s^{\chi})(wb_{s,t}/wa_{s,t})$ | $w_{s,t}^{*1+\phi\chi} = (\mu_w/S_w)(\kappa/n_s^{\chi})(wb_{s,t}/wa_{s,t})$

(11) $wb_{s,t} = \omega_s\beta E_t[(W_{s,t+1}/W_{s,t})^{\phi(1+\chi)}wb_{s,t+1}] + N_{s,t}^{1+\chi}$ | $wb_s = (1-\omega_s\beta)^{-1}N_s^{1+\chi}$

(12) $wa_{s,t} = \omega_s\beta E_t[(W_{s,t+1}/W_{s,t})^{\phi-1}wa_{s,t+1}] + \Lambda_t N_{s,t}w_{s,t}$ | $wa_s = (1-\omega_s\beta)^{-1}\Lambda N_s w_s$

(13) $1 = (1-\omega_s)w_{s,t}^{*1-\phi} + \omega_s(W_{s,t-1}/W_{s,t})^{1-\phi}$

(14) $w_{s,t}/w_{s,t-1} = (W_{s,t}/W_{s,t-1})/(P_t/P_{t-1})$

(15) $I_t^{-1} = \beta E_t[(\Lambda_{t+1}/\Lambda_t)(P_t/P_{t+1})]$ | $I^{-1} = \beta$

where $w_{s,t}$ is still the real wage, but $w_{s,t}^* = W_{s,t}^*/W_{s,t}$, $wa_{s,t} = WA_{s,t}/W_{s,t}^{\phi-1}$ and $wb_{s,t} = WB_{s,t}/W_{s,t}^{\phi(1+\chi)}$; as before, $\sum_{s=1}^{S}\gamma_s = 1$, $\mu_p = \sigma/(\sigma-1)$, $\mu_w = \phi/(\phi-1)$, and $\Lambda_t = P_t\lambda_t$

Remarks:

1. We have divided (10) in the "nominal" model by $W_{s,t}^{1+\phi\chi}$.
2. Since we add a new variable, wage inflation = $W_{s,t+1}/W_{s,t}$, we need a new equation, (14).
3. The calculation of DW is also modified:

$$DW_{s,t} = \int_{Ns}L_{s,t}(h)^{1+\chi}dh = (N_{s,t}/n_s)^{1+\chi}\int_{Ns}(W_{s,t}(h)/W_{s,t})^{-\phi(1+\chi)}dh$$

$$= (N_{s,t}/n_s)^{1+\chi}\{n_s\textstyle\sum_{j=0}^{\infty}(1-\omega_s)\omega_s^j(W_{s,t-j}^*(h)/W_{s,t})^{-\phi(1+\chi)}\} = \theta_t\sum_{j=0}^{\infty}\omega_s^j(W_{s,t-j}^*(h)/W_{s,t})^{-\phi(1+\chi)}$$

where $\theta_t \equiv (1-\omega_s)N_{s,t}^{1+\chi}n_s^{-\chi}$

$$D_{s,t} \equiv \textstyle\sum_{j=0}^{\infty}\omega_s^j(W_{s,t-j}^*(h)/W_{s,t})^{-\phi(1+\chi)} = w_{s,t}^*(h)^{-\phi(1+\chi)} + \sum_{j=1}^{\infty}\omega_s^j(W_{s,t-j}^*(h)/W_{s,t})^{-\phi(1+\chi)}$$

$$= w_{s,t}^*(h)^{-\phi(1+\chi)} + (W_{s,t-1}/W_{s,t})^{-\phi(1+\chi)}\textstyle\sum_{j=0}^{\infty}\omega_s^{j+1}(W_{s,t-j-1}^*(h)/W_{s,t-1})^{-\phi(1+\chi)}$$

$$= w_{s,t}^*(h)^{-\phi(1+\chi)} + \omega_s(W_{s,t}/W_{s,t-1})^{\phi(1+\chi)}D_{s,t-1}$$

Parameterization:

Intertemporal elasticity: ρ

RBC typically takes $\rho = 1$. GG&LS do 1, and then 5.

We use $\rho = 1$.

Weight on disutility of labor: κ

With NNS preferences, we can't calibrate it to average hours worked.

CEE (AER, RBC with real rigidity) have a similar problem; choose their version of κ to make employment = 1 in SS.

Bayoumi, Laxton and Pesenti (pg. 10): no discussion that I saw, set $\kappa = 1$.

We use $\kappa = 1$. Do robustness.

Labor Supply elasticity: $1/\chi$

King & Rebelo: in their RBC model, they used an elasticity of 4; dropped it to 1, and could not explain output and (especially) employment variation.

Gali, Gertler and L-S (pg. 6, ft. 8): Frisch elasticity (λ_t held constant) estimates range from 0.05 - 0.3; they use $\chi = 5$. Cite several recent papers leading to 0.2.

Bayoumi, Laxton and Pesenti: Say Frisch elasticity estimates vary from .05 - .35; use .33 as baseline case, but also .15 ($\chi = 6.7$) as an alternative, saying it is closer to mean estimate.

We split difference between GG&LS and BL&P and use $\chi = 6$ (or an elasticity of .17) as our baseline case.

Elasticity of substitution in consumption aggregator: σ

Bayoumi, Laxton and Pesenti: say Bradford and Lawrence (2003) $\Rightarrow \mu_p \in [1.15, 1.20]$; but, they use $\mu_p = 1.10$ (or $\sigma = 10$) ???? Pg. 47: they let μ_p and μ_w vary in [1.055, 1.555].

Rotemberg and Woodford (Taylor book): $\sigma = 7.88$ or $\mu_p = 1.145$

G&LS&V (both fiscal policy papers): $\sigma = 6$

EHL: $\mu_p = 1.333 \Rightarrow \sigma = 4$.

We use $\sigma = 6$.

Elasticity of substitution in the labor aggregator: ϕ

Bayoumi, Laxton and Pesenti: say Sebastian & Nicoletti (2002) $\Rightarrow \mu_w = 1.15$ (or $\phi = 7.7$), which they use. Pg. 47: they let μ_p and μ_w vary in [1.055, 1.555]

EHL: $\mu_w = \mu_p = 1.333 \Rightarrow \phi = \sigma = 4$.

Wooders and Smets (via Dellas?):

We use $\phi = \sigma = 6$.

Productivity shocks:

Cooley & Prescott: 0.95 (AR1 coef), 0.007 (std error) for filtered Solow Residual.

King & Rebelo: 0.979 (AR1 coef), 0.0072 (std error) for filtered Solow Residual.

Bob Cumby: 0.72 (AR1 coef), 0.0076 (std error) for filtered hours productivity.

0.77 (AR1 coef), 0.0086 (std error) for filtered worker productivity.

We use: 0.8 (AR1 coef), 0.009 (std error) for filtered worker productivity.

Taylor Rule: (Bob Cumby's estimate) **this is still now quite right below**

For Volcker and Greenspan years (1979.3 - 2003.2) –

$FFR_t = 0.824FFR_{t-1} + (1 - 0.824)*2.020*(\pi_t - \pi^*) + (1 - 0.824)*0.184*gap_t$

$FRR_t = 0.222 + 0.824*FFR_{t-1} + (1 - 0.824)*2.020*\pi_t + (1 - 0.824)*0.184*gap_t$

std error = .00245, variance = 0.000006

correlation with worker productivity = .175, cov = .00000267

correlation with hours productivity = .092, cov = .00000124

For Greenspan years (1987.3 - 2003.2) –

$FRR_t = 0.238 + 0.884*FFR_{t-1} + (1 - 0.884)*1.720*\pi_t + (1 - 0.884)*0.352*gap_t$

std error = .00108

Remarks: (we probably don't want to make much of these)

1. gap = $y_t - \bar{y} = (y_t - y_t^*) + (y_t^* - \bar{y})$, where $\bar{y}$ is SS output and y_t^* is potential output.
2. Some would want us to use gap* = $(y_t - y_t^*) = y_t - \bar{y} + (\bar{y} - y_t^*)$ = gap + $(\bar{y} - y_t^*)$. In their view, $(\bar{y} - y_t^*)$ would be in the residual of our Taylor Rule. This would induce a negative correlation between the Taylor Rule residual and the productivity shock innovations. Empirically, we observe a positive correlation??
3. Bob mentioned real interest rates, but this seems model dependent.

Model 2: a one sector model with capital and habit.

Here we simplify Model 1 to one sector, but we add capital, habit and a laziness shock, and show how to model 'rule of thumbers'.

Remarks:

1. Capital, K_{t-1}, is owned by the households, supplied to firms at the (nominal) rental rate R_t, and depreciates at the rate δ. Production is now Cobb-Douglas, not linear.
2. We assume capital adjustment costs that are common in the RBC literature.

The Household's Intertemporal Maximization Problem:

Utility of household h:

(1) $U_t(h) = E_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}[\log(C_\tau(h) - bC_{\tau-1}(h)) - \kappa_\tau(1+\chi)^{-1}L_\tau(h)^{1+\chi} + \nu\cdot v((M_\tau(h)/P_\tau)]$

Budget constraint of household h:

(2) $M_\tau(h) + E_\tau[\Delta_{\tau,\tau+1}B_{\tau+1}(h)] + P_\tau[C_\tau(h)+I_\tau(h)+T_\tau] = S_wW_{s,\tau}(h)L^d_{s,\tau}(h) + S_mM_{\tau-1}(h) + B_\tau(h) + R_\tau K_{\tau-1}(h) + D_\tau(h)$

where (as before) $B_{\tau+1}(h)$ is a state contingent claim, $\Delta_{\tau,\tau+1}$ is the stochastic discount factor; $D_\tau(h)$ are dividends, T_τ is a lump sum tax (used to balance it's budget each period), S_w is a wage subsidy, and S_m is a subsidy on money holdings. (*Notation change*: Δ in place of δ.)

What's new: I_τ is investment, κ_τ, is a laziness shock, and b (if > 0) introduces "habit".

Capital constraint for household h:

(3) $K_\tau(h) = (1 - \delta)K_{\tau-1}(h) + I_\tau(h) - ½\psi[(I_\tau(h)/K_{\tau-1}(h)) - \delta]^2K_{\tau-1}(h)$

where the last term represents the capital adjustment cost.

Note: K_t is the end of period stock, the capital that will be available for use in period in t+1.

<u>Household's problem</u>: choose I_t, K_t, $B_{t+1}(h)$, $C_t(h)$, $M_t(h)$, and $W_{s,t}(h)$ to maximize (1) subject to (2), (3) and the labor demand curve. Only the C_t, I_t and K_{t+1} decisions are new.

Letting λ_t and ξ_t be the multipliers for constraints (2) and (3), the new FOC are:

$C_t(h)$: (4a) $(C_t(h) - bC_{t-1}(h))^{-1} - \beta b(C_{t+1}(h) - bC_t(h))^{-1} = \lambda_tP_t$

$I_t(h)$: (4b) $\lambda_tP_t = \xi_t - \xi_t\psi[(I_t(h)/K_{t-1}(h)) - \delta]$

$K_t(h)$: (4c) $\xi_t = \beta E_t\{\lambda_{t+1}R_{t+1} + \xi_{t+1}[(1-\delta) - ½\psi[(I_{t+1}(h)/K_t(h))-\delta]^2 - \psi K_t(h)[(I_{t+1}(h)/K_t(h))-\delta](-I_{t+1}(h)/K_t(h)^2)\}$

$= \beta E_t\{\lambda_{t+1}R_{t+1} + \xi_{t+1}[(1-\delta) - ½\psi[(I_{t+1}(h)/K_t(h)) - \delta]^2 + \psi[(I_{t+1}(h)/K_t(h)) - \delta](I_{t+1}(h)/K_t(h))\}$

Remarks:

1. The household's wage setting equations are the same as in Model 1.
2. The firm's price setting equations are different because the production function and MC have changed. With the linear production function in Model 1, nominal marginal cost = W_t/Z_t. Here, production is Cobb Douglass, and we have to find an new expression for MC.

Firm f's cost minimization problem and nominal marginal cost:

Firm f's production function is:

(5) $Y_t(f) = Z_t K(f)_{t-1}^{\Theta} N(f)_t^{\Theta-1}$

where (as before) $N_t(f) \equiv [\int_H L_t(h,f)^{(\phi-1)/\phi} dh]^{\phi/(\phi-1)}$ and Z_t is a productivity shock.

(For the derivations in this section, we drop the firm index, f, and the time subscript, t.)

Firm's Cost Minimization Problem:

Choose K and N to minimize TC = RK + WN s.t. $\bar{Y} = ZK^{\Theta}N^{1-\Theta}$

FOC are (where *'s denote cost minimizing values):

$$R/P = MPK = Z\Theta(N^*/K^*)^{1-\Theta}$$
$$\Rightarrow \quad R/W = [\Theta/(1-\Theta)](N^*/K^*)$$
$$W/P = MPL = Z(1-\Theta)(K^*/N^*)^{\Theta}$$

Express TC = RK* + WN* in terms of Y:

$$Y = ZK^{*\Theta}N^{*1-\Theta} = ZK^{*\Theta}[(R/W)(\Theta-1)/\Theta]^{1-\Theta}K^{*1-\Theta} = ZK^*[(R/W)(\Theta-1)/\Theta]^{1-\Theta}$$

$$\Rightarrow K^* = (Y/Z)[(W/R)\Theta/(1-\Theta)]^{1-\Theta}$$

$$Y = ZK^{*\Theta}N^{*1-\Theta} = Z[(W/R)\Theta/(1-\Theta)]^{\Theta}N^{*\Theta}N^{*1-\Theta} = ZN^*[(W/R)\Theta/(1-\Theta)]^{\Theta}$$

$$\Rightarrow N^* = (Y/Z)[(W/R)\Theta/(1-\Theta)]^{-\Theta}$$

$$TC = RK^* + WN^* = (Y/Z)\{R[(W/R)(\Theta/(1-\Theta))]^{1-\Theta} + W[(W/R)\Theta/(1-\Theta)]^{-\Theta}\}$$
$$= (Y/Z)\{R^{\Theta}W^{1-\Theta}[\Theta/(1-\Theta)]^{1-\Theta} + W^{1-\Theta}R^{\Theta}[\Theta/(1-\Theta)]^{-\Theta}\}$$
$$= (Y/Z)(R^{\Theta}W^{1-\Theta})\{[\Theta/(1-\Theta)]^{1-\Theta} + [\Theta/(1-\Theta)]^{-\Theta}\}$$

$$[\Theta/(1-\Theta)]^{1-\Theta} + [\Theta/(1-\Theta)]^{-\Theta} = \Theta^{-\Theta}(1-\Theta)^{-(1-\Theta)}[\Theta + (1-\Theta)] = 1/[\Theta^{\Theta}(1-\Theta)^{(1-\Theta)}]$$

$$TC(Y) = (Y/Z)(R^{\Theta}W^{1-\Theta})/[\Theta^{\Theta}(1-\Theta)^{(1-\Theta)}] \quad \text{and} \quad MC_t = R_t^{\Theta}W_t^{1-\Theta}/[\Theta^{\Theta}(1-\Theta)^{(1-\Theta)}]Z_t$$

So, finally, real marginal cost is:

(6) $mc_t(f) = r_t^{\Theta}w_t^{1-\Theta}/[\Theta^{\Theta}(1-\Theta)^{(1-\Theta)}]Z_t$

where $r_t = R_t/P_t$ and $w_t = W_t/P_t$

A computable expression for aggregate sectoral output (or employment) –

This goes much the same as in Model 1.

Recall:

(8a) $Y(f)_t = Z_t K(f)_{t-1}{}^{\Theta} N(f)_t{}^{\Theta-1}$

(8b) $N(f)_t \equiv [\int_H L(h,f)_t{}^{(\phi-1)/\phi} dh]^{\phi/(\phi-1)}$

(8c) $Y_t \equiv [\int_F Y(f)_t{}^{(\sigma-1)/\sigma} df]^{\sigma/(\sigma-1)}$

(8d) $Y^d(f)_t = (P(f)_t/P_t)^{-\sigma} Y_t$

Recall: $R/W = [\Theta/(1-\Theta)](N^*/K^*) \Rightarrow K(f)_{t-1} = (W_t/R_t)(\Theta/(1-\Theta))N(f)_t \Rightarrow$

$$Y(f)_t = Z_t[(W_t/R_t)(\Theta/(1-\Theta)]^{\Theta} N(f)_t{}^{\Theta} N(f)_t{}^{1-\Theta} = Z_t[(W_t/R_t)(\Theta/(1-\Theta)]^{\Theta} N(f)_t$$

So, the aggregate demand for the composite labor input is:

(9a) $N_t = \int_F N_t(f)_t df = (1/Z_t)[(R_t/W_t)(1-\Theta)/\Theta]^{\Theta} \int_F Y(f)_t df$

$= (1/Z_t)[(R_t/W_t)(1-\Theta)/\Theta]^{\Theta} \int_F (P(f)_t/P_t)^{-\sigma} Y_t df = (1/Z_t)[(R_t/W_t)(1-\Theta)/\Theta]^{\Theta} Y_t DP_t$

where $DP_t \equiv \int_F (P(f)_t/P_t)^{-\sigma} df$

Or equivalently, the aggregate output is:

(9b) $Y_t = [\Theta/(1-\Theta)]^{\Theta} Z_t (W_t/R_t)^{\Theta} N_t/DP_t$

And, since aggregate capital is:

$K_{t-1} = \int_F K(f)_{t-1} df = (W_t/R_t)[\Theta/(1-\Theta)] \int_F N(f)_t = (W_t/R_t)[\Theta/(1-\Theta)]N_t$

(9b) can be rewritten as: $Y_t = Z\{[\Theta/(1-\Theta)](W_t/R_t)N_t\}^{\Theta} N_t{}^{1-\Theta}/DP_t$ where $\{...\} = K_{t-1}$

So, aggregate output can also be written as:

(9c) $Y_t = Z_t K_{t-1}{}^{\Theta} N_t{}^{1-\Theta}/DP_t$

Finally, DP_t can be calculated in the same way as in Model 1.

Model 2: (the Wage & Price inflation version)

(1) $\Lambda_t = (C_t - bC_{t-1})^{-1} - \beta b(C_{t+1} - bC_t)^{-1}$	$\Lambda = (1-\beta b)/(1-b)C$
(2) $K_t = (1 - \delta)K_{t-1} + I_t - ½\psi[(I_t/K_{t-1}) - \delta]^2K_{t-1}$	$I_t = \delta K_{t-1}$
(3) $r_t/w_t = [\Theta/(1-\Theta)](N_t/K_{t-1})$	$r/w = [\Theta/(1-\Theta)]N/K$
(4) $mc_t = r_t^{\Theta}w_t^{1-\Theta}/[\Theta^{\Theta}(1-\Theta)^{(1-\Theta)}]Z_t$	$r^{\Theta}w^{1-\Theta}/[\Theta^{\Theta}(1-\Theta)^{(1-\Theta)}]$
(5) $\Lambda_t = \xi_t - \xi_t\psi[(I_t/K_{t-1}) - \delta]$	$\Lambda = \xi$
(6) $\xi_t = \beta E_t\{\Lambda_{t+1}r_{t+1} + \xi_{t+1}[(1-\delta) - ½\psi[(I_{t+1}/K_t) - \delta]^2 + \psi[(I_{t+1}/K_t) - \delta](I_{t+1}/K_t)\}$	
(7) $p_t^* = (\mu_p/S_p)(pb_t/pa_t)$	$p^* = (\mu_p/S_p)(pb/pa)$
(8) $pb_t = \alpha\beta E_t[(P_{t+1}/P_t)^{\sigma}pb_{t+1}] + \Lambda_t mc_t Y_t$	$pb = (1-\alpha\beta)^{-1}\Lambda_t mcY$
(9) $pa_t = \alpha\beta E_t[(P_{t+1}/P_t)^{\sigma-1}pa_{t+1}] + \Lambda_t Y_t$	$pa_t = (1-\beta\alpha)^{-1}\Lambda Y$ 9
(10) $1 = (1-\alpha)p_t^{*1-\sigma} + \alpha(P_{t-1}/P_t)^{1-\sigma}$	$p^* = 1$
(11) $Y_t = Z_tK_{t-1}^{\Theta}N_t^{1-\Theta}/DP_t$	$Y = ZK^{\Theta}N^{1-\Theta}/DP$
(12) $DP_t = (1-\alpha)(1/p_t^*)^{\sigma} + (P_t/P_{t-1})^{\sigma}\alpha DP_{t-1}$	$DP_t = 1$
(13) $C_t + I_t + G_t = Y_t$	$C + I + G = Y$
(14) $w_t^{*1+\phi\chi} = (\mu_w/S_w)\kappa(wb_t/wa_t)$	$w_t^{*1+\phi\chi} = (\mu_w/S_w)\kappa(wb/wa)$
(15) $wb_t = \omega\beta E_t[(W_{t+1}/W_t)^{\phi(1+\chi)}wb_{t+1}] + N_t^{1+\chi}$	$wb = (1-\omega\beta)^{-1}N^{1+\chi}$
(16) $wa_t = \omega\beta E_t[(W_{t+1}/W_t)^{\phi-1}wa_{t+1}] + \Lambda_tN_tw_t$	$wa = (1-\omega\beta)^{-1}\Lambda Nw$
(17) $1 = (1-\omega)w_t^{*1-\phi} + \omega(W_{t-1}/W_t)^{1-\phi}$	$w^* = 1$
(18) $w_t/w_{t-1} = (W_t/W_{t-1})/(P_t/P_{t-1})$	
(19) $I_t^{-1} = \beta E_t[(\Lambda_{t+1}/\Lambda_t)(P_t/P_{t+1})]$	$I = 1/\beta$

where $\mu_p = \sigma/(\sigma-1)$, $\mu_w = \phi/(\phi-1)$, and $\Lambda_t = P_t\lambda_t$.

Adding 'rule of thumb' or 'myopic' or 'liquidity constrained' consumers:

Remarks:

1. G impulses lower both C and I in standard models like the one above. Some VAR studies and traditional wisdom suggest this is counterfactual. Many institutional models incorporate 'rule of thumb' households to try to overcome this perceived deficiency.
2. 'Liquidity Constrained' consumers (denoted by L) do not save or optimize intertemporally; they just work and consume what they earn each period. 'Optimizing' consumers (denoted by O) behave as previously modeled households.
3. Framework: There is a unit mass of L households and a unit mass of O households. L households are less productive (and receive lower wages) than O households. For simplicity, we assume that L household wages are proportional to O households wage.

The effective labor input entering the firms' production functions is:

(1) $N_t = [\zeta N_{O,\,t}^{(\eta-1)/\eta} + (1 - \zeta)N_{L,\,t}^{(\eta-1)/\eta}\,]^{\eta/(\eta-1)}, \quad 0.5 < \zeta < 1$

where $N_{O,\,t}$ is the labor input bundle of O households (the CES aggregate defined above), and $N_{L,\,t}$ is the labor input of any given L households.

The bundler's cost minimization implies:

(2) $[N_{L,\,t} / N_{O,\,t}]^{1/\eta} = (W_{O,\,t} / W_{L,\,t})\,[(1 - \zeta\,) / \zeta\,]$

and the bundlers' price (or wage) for the aggregate labor input is:

(3) $W_t = [\zeta^\eta\, W_{O,\,t}^{1-\eta} + (1 - \zeta)^\eta\, W_{L,\,t}^{1-\eta}\,]^{1/(1-\eta)}$

L household wages are proportional to O household wages (which are determined as before)

(4) $W_{L,\,t} / W_{O,\,t} = (1 - \zeta\,) / \zeta$

Remarks:

1. This implies that hours equalize across L and O households, and coincide with aggregate hours.
2. Given the new definitions of N and W, the firms' behavior follows the same equation (for marginal cost, etc) as before.

L households consume their entire disposable income (including a transfer (TR_t) each period):

(5) $C_{L,t} = (1 - \tau_{w,t})(W_{L,t} / P_t) N_{L,t} + TR_t$

Since both groups of households have unit mass, aggregate consumption (C_t) is

(6) $C_t = C_{O,t} + C_{L,t}$

Remarks:

1. The size of the elasticity η does not matter for ??
2. ζ and TR can be used to calibrate the importance of L households' behavior relative to O households' behavior.

Model 3a: a Two Country Model of a Monetary Union.

The Basic Framework:

1. We model two (basically) symmetric countries, each with a single sector, and capital formation; model 3b will generalize to traded and non-traded goods..
2. We introduce home bias to be able to discuss inflation differentials.
3. We use distortionary labor and sales taxes (with a European application in mind).
4. Capital is owned in by the households, and is rented to firms within the household's country.
5. Notation: Home and Foreign goods, and their prices, will be differentiated by subscript H's and F's; other wise F variables will be differentiated by *'s.
6. Many of the derivations can be seen directly from the multi sector Model 1; all that is required is a change of notation.

Aggregators and Price Indexes:

Here, for simplicity, we suppress time subscripts here.

The Home good is a CES aggregate of the home firms' product:

(1) $Y_H = [\int_0^1 Y_H(f)^{(\sigma-1)/\sigma} df]^{\sigma/(\sigma-1)}$ Bundler's aggregate

See earlier bundler's problem (page 3):

(2) $P_H = [\int_0^1 P_H(f)^{1-\sigma} df]^{1/(1-\sigma)}$ Bundler's price for Home good

(3) $Y_H{}^d(f) = (P_H/P_H(f))^\sigma Y_H$ Bundler's demand for output of home firm f

Similarly, the Foreign good is a CES aggregate of the foreign firms' product:

(1)* $Y_F = [\int_0^1 Y_F(f)^{(\sigma-1)/\sigma} df]^{\sigma/(\sigma-1)}$ Bundler's aggregate

(2)* $P_F = [\int_0^1 P_F(f)^{1-\sigma} df]^{1/(1-\sigma)}$ Bundler's price for Home good

(3)* $Y_F{}^d(f) = (P_F/P_F(f))^\sigma Y_F$ Bundler's demand for output of foreign firm f

Home consumption is a CES aggregate of the home and foreign goods:

(4) $C = [\mu^{1/\eta}C_H^{(\eta-1)/\eta} + (1-\mu)^{1/\eta}C_F^{(\eta-1)/\eta}]^{\eta/(1-\eta)}$

and similarly, the Foreign consumption is a CES aggregate of home and foreign goods:

(4)* $C^* = [\mu^{*1/\eta}C^{*(\eta-1)/\eta}_F + (1-\mu)^{*1/\eta}C^{*(\eta-1)/\eta}_H]^{\eta/(1-\eta)}$

Note: μ and μ^* (> 0.5) indicate the degree of home bias.

The Home consumption good bundler's problem is to:

$\text{Min}_{YH,YF}\quad P_HY_H + P_FY_F$ subject to $[\mu^{1/\eta}Y_H^{(\eta-1)/\eta} + (1-\mu)^{1/\eta}Y_F^{(\eta-1)/\eta}]^{\eta/(\eta-1)} = Y$

$\mathcal{L} = P_HY_H + P_FY_F + P\{Y - [\mu^{1/\eta}Y_H^{(\eta-1)/\eta} + (1-\mu)^{1/\eta}Y_F^{(\eta-1)/\eta}]^{\eta/(\eta-1)}\}$

FOC for Y_H:

$P_H = P\eta/(\eta-1)[...]^{[\eta/(\eta-1)]-1}[(\eta-1)/\eta]\mu^{1/\eta}Y_H^{[(\eta-1)/\eta]-1} = P[...]^{[\eta/(\eta-1)]-1}\mu^{1/\eta}Y_H^{[(\eta-1)/\eta]-1}$

(and since $[...] = Y^{(\eta-1)/\eta}$) $= P[Y^{(\eta-1)/\eta}]^{1/(\eta-1)}\mu^{1/\eta}Y_H^{-1/\eta} = P\mu^{1/\eta}(Y/Y_H)^{1/\eta}$

the Home bundler's demand for the Home good is:

$Y_H = \mu(P/P_H)^\eta Y$

and similarly, the Home bundler's demand for the Foreign good is:

$Y_F = (1-\mu)(P/P_F)^\eta Y$

to find the bundler's price:

$P = \min P_HY_H + P_FY_F$ (to produce Y = 1)

$= P_H\mu(P/P_H)^\eta + P_F(1-\mu)(P/P_F)^\eta$

$P^{1-\eta} = \mu P_H^{1-\eta} + (1-\mu)P_F^{1-\eta}$

$P = [\mu P_H^{1-\eta} + (1-\mu)P_F^{1-\eta}]^{1/(1-\eta)}$

So, the Home consumption bundler's demands and prices are given by:

(5) $Y_H = \mu(P/P_H)^\eta Y$ and $Y_F = (1-\mu)(P/P_F)^\eta Y \Rightarrow C_H/C_F = [\mu/(1-\mu)](P_F/P_H)^\eta$

(6) $P = [\mu P_H^{1-\eta} + (1-\mu)P_F^{1-\eta}]^{1/(1-\eta}$

and the Foreign consumption bundler's demands and price are given by:

(5)* $Y_H = (1-\mu^*)(P/P_H)^\eta Y$ and $Y_F = \mu^*(P/P_F)^\eta Y \Rightarrow C^*_F/C^*_H = [\mu^*/(1-\mu^*)](P^*_H/P^*_F)^\eta$

(6)* $P^* = [(1-\mu^*)P_H^{1-\eta} + \mu^*P_F^{1-\eta}]^{1/(1-\eta)}$

and note that:

(6a) $P^*/P = [P^{-(1-\eta)}(1-\mu^*)P_H^{1-\eta} + P^{-(1-\eta)}\mu^*P_F^{1-\eta}]^{1/(1-\eta)} = [(1-\mu^*)(P_H/P)^{1-\eta} + \mu^*(P_F/P)^{1-\eta}]^{1/(1-\eta)}$

Home investment is a CES aggregate of the home and foreign goods:

(7) $I = [\mu_I^{1/\eta} I_H^{(\eta-1)/\eta} + (1-\mu_I)^{1/\eta} I_F^{(\eta-1)/\eta}]^{\eta/(1-\eta)}$

and similarly, the Foreign investment is a CES aggregate of home and foreign goods:

(7)* $I^* = [\mu_I^{*1/\eta} I^*_F{}^{(\eta-1)/\eta} + (1-\mu_I^*)^{1/\eta} I^*_H{}^{(\eta-1)/\eta}]^{\eta/(1-\eta)}$

This is just like the consumption aggregations, but with different bias parameters; so,

(8) $P_I = [\mu_I P_H^{1-\eta} + (1-\mu_I) P_F^{1-\eta}]^{1/(1-\eta)}$ and $p_I = [\mu_I p_H^{1-\eta} + (1-\mu_I) p_F^{1-\eta}]^{1/(1-\eta)}$ plus ratio equation

(8)* $P_{I^*} = [(1-\mu_I^*) P_H^{1-\eta} + \mu_I^* P_F^{1-\eta}]^{1/(1-\eta)}$ and $p_{I^*} = [(1-\mu_I^*) p_H^{1-\eta} + \mu_I^* p_F^{1-\eta}]^{1/(1-\eta)}$ plus ratio equation

where $p_x = P_x/P$ is a price normalized on the Home CPI

As before, each Home household h works at each Home firm f. W(h) is the household's wage (which does not depend on the firm f). Each Home firm f has a labor bundler:

(9) $N(f) \equiv [\int_0^1 L(h,f)^{(\phi-1)/\phi} dh]^{\phi/(\phi-1)}$, $\phi > 1$ (labor input of firm f)

(10) $L(h,f) = (W/W(h))^{\phi} N(f)$ (labor demand of bundler for firm f)

(11) $W = [\int_0^1 W(h)^{1-\phi} dh]^{1/(1-\phi)}$ (wage charged by bundler to any firm f)

(12) $L(h) = \int_0^1 L(h,f) df$ and $N = \int_0^1 N(f) df$ (aggregate hours worked and labor input)

(13) $L(h)^d = (W/W(h))^{\phi} N$ (integrating demands over f in (10))

Similar equations hold for the composite labor input in the Foreign country.

Home Household Intertemporal Maximization Problem:

(14) $U_t(h) = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} [(1-\Theta)(C_\tau(h))^{1-\Theta} - (1+\chi)^{-1} L_\tau(h)^{1+\chi} + v((M_\tau(h)/P_\tau)]$

Budget constraint of household h:

(15) $M_\tau(h) + E_\tau[\Delta_{\tau,\tau+1} B_{\tau+1}(h)] + S_c P_\tau C_\tau(h) + P_{k,\tau} I_\tau(h) + P_\tau T_\tau]$

$= S_w W_\tau(h) L^d_\tau(h) + S_m M_{\tau-1}(h) + B_\tau(h) + R_\tau K_{\tau-1}(h) + D_\tau(h)$

where (as before) $B_{\tau+1}(h)$ is a state contingent claim, $\Delta_{\tau,\tau+1}$ is the stochastic discount factor; $D_\tau(h)$ are dividends, T_τ is a lump sum tax, S_c, S_w and S_m are tax/subsidies on consumption, labor and money holdings (eg $S_w = (1 - \tau_w)$). R_τ is the rental rate on domestic capital.

Capital constraint for home household h:

(16) $K_\tau(h) = (1 - \delta) K_{\tau-1}(h) + I_\tau(h) - \tfrac{1}{2}\psi[(I_\tau(h)/K_{\tau-1}(h)) - \delta]^2 K_{\tau-1}(h)$

where the last term represents the capital adjustment cost; note: K_t is the end of period stock.

<u>Home household's problem</u>: choose I_t, K_t, $B_{t+1}(h)$, $C_t(h)$, $M_t(h)$, and $W_t(h)$ to maximize (14) subject to (15), (16) and the labor demand curve (13).

$$\mathcal{L} = E_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}\{[(1-\Theta)(C_\tau(h))^{1-\Theta} - (1+\chi)^{-1}L_\tau(h)^{1+\chi} + ...]$$
$$- \lambda_\tau[E_\tau[\Delta_{\tau,\tau+1}B_{\tau+1}(h)] + S_{c,\tau}P_\tau C_\tau(h) + P_{I,\tau}I_\tau(h) + P_\tau T_\tau] - S_{w,\tau}W_\tau(h)L^d_\tau(h) - B_\tau(h) - R_\tau K_{\tau-1}(h) + ...]$$
$$+ \xi_\tau[(1 - \delta)K_{\tau-1}(h) + I_\tau(h) - \tfrac{1}{2}\psi[(I_\tau(h)/K_{\tau-1}(h)) - \delta]^2K_{\tau-1}(h) - K_\tau(h)\}$$

The FOC include:

$C_t(h)$: $C_t(h)^{-\Theta} = \lambda_t S_{c,t}P_t$

$I_t(h)$: $\lambda_t P_{I,t} = \xi_t - \xi_t\psi[(I_t(h)/K_{t-1}(h)) - \delta]$

$K_t(h)$: $\xi_t = \beta E_t\{\lambda_{t+1}R_{t+1} + \xi_{t+1}[(1-\delta) - \tfrac{1}{2}\psi[(I_{t+1}(h)/K_t(h))-\delta]^2 - \psi K_t(h)[(I_{t+1}(h)/K_t(h))-\delta](-I_{t+1}(h)/K_t(h)^2)]\}$

$= \beta E_t\{\lambda_{t+1}R_{t+1} + \xi_{t+1}[(1-\delta) - \tfrac{1}{2}\psi[(I_{t+1}(h)/K_t(h)) - \delta]^2 + \psi[(I_{t+1}(h)/K_t(h)) - \delta](I_{t+1}(h)/K_t(h))]\}$

$B_{t+1}(h)$: $\Delta_{t,t+1} = \beta\lambda_{t+1}/\lambda_t \Rightarrow R_t^{-1} = E_t[\Delta_{t,t+1}] = \beta E_t[\lambda_{t+1}/\lambda_t]$

Note: as before, complete markets mean that $\lambda_t(h) = \lambda_t$ for all h.

Converting to the "real" model: Here we normalize on P_t, the Home CPI. As before, let

$\partial U_t/\partial C_t = S_{c,t}P_t\lambda_t = S_{c,t}\Lambda_t$; so, $\Lambda_t = P_t\lambda_t$ is the marginal utility of the final Home C_t.

Writing the FOC in aggregate values:

(17) $C_t^{-\Theta} = S_{c,t}\Lambda_t$

(18) $\Lambda_t p_{I,t} = \xi_t - \xi_t\psi[I_t/K_{t-1} - \delta]$

(19) $\xi_t = \beta E_t\{\Lambda_{t+1}r_{t+1} + \xi_{t+1}[(1-\delta) - \tfrac{1}{2}\psi[(I_{t+1}/K_t) - \delta]^2 + \psi[(I_{t+1}/K_t) - \delta](I_{t+1}/K_t)]\}$

(20) $R_t^{-1} = \beta E_t[\lambda_{t+1}/\lambda_t] = \beta E_t[(\Lambda_{t+1}/\Lambda_t)(P_t/P_{t+1})]$

Further implications of Consumption Risk Sharing:

Since all households in both countries face the same contingent claims prices (and probabilities):

$\beta\lambda(h)_{t+1}/\lambda(h)_t = \Delta_{t,t+1} = \beta\lambda^*(h^*)_{t+1}/\lambda^*(h^*)_t$ for all h and h*.

$S_{c,t}P_tC_t^{\Theta}/S_{c,t+1}P_{t+1}C_{t+1}^{\Theta} = \Delta_{t,t+1}/\beta = S^*_{c,t}P^*_tC^{*\Theta}_t/S^*_{c,t+1}P^*_{t+1}C^{*\Theta}_{t+1}$

$S_{c,t}P_tC_t^{\Theta} = \Gamma\, S^*_{c,t}P^*_tC^{*\Theta}_t$ where $\Gamma = S_{c,0}P_0C_0^{\Theta}/S^*_{c,0}P^*_0C^{*\Theta}_0$ (= 1?), or

(18) $(C_t/C^*_t)^{\Theta}(S_{c,t}/S^*_{c,t}) = \Gamma\, RER_t$ where $RER_t = P^*_t/P_t$

Remark: $S_{c,t}/S^*_{c,t}$ is perhaps worse than the Ireland shock problem in the risk sharing equation!

Note: $p_H = P_H/P = (P^*/P)(P_H/P^*) = RER\; p^*_H$ and similarly $p_F = RER\; p^*_F$

Note: in the "real" model, price levels are suppressed, and inflation rates appear in the coding. We need an equation that relates the home and foreign inflation rates.

Converting (or replacing) an Euler equation in the "real" model:

Note: by definition, $RER_{t+1}/RER_t = \Pi^*_{t+1}/\Pi_{t+1}$, where $\Pi_t = P_t/P_{t-1}$ and $\Pi^*_t = P^*_t/P^*_{t-1}$

So,

(20) $R_t^{-1} = \beta E_t[(\Lambda_{t+1}/\Lambda_t)/\Pi_{t+1}]$

(20)* $R_t^{-1} = \beta E_t[(\Lambda^*_{t+1}/\Lambda^*_t)/\Pi^*_{t+1}]$

can be replaced by:

(20) $R_t^{-1} = \beta E_t[(\Lambda_{t+1}/\Lambda_t)/\Pi_{t+1}]$

(20a)* $E_t(RER_{t+1}/RER_t) = E_t(\Pi^*_{t+1}/\Pi_{t+1})$

since (20) and (20a)* $\Rightarrow$ (20)*

To see this, recall (17) $C_t^{-\Theta} = S_{c,t}\Lambda_t$, (17)* $C^{*\,-\Theta}_t = S^*_{c,t}\Lambda^*_t$ and (18) $(C_t/C^*_t)^{\Theta}(S_{c,t}/S^*_{c,t}) = RER_t$

These equations $\Rightarrow$

$$RER_t = (C_t/C^*_t)^{\Theta}(S_{c,t}/S^*_{c,t}) = \Lambda^*_t/\Lambda_t$$

So,

$$(\Lambda^*_{t+1}/\Lambda^*_t)/(\Lambda_{t+1}/\Lambda_t) = RER_{t+1}/RER_t = \Pi^*_{t+1}/\Pi_{t+1} \quad \text{or} \quad (\Lambda^*_{t+1}/\Lambda^*_t)/\Pi^*_{t+1} = (\Lambda_{t+1}/\Lambda_t)/\Pi_{t+1}$$

And,

$$R_t^{-1} = \beta E_t[(\Lambda_{t+1}/\Lambda_t)\Pi_{t+1}] = \beta E_t[(\Lambda^*_{t+1}/\Lambda^*_t)\Pi^*_{t+1}]$$

Optimal wage setting in period t –

This is the same as above except that $S_{w,t}$ now has a time subscript and must be kept inside the summation sign.

(19) $W^n_t(h)^{1+\phi\chi} = \mu_w\kappa(WB_t/WA_t)$

where $\mu_w \equiv \phi/(\phi\text{-}1)$

$WB_t \equiv E_t\sum_{j=t}^{\infty}(\omega\beta)^{j-t}N_j^{1+\chi}W_j^{\phi(1+\chi)} = \omega\beta E_tWB_{t+1} + N_t^{1+\chi}W_t^{\phi(1+\chi)}$

$WA_t \equiv E_t\sum_{j=t}^{\infty}(\omega\beta)^{j-t}\lambda_jS_{w,j}N_jW_j^{\phi} = \omega\beta E_tWA_{t+1} + \lambda_tS_{w,t}N_tW_t^{\phi}$

(20) $W_t^{1-\phi} = (1\text{-}\omega)W^{n\ 1-\phi}_t + \omega(W_{t-1})^{1-\phi}$

Converting to the "real" model – as before (page 17)

(19) $w^n_t{}^{1+\phi\chi} = \mu_w\kappa(wb/wa)$ $\qquad$ $w^{n\ 1+\phi\chi} = \mu_w\kappa(wb/wa)$

$wb_t = \omega\beta E_t[(W_{t+1}/W_t)^{\phi(1+\chi)}wb_{,t+1}] + N_t^{1+\chi}$ $\qquad$ $wb = (1\text{-}\omega\beta)^{-1}N^{1+\chi}$

$wa_t = \omega\beta E_t[(W_{t+1}/W_t)^{\phi-1}wa_{t+1}] + S_{w,t}\Lambda_tN_tw_t$ $\qquad$ $wa = (1\text{-}\omega\beta)^{-1}S_w\Lambda Nw$

(20) $1 = (1\text{-}\omega)w^n_t{}^{1-\phi} + \omega(W_{t-1}/W_t)^{1-\phi}$

where w_t is the real wage, but $w^n_t = W^n_t/W_t$, $wa_t = WA_t/W_t^{\phi-1}$ and $wb_t = WB_t/W_t^{\phi(1+\chi)}$

and $w_t/w_{t-1} = (W_t/W_{t-1})/(P_t/P_{t-1})$

Home firm f's cost minimization problem and nominal marginal cost:

Here the development is the same; see above.

Firm f's production function is:

(21) $Y_{H,t}(f) = Z_tK_{t-1}(f)^{\Theta}N(f)_t^{\Theta-1}$

where (as before) $N_t(f) \equiv [\int_0^1L_t(h,f)^{(\phi-1)/\phi}dh]^{\phi/(\phi-1)}$ and Z_t is a productivity shock.

(22) $r_t/w_t = [\Theta/(1\text{-}\Theta)](N_t/K_{t-1})$

(23) $mc_t(f) = r_t^{\Theta}w_t^{1-\Theta}/[\Theta^{\Theta}(1\text{-}\Theta)^{(1-\Theta)}]Z_t$

where $r_t = R_t/P_t$ and $w_t = W_t/P_t$

Note: here again, this is real marginal cost in terms of the CPI, and not P_H.

Optimal price setting in period t –

Firm-f seeks to maximize it's market value for Home households:

$MV_t = E_t\sum_{j=t}^{\infty}\beta^{j-t}\lambda_j[S_pP_{H,j}(f)Y_{H,j}(f) - TC_j(Y_{H,j}(f))]$, where TC is total cost and S_p is a (constant) price subsidy.

Remark:

Using the Home λ to value the firm is suspicious in a two country setting, especially if the risk sharing assumption is eliminated. There are a host of issues here.

With probability α^{j-t}, the new price $P^n_{H,t}(f)$ will be in effect in period j; so, a Home firm-f (which gets to reset its price) sets $P^n_{H,t}(f)$ to maximize:

$$MV_t = E_t\sum_{j=t}^{\infty}(\alpha\beta)^{j-t}\lambda_j[S_pP^n_{H,t}(f)Y_{H,j}(f) - TC_j(Y_{H,j}(f))], \text{ where } Y_{H,j}(f) = (P^n_{H,t}(f)/P_{H,j})^{-\sigma}Y_{H,j}$$

$$= E_t\sum_{j=t}^{\infty}(\alpha\beta)^{j-t}\lambda_j[S_pP^n_{H,t}(f)^{1-\sigma}(1/P_{H,j})^{-\sigma}Y_{H,j} - TC_j(P^n_{H,t}(f)^{-\sigma}(1/P_H)^{-\sigma}Y_{H,j})]$$

As before (see Model 1), we have:

(24) $P^n_{H,t}(f) = (\mu_p/S_p)(PB_t/PA_t)$

where

$$PB_t = \alpha_H\beta E_tPB_{t+1} + \lambda_tMC_tP_{H,t}{}^{\sigma}Y_{H,t}$$

$$PA_t \equiv E_t\sum_{\tau=t}^{\infty}(\alpha\beta)^{\tau-t}\lambda_\tau P_{H,}\sigma Y_\tau = \alpha_H\beta E_tPA_{t+1} + \lambda_tP_{H,t}{}^{\sigma}Y_{H,t}$$

(25) $P_{H,t}{}^{1-\sigma} = (1-\alpha_H)P^n_{H,t}{}^{1-\sigma} + \alpha_H(P_{H,t-1})^{1-\sigma}$

Converting to the "real" model – Here again we normalize on P_t; let $P_t\lambda_t = \Lambda_t$ and $p^n_{H,t} = P^n_{H,t}/P_t$

$(24)_{real}$ $p^n_{H,t} = (\mu_p/S_p)(pb_t/pa_t)$

$$pb_t = \alpha_H\beta E_t[(P_{t+1}/P_t)^{\sigma}pb_{t+1}] + \Lambda_tmc_tp_{H,t}{}^{\sigma}Y_{H,t}$$

$$pa_{s,t} = \alpha_H\beta E_t[(P_{t+1}/P_t)^{\sigma-1}pa_{t+1}] + \Lambda_tp_{H,t}{}^{\sigma}Y_{H,t}$$

And finally, the aggregate sectoral price (10) becomes:

$(25)_{real}$ $p_{H,t}{}^{1-\sigma} = (1-\alpha_H)p^n_{H,t}{}^{1-\sigma} + \alpha_Hp_{H,t-1}{}^{1-\sigma}(P_{t-1}/P_t)^{1-\sigma}$

A computable expression for aggregate sectoral output (or employment) –

This too goes the same as in Model 2; see that development.

Recall:

$$Y_{H,t}(f) = Z_t K(f)_{t-1}{}^{\Theta} N(f)_t{}^{\Theta-1} \qquad N(f)_t \equiv [\int_0^1 L(h,f)_t{}^{(\phi-1)/\phi} dh]^{\phi/(\phi-1)}$$

$$Y_{H,t} \equiv [\int_0^1 Y_{H,t}(f)^{(\sigma-1)/\sigma} df]^{\sigma/(\sigma-1)} \qquad Y^d_{H,t}(f) = (P_{H,t}(f)/P_{H,t})^{-\sigma} Y_t$$

The aggregate output can be written as:

(26) $Y_{H,t} = Z_t K_{t-1}{}^{\Theta} N_t{}^{1-\Theta}/DP_{H,t}$

Note: in the coding we replace Θ with ν.

and $DP_{H,t}$ can be calculated in the same way as in Model 1.

$(27)_{nominal}$ $DP_{H,t} = (1-\alpha_H)(P_{H,t}/P^n_{H,t})^{\sigma} + (P_{H,t}/P_{H,t-1})^{\sigma}\alpha_H DP_{H,t-1}$

Converting to the "real" model – Here we normalize nominal variables on P_t.

$(27)_{real}$ $DP_{H,t} = (1-\alpha_H)(p_{H,t}/p^n_{H,t})^{\sigma} + (p_{H,t}/p_{H,t-1})^{\sigma}(P_t/P_{t-1})^{\sigma}\alpha_H DP_{H,t-1}$

Market Equilibrium Conditions:

(28) $Y_{H,t} = C_{H,t} + C^*_{H,t} + I_{H,t} + I^*_{H,t} + G_t$

(28)* $Y_{F,t} = C_{F,t} + C^*_{F,t} + I_{F,t} + I^*_{F,t} + G^*_t$

Government budget constraint and policy:

Equations to code up –

aggregator and price index block:

$$C = [\mu^{1/\eta}C_H^{(\eta-1)/\eta} + (1-\mu)^{1/\eta}C_F^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$$

$$1 = [\mu p_H^{1-\eta} + (1-\mu)p_F^{1-\eta}]^{1/(1-\eta)}$$

$$C_H/C_F = [\mu/(1-\mu)](p_F/p_H)^\eta$$

$$I = [\mu_I^{1/\eta}I_H^{(\eta-1)/\eta} + (1-\mu_I)^{1/\eta}I_F^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$$

$$p_I = [\mu_I p_H^{1-\eta} + (1-\mu_I)p_F^{1-\eta}]^{1/(1-\eta)}$$

$$I_H/I_F = [\mu_I/(1-\mu_I)](p_F/p_H)^\eta$$

$$C^* = [\mu^{*1/\eta}C^*_F{}^{(\eta-1)/\eta} + (1-\mu^*)^{1/\eta}C^*_H{}^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$$

$$1 = [\mu^* p^*_F{}^{1-\eta} + (1-\mu^*)p^*_H{}^{1-\eta}]^{1/(1-\eta)}$$

$$C^*_F/C^*_H = [\mu^*/(1-\mu^*)](p^*_H/p^*_F)^\eta$$

$$I^* = [\mu_I^{*1/\eta}I^*_F{}^{(\eta-1)/\eta} + (1-\mu_I^*)^{1/\eta}I^*_H{}^{(\eta-1)/\eta}]^{\eta/(\eta-1)}$$

$$p_{I*} = [\mu_I^* p^*_F{}^{1-\eta} + (1-\mu_I^*)p^*_H{}^{1-\eta}]^{1/(1-\eta)}$$

$$I^*_F/I^*_H = [\mu_I^*/(1-\mu_I^*)](p^*_H/p^*_F)^\eta$$

$$Y_H = C_H + C^*_H + I_H + I^*_H + G$$

$$Y_F = C_F + C^*_F + I_F + I^*_F + G^*$$

Euler equation, risk sharing and investment block:

$$C^{-\Theta} = S_c\Lambda$$

$$\Lambda p_I = \xi - \xi\psi[I/K(-1) - \delta]$$

$$\xi = \beta\{\Lambda(+1)r(+1) + \xi(+1)[(1-\delta) - \tfrac{1}{2}\psi[(I(+1)/K) - \delta]^2 + \psi[(I(+1)/K) - \delta](I(+1)/K)]\}$$

$$K = (1 - \delta)K(-1) + I - \tfrac{1}{2}\psi[(I/K(-1)) - \delta]^2K(-1)$$

$$R^{-1} = \beta(\Lambda(+1)/\Lambda)(1/\Pi(+1))$$

$$(C/C^*)^\Theta(S_c/S^*_c) = \Gamma\, RER \quad \text{where } RER = P^*/P$$

$$p_H = RER\ p^*_H$$

$$p_F = RER\ p^*_F$$

$$C^{*-\Theta} = S^*_c\Lambda^*$$

$$\Lambda^* p^*_K = \xi^* - \xi^*\psi^*[I^*/K^*(-1) - \delta^*]$$

$$\xi^* = \beta\{\Lambda^*(+1)r^*(+1) + \xi^*(+1)[(1-\delta^*) - \tfrac{1}{2}\psi^*[(I^*(+1)/K^*) - \delta^*]^2 + \psi^*[(I^*(+1)/K^*) - \delta^*](I^*(+1)/K^*)]\}$$

$$RER_t/RER_{t-1} = \Pi^*_t/\Pi_t$$ (replacing the foreign Euler equation, as explained on pg 29)

$$K^* = (1 - \delta)K^*(-1) + I^* - \tfrac{1}{2}\psi[(I^*/K^*(-1)) - \delta]^2K^*(-1)$$

Production function block:

equations for Z and Z* processes

$Y_H = ZK(-1)^{\nu}N^{1-\nu}/DP_H$

$DP_H = (1-\alpha)(p_H/p_H^n)^{\sigma} + (p_H/p_H(-1))^{\sigma}\Pi^{\sigma}\alpha DP_H(-1)$

$r/w = [\nu/(1-\nu)](N/K(-1))$

$mc = r^{\nu}w^{1-\nu}/[\nu^{\nu}(1-\nu)^{(1-\nu)}]Z$

$Y_F = Z^*K^*(-1)^{\nu}N^{*1-\nu}/DP_F$

$DP_F = (1-\alpha^*)(p_F/p_F^n)^{\sigma} + (p_F/p_F(-1))^{\sigma}\Pi^{*\sigma}\alpha^* DP_F(-1)$

$r^*/w^* = [\nu/(1-\nu)](N^*/K^*(-1))$

$mc^* = r^{*\nu}w^{*1-\nu}/[\nu^{\nu}(1-\nu)^{(1-\nu)}]Z^*$

Calvo price setting block:

$p_H^n = (\mu_p/S_p)(pb/pa)$

$pb = \alpha\beta(\Pi(+1))^{\sigma}pb(+1) + \Lambda mc p_H{}^{\sigma}Y_H$

$pa = \alpha\beta(\Pi(+1))^{\sigma-1}pa(+1) + \Lambda p_H{}^{\sigma}Y_H$

$p_H{}^{1-\sigma} = (1-\alpha)p_H^{n\ 1-\sigma} + \alpha p_H(-1)^{1-\sigma}(1/\Pi)^{1-\sigma}$

$p_F^n = (\mu^*_p/S^*_p)(pb^*/pa^*)$

$pb^* = \alpha^*\beta(\Pi^*(+1))^{\sigma}pb^*(+1) + \Lambda^* mc^* p_F{}^{\sigma}Y_F$

$pa^* = \alpha^*\beta(\Pi^*(+1))^{\sigma-1}pa^*(+1) + \Lambda^* p_F{}^{\sigma}Y_F$

$p_F{}^{1-\sigma} = (1-\alpha^*)p_F^{n\ 1-\sigma} + \alpha p_F(-1)^{1-\sigma}(1/\Pi^*)^{1-\sigma}$

Calvo wage setting block:

$w^{n\ 1+\phi\chi} = \mu_w\kappa(wb/wa)$

$wb = \omega\beta\Pi_w(+1)^{\phi(1+\chi)}wb(+1) + N^{1+\chi}$

$wa = \omega\beta\Pi_w(+1)^{\phi-1}wa(+1) + S_w\Lambda Nw$

$1 = (1-\omega)w^{n\ 1-\phi} + \omega(1/\Pi_w)^{1-\phi}$

$w/w(-1) = \Pi_w/\Pi$

$w^{*n\ 1+\phi\chi} = \mu_w\kappa(wb^*/wa^*)$

$wb^* = \omega^*\beta\Pi^*_w(+1)^{\phi(1+\chi)}wb^*(+1) + N^{*1+\chi}$

$wa^* = \omega^*\beta\Pi^*_w(+1)^{\phi-1}wa^*(+1) + S^*_w\Lambda^* N^* w^*$

$1 = (1-\omega^*)w^{*n\ 1-\phi} + \omega^*(1/\Pi^*_w)^{1-\phi}$

$w^*/w^*(-1) = \Pi^*_w/\Pi$

fiscal policy block:

equations for G and G* processes

$S_c = S_w = S_c = S_w = 1$

monetary policy block:

$0.5\Pi + 0.5\Pi^* = 1$

References:

Bayoumi, Tamim, Douglas Laxton, and Paolo Pesenti, "When Leaner Isn't Meaner: Measuring Benefits and Spillovers of Greater Competition in Europe," mimeo, April 2003.

Erceg, Christopher, Dale Henderson, and Andrew Levin, "Optimal Monetary Policy with Staggered Wage and Price Contracts", JME, 46, 2000, pg. 281 - 313.

Gali, Jordi, Mark Gertler and J. David Lopez-Salido, "Markups, Gaps, and the Welfare Costs of Business Cycles," NBER Working Paper #8850, March 2002.

King, Robert, and Sergio Rebelo, "Resuscitating Real Business Cycles," in (Ch. 14) Taylor, John and Michael Woodford (eds), Handbook of Macroeconomic, Volume 1B, Elsevier, 1999, Amsterdam.