NOTES ON MONOPOLISTIC COMPETITION AND NOMINAL INERTIA

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This set of notes complements Canzoneri, Cumby and Diba (2002b). Canzoneri, Cumby and Diba (2001) extends the analysis to two country models, and Canzoneri, Cumby and Diba (2002a) adds fiscal policy.

Note 1: The Yeoman Farmer Model

The basic fundamentals are more easily understood in the Yeoman Farmer Model, where no distinction is made between households and firms. Students may be well advised to read these notes before Canzoneri, Cumby and Diba (2002b).

Note 2: The "New Keynesian Phillips Curve"

Using "Calvo contracts", we log linearize the price setting equation and derive what is being called the New Keynesian Phillips Curve. We note some important caveats as to how this Phillips Curve should be used; we think it is being widely abused in the literature.

Note 3: Linearizing the Demand Side

We log linearize the Euler equation and note some caveats as to its use.

Note 4: Unbundling the Bundler

In Canzoneri, Cumby and Diba (2...), we used the popular artifice of a "bundler" to short circuit the algebra involved with the composite goods and their prices. Here, we provide some of the missing algebra. In particular, we show how to separate the consumer's maximization into a "temporal" problem and an "intertemporal" problem.

Note 5: Interest Rate Rules

In the Canzoneri, Cumby and Diba (2...), we assumed the central bank set nominal income. This allowed us to solve models without worrying about the dynamics inherent in the Euler equation. However, the literature has (rightly) focused on interest rate rules instead of money targeting. Here, we show how the method of undetermined coefficients can be used to solve the Yeoman Farmer Model under interest rate targeting.

Note 6: Sticky Wage/Flexible Price Model with Non-linear Production

In the notes, we assumed that production was linear. This fixed the flex-price real wage, with strong implications for monetary policy. Here, we see that stabilization policy affects the average level of real wages and output. In addition, monetary innovations affect the current price level and current real wage rate.

Note 1: ***The Yeoman-Farmer Model –***

Household j's Utility –

(1) $U^j_t = E_t\sum^{\infty}_{s=t}\beta^{s-t}[u(C^j_s) - h(N^j_s)] = E_t\sum^{\infty}_{s=t}\beta^{s-t}[(1-\gamma)^{-1}C^{j\,1-\gamma}_s - (1+\chi)^{-1}A_s(N^j_s)^{1+\chi}]$

where $Y^j_s(j) = Z_sN^j_s$, $C^j_s \equiv [\int^1_0 C^j_s(f)^{(\theta-1)/\theta}df]^{\theta/(\theta-1)}$ and $\theta > 1$.

Remarks:

1. There is a continuum of households (indexed by $j \in [0,1]$); each produces a differentiated product (indexed by f); C^j_s is a consumption "bundle"(or composite good); production is linear in labor, and $Y^j_s(j)$ is household j's supply of product f = j; Z_s is an aggregate productivity shock; A_s is an aggregate preference shock. θ will be the elasticity of demand for good f; θ must be > 1 for an interior solution. (Do we need the (j) index?)
2. u(C) is the utility of consumption, and - h(N) is the disutility of work. Sometimes, we will specialize to the constant elasticity specifications given in (1).
3. Production is linear in our Yeoman-Farmer Model. This simplifies the algebra greatly, but it also has implications for the interpretation of the "supply"shocks, A_s and Z_s:
   a. Using the production function to eliminate N^j_s in the utility function – so that $(1+\chi)^{-1}A_s(N^j_s)^{1+\chi}$ becomes $(1+\chi)^{-1}A_s(Y^j_s(j)/Z_s)^{1+\chi}$ – we see that only the combination shock, $A/Z^{1+\chi}$, will matter in the solution. So, in the Yeoman-Farmer Model, we will set $A_s \equiv 1$; this simplifies the algebra. We will reintroduce the labor supply shock in later sections, when we decentralized the economy and the distinction between labor supply shocks and productivity shocks can be important.
   b. We will see that the analysis of the optimal monetary policy response to productivity shocks is also somewhat limited when production is linear.
   c. We will let production be concave in the Sticky Wage Model of Appendix 5.

Bundling:

Remarks:

1. $C^j \equiv [\int_0^1 C^j(f)^{(\theta-1)/\theta} df]^{\theta/(\theta-1)}$ is a composite good with implicit price deflator $P \equiv [\int_0^1 P(f)^{1-\theta} df]^{1/(1-\theta)}$. See Appendix 3 for a full discussion of how to decompose the household's maximization problem into "intratemporal" and an "intertemporal" problems, and how to derive P.
   Other references include:
   Frenkel and Razin, Fiscal Policies in the World Economy, 1987, Chapter 6.
   Blanchard and Fischer, Lectures in Macroeconomics, 1989, Chapter 8.
2. Chari, Kehoe and McGrattan (1996) assume the artifice of a "bundler" to shorten the discussion and lighten the algebra.

The "bundler" is a competitive (or zero profit) firm that buys the Y(f) at price P(f), bundles them into the composite good, $Y \equiv [\int_0^1 Y(f)^{(\theta-1)/\theta} df]^{\theta/(\theta-1)}$, and then sells the composite good at price P. Here, we derive P and the bundler's demand for Y(f).

The bundler minimizes the cost of "producing" a given amount of Y:

$\min_{Y(f)} \int_0^1 P(f)Y(f)df \quad \text{s.t.} \quad Y = [\int_0^1 Y(f)^{(\theta-1)/\theta} df]^{\theta/(\theta-1)}$

$\mathcal{L} = \int_0^1 P(f)Y(f)df + P\{Y - [\int_0^1 Y(f)^{(\theta-1)/\theta} df]^{\theta/(\theta-1)}\}$

Note: P = MC (= Lagrangian multiplier) since the bundler is a competitive producer.

First Order Condition –

$P(f) = P[\int_0^1 Y(f)^{(\theta-1)/\theta} df]^{[\theta/(\theta-1)]-1} Y(f)^{[(\theta-1)/\theta]-1} = PY^{1/\theta}Y(f)^{-1/\theta} = P(Y(f)/Y)^{-1/\theta} \Rightarrow Y(f) = (P(f)/P)^{-\theta}Y$

To find P, use FOC to eliminate Y(f) in $Y = [\int_0^1 Y(f)^{(\theta-1)/\theta} df]^{\theta/(\theta-1)}$

$Y = [\int_0^1 Y(f)^{(\theta-1)/\theta} df]^{\theta/(\theta-1)} = [\int_0^1 [(P(f)/P)^{-\theta}Y]^{(\theta-1)/\theta} df]^{\theta/(\theta-1)} = (YP^{\theta})[\int_0^1 [P(f)^{-(\theta-1)} df]^{\theta/(\theta-1)}$

$P^{-\theta} = [\int_0^1 [P(f)^{-(\theta-1)} df]^{\theta/(\theta-1)} \Rightarrow P = [\int_0^1 P(f)^{1-\theta} df]^{1/(1-\theta)}$

Collecting results:

(2) $P = [\int_0^1 P(f)^{1-\theta} df]^{1/(1-\theta)}$ Price of composite good Y (or C)

(3) $Y^d(f) = (P(f)/P)^{-\theta}Y$ Demand for good Y(f)

Household j’s cash in advance (CIA) constraint in period s –

(4) $M^j_s + \upsilon_s P_s Y_s = P_s C^j_s$

Remark:

1. Following Canzoneri and Dellas (JME, 42, 1998), we assume that the fraction υ_s of per capita aggregate income Y_s can be bought on credit. The basic idea goes back to Woodford (1991), who takes υ as a measure of the sophistication of the financial system.
2. We let υ_s be a random variable; this (we will see) is a way of introducing a velocity shock.

Household j's BC in period s –

(5) $M^j_s + B^j_s + P_s\overline{\tau}_s = \tau_s P_{s-1}(j)Y^j_{s-1}(j) + I_{s-1}B^j_{s-1} = \tau_s P_{s-1}(j)^{1-\theta}(1/P_{s-1})^{-\theta}Y_{s-1} + I_{s-1}B^j_{s-1}$

Remarks:

1. Household-j maximizes (1) s.t. (4) and (5), and $Y^j_{s-1}(j) = Y^d_{s-1}(j)$, where $Y^d_s(j)$ is given by (3).
2. $\overline{\tau}_s$ is a head tax; τ_s is a subsidy (= 0 when $\tau_s = 1$) to household receipts, used later to offset various distortions.

Household j's intertemporal maximization problem –

$$\mathcal{L} = E_t\sum_{s=t}^{\infty}\beta^{s-t}\{[u(C^j_s) - h((P_s(j)/P_s)^{-\theta}C_s/Z_s)] + \lambda^j_s[\tau_s P_{s-1}(j)(P_{s-1}(j)/P_{s-1})^{-\theta}Y_{s-1} + I_{s-1}B^j_{s-1}$$
$$- (P_sC^j_s - \upsilon_s P_s Y_s) - B^j_s - P_s\overline{\tau}_s]\}$$

The following FOC's hold for any pricing assumption:

C^j_t: $u'(C^j_t) = \lambda^j_t P_t$ or $1/C^{j\gamma}_t - \lambda^j_t P_t = 0$

B^j_t: $-\beta I_t E_t\lambda^j_{t+1} + \lambda^j_t = 0$

Remarks on symmetry and aggregation:

1. The FOC for C^j_t, M^j_t and λ^j_t are the same for all households. So, $C_t \equiv \int_0^1 C^j_t dj = C^j_t\int_0^1 dj = C^j_t$ and $M_t \equiv \int_0^1 M^j_t dj = M^j_t\int_0^1 dj = M^j_t$; C_t and M_t simultaneously represent “aggregate”, “individual” and “representative” values of consumption and money.
2. Similarly, supplies and demands for each household's good will be the same. All suppliers will set the same relative price, $P_t(j)/P_t$, and output, Y^j_t, in equilibrium. So, $P_t \equiv [\int_0^1 P_t(j)^{1-\theta}dj]^{1/(1-\theta)} = P_t(j)$ and $Y_t \equiv \int_0^1 Y^j_t dj = Y^j_t$. Y_t represent “aggregate”, “representative” and “individual” output.

Therefore, in a symmetric equilibrium, the FOC's and CIA constraint become:

(6) $\beta I_t E_t[u'(C_{t+1})P_t/u'(C_t)P_{t+1}] = 1$

(7) $M_t = (1 - \upsilon_t)P_tY_t = (1/V_t)P_tY_t$

where $V_t \equiv 1/(1 - \upsilon_t)$ is a velocity shock.

Recall: $\mathscr{L} = E_t\sum_{s=t}^{\infty}\beta^{s-t}\{[u(C_s^j) - h((P_s(j)/P_s)^{-\theta}Y_s/Z_s)] + \lambda_s[\tau_sP_{s-1}(j)^{1-\theta}(1/P_{s-1})^{-\theta}Y_{s-1} + I_{s-1}B_{s-1}^j$
$- (P_sC_s^j - \upsilon_sP_sY_s) - B_s^j - P_s\overline{\tau}_s)]\}$

if prices are flexible:

$P_t(j)$: $\theta h'(\cdot)P_t(j)^{-1-\theta}(1/P_t)^{-\theta}(Y_t/Z_t) + \beta(E_t\lambda_{t+1})\tau_{t+1}(1-\theta)P_t(j)^{-\theta}(1/P_t)^{-\theta}Y_s = 0$

$\theta h'(\cdot)P_t(j)^{-1-\theta}(1/P_t)^{-\theta}(Y_t/Z_t) + \lambda_t\beta E_t(\lambda_{t+1}/\lambda_t)\tau_{t+1}(1-\theta)P_t(j)^{-\theta}(1/P_t)^{-\theta}Y_s = 0$

$\theta h'(\cdot)P_t(j)^{-1-\theta}(1/P_t)^{-\theta}(Y_t/Z_t) = (u'(C_t^j)/P_t)I_t^{-1}\tau_{t+1}(\theta-1)P_t(j)^{-\theta}(1/P_t)^{-\theta}Y_t$

in equilibrium, $C_t^j = C_t = Y_t = Y_t^j(j)$ and $P_t(j) = P_t$, so

$h'(Y_t/Z_t)P_t^{-1}(Y_t/Z_t) = \tau_{t+1}I_t^{-1}[(\theta-1)/\theta]u'(Y_t)P_t^{-1}Y_t$

and canceling the $P_t^{-1}Y_t$ (we can't do this in the fixed price case)

(8a)*flex* $h_t'(Y_t/Z_t)/Z_t = \tau_{t+1}I_t^{-1}[(\theta-1)/\theta]u'(Y_t)$

or using the constant elasticity functions: $(Y_t/Z_t)^{\chi}/Z_t = \tau_{t+1}I_t^{-1}[(\theta-1)/\theta]Y_t^{-\gamma}$ and

(8b)*flex* $Y_t = [\tau_{t+1}I_t^{-1}(\theta-1)/\theta]^{1/(\gamma+\chi)}Z_t^{(1+\chi)/(\gamma+\chi)}$

Remarks: Interpretations of (8) & distortions of the Labor-Leisure decision –

1. A straightforward interpretation of equations (8):

Recall that –

$Y^d(j) = (P(j)/P)^{-\theta}Y$	(Demand for good Y(j); dividing by Z gives work effort)
$(\tau/I)P(j)^{1-\theta}(1/P)^{-\theta}Y$	(Revenue for sale of Y(j))

A marginal P(f)↓ raises the work effort by $\theta P(j)^{-1-\theta}P^{\theta}Y/Z$

and increases revenue by $(\tau/I)(\theta-1)P(j)^{-\theta}P^{\theta}Y$

The third line in the derivation says that the marginal disutility of the increased work effort is equal to the marginal utility of spending the increased revenue.

2. The interest rate – I_t^{-1} – in (8) is the seigniorage tax distortion. Payment for an increase in today's work effort comes next period. In calculating today's work-consumption tradeoff, payments for additional work must be discounted by I_t^{-1}. Having to hold money (to satisfy the CIA constraint) is costly unless $I_t = 1$. The household works too little if $I_t > 1$.

3. What is the monopoly distortion? A more interesting interpretation of (8a):

   The LHS of (8a) is the disutility of working enough to get one more Y(f).

   If the household produces one more Y(f), how much more revenue does it get?

   Recall that: $Y^d(j) = (P(j)/P)^{-\theta}Y$, so price falls as Y(f) rises!

   $d(P(j)Y(j))/dY(j) = [(P(j)Y(j))/dP(j)]\cdot[dP(j)/dY(j)] = [d(P(j)Y(j))/dP(j)]/[dY(j)/dP(j)]$

   $= [(1-\theta)P(j)^{-\theta}P^{\theta}Y]/[-\theta P(j)^{-1-\theta}P^{\theta}Y] = [(\theta-1)/\theta]P(j)$

   $[d(P(j)Y(j))]/P(j) = [(\theta-1)/\theta]dY(j) < dY(j)$ (would be = dY(j) if the price did not fall)

   this new revenue becomes available next period, and it is subsidized; adds the $I^{-1}\tau$ factor.

   So, the RHS of (8a) is the utility of spending the (real) proceeds, or $\tau I^{-1}[(\theta-1)/\theta]$.

   So, what is the distortion?

   The resource constraint is 1 for 1; one more unit of output ⇒ one more unit of consumption.

   When monopolistic price setters increase output, (real) revenue goes up less than 1 for 1 since the P(j) falls. $\mu = \theta/(\theta-1) > 1$ is the distortion (or markup) created by monopolistic competition. It makes households produce too little. As $\theta \rightarrow \infty$ (and $\mu \rightarrow 1$), the demand curves become infinitely elastic, and the distortion is eliminated, leaving the private marginal benefit of work equal to the marginal cost of work. This is the solution a social planner would try to achieve.

4. *Fiscal policy can eliminate either (or both) of the distortions*: Most of this literature assumes that the welfare costs of seigniorage are small, and ignores them. (Most of the literature uses "money in the utility function", but ignores the money term in the welfare analysis.) We will follow that tradition by setting the subsidy τ_{t+1} equal to I_t. (Therefore τ_{t+1} is known at date t and can be pulled out of the expectation $E_t(\cdot)$, as was done above.) Much of the literature ignores the monopoly distortion as well. Monopolistic competition is viewed as a device for rationalizing sticky prices; it's distortions are not taken seriously. We could eliminate the monopoly distortion as well by setting $\tau_{t+1} = \mu I_t$. (See Henderson and Kim (1999).) Generally, we will keep the monopoly distortion, and just set $\tau_{t+1} = I_t$.

5. Setting $\tau_{t+1} = I_t$ is not equivalent to paying interest on money. (Paying interest on money may lead to indeterminacy by making the CIA constraint non-binding.) Excess money balances are not subsidized. Receipts generated by new sales (or work) are subsidized, eliminating the CIA constraint's distortion of the labor-leisure decision.

Fiscal Policy: a balanced budget rule –

(9) $(\tau_t - 1)P_{t-1}Y_{t-1} = P_t\bar{\tau}_t + (M_t - M_{t-1})$

Remarks:

1. Generally, we will be assume that $\tau_t = I_{t-1}$; this eliminates the seigniorage tax distortion.
2. $\bar{\tau}_t$ is a lump-sum tax; fiscal policy is "Ricardian" in Woodford's sense.

if prices are set one period ahead (and letting $\tau_{t+1} = I_t$ and $\mu = \theta/(\theta-1)$)*:*

$P_t(j)$: $\mu E_{t-1}[h'(Y_t/Z_t)P_t^{-1}(Y_t/Z_t)] = E_{t-1}[\beta I_t\lambda_{t+1}Y_t] = E_{t-1}[E_t(\beta I_t\lambda_{t+1}Y_t)] = E_{t-1}[\beta I_tY_tE_t(\lambda_{t+1})] \Rightarrow$

$\mu E_{t-1}[h'(Y_t/Z_t)P_t^{-1}(Y_t/Z_t)] = E_{t-1}[u'(Y_t)P_t^{-1}Y_t]$ or $\mu E_{t-1}[(Y_t/Z_t)^{\chi}P_t^{-1}(Y_t/Z_t)] = E_{t-1}[Y_t^{-\gamma}P_t^{-1}Y_t]$

we can cancel the $1/P_t$, since it is known (or being set), so

$(8)_{sticky}$ $\mu E_{t-1}[h'(\cdot)(Y_t/Z_t)] = E_{t-1}[u'(\cdot)Y_t]$ or $\mu E_{t-1}[(Y_t/Z_t)^{1+\chi}] = E_{t-1}[Y_t^{-(\gamma-1)}]$ (= 1 if $\gamma = 1$)

Remarks: Demand determination of work and output & and the zero interest rate bound –

1. When prices are set in advance, they can not move to clear the market. We revert to the "Keynesian" notion that employment and output are demand determined. Suppliers expand to meet an increase in demand (as given by equations (6) and (7)).
2. *Incentive compatibility constraint* (for the demand determination assumption) –

   Competitive suppliers would not find it profitable (or utility enhancing) to increase supply as demand increases. This is why the new "Keynesian" models assume a monopoly rent. From the discussion in the last set of remarks, it is clear that households will want to increase supply as long as:

   (8c) $h'(Y_t/Z_t)/Z_t < u'(Y_t)$ or $(Y_t/Z_t)\chi/Z_t < Y_t^{-\gamma}$

   P_t^j is now fixed; so household revenue goes up 1 for 1 with dY(j). If these inequalities hold, the household can increase its utility by working more. Note that (9) puts a limit on how far households are will to go in responding to expansionary demand shocks.
3. *Zero interest rate bound* ($I_t = 1 + i_t > 1$) –

   Nominal interest rates can not be negative. And if they are zero, we have well known indeterminacy problems (since there is no cost to holding real money balances). Here again, large shocks might violate the zero interest rate bound. Japan is an example.

   Note: this is a problem in both flexible price solutions and sticky price solutions.

4. Dealing with the incentive compatibility constraint and the zero interest rate bound –
   If we want to take these constraints seriously, we have to: (a) incorporate the constraints and their implications in our analysis, or (b) limit the support of the distributions of random variables (shocks and policy decisions) so that the constraints are never binding.
   In practice, this never seems to be done. "There are only so many hours in the research day."
5. Some think that the macroeconomic problems caused by "Harberger Triangles" are small compared to those created by macroeconomic shocks. However, if distortions are small, then the shocks we can accommodate (see Remark 3) are also small. We may ultimately be forced to deal with the incentive compatibility constraint and the zero interest rate bound in a more direct way. Bummer!

Summarizing, setting $\tau_t = I_{t-1}$ and letting $\mu = \theta/(\theta-1) > 1$ –

Flexible Price Yeoman-Farmer Model:

(11) $\beta I_t E_t[u'(Y^*_{t+1})P_t/u'(Y^*_t)P_{t+1}]$ or $\beta I_t E_t[(Y^*_t/Y^*_{t+1})^\gamma(P_t/P_{t+1})] = 1$

(12) $M_t V_t = P_t Y^*_t$

$(13)_{flex}$ $\mu h_t'(Y^*_t/Z_t)/Z_t = u'(Y^*_t)$ or $Y^*_t = (1/\mu)^{1/(\gamma+\chi)} Z_t^{(1+\chi)/(\gamma+\chi)}$

Sticky Price Yeoman-Farmer Model:

(11) $\beta I_t E_t[u'(Y_{t+1})P_t/u'(Y_t)P_{t+1}]$ or $\beta I_t E_t[(Y_t/Y_{t+1})^\gamma(P_t/P_{t+1}) = 1$

(12) $M_t V_t = P_t Y_t$

$(13)_{sticky}$ $\mu E_{t-1}[h'(\cdot)(Y_t/Z_t)] = E_{t-1}[u'(\cdot)Y_t]$ or $\mu E_{t-1}[(Y_t/Z_t)^{1+\chi}] = E_{t-1}[Y_t^{-(\gamma-1)}]$

Remarks:

1. $Y^*_t = (1/\mu)^{1/(\gamma+\chi)} Z_t^{(1+\chi)/(\gamma+\chi)}$ is independent of both M_t and V_t. Monopolistic price setting *per se* does not imply a stabilization role for monetary policy; we have the usual dichotomy between real and monetary sides. We still need some kind of wage or price stickiness.
2. When uncertainty goes to zero, $(13)_{sticky} \rightarrow (13)_{flex}$. Price setters know how to set their prices in advance, and we have the flex price solution.
3. M-targeting: use (12) & (13) to find P_t and Y_t; then (11) determines I_t.
   I-targeting: use (11) & (13) to find P_t and Y_t; then (12) determines M_t.

4. $(13)_{sticky}$ contains something very new. Expected or average level of economic activity now depends on second moments of policy variables as well as first moments. See below.

Monetary Policy Question 1.1: Does monetary uncertainty decrease average output (or $E_{t-1}Y_t$), as might be expected?

The flexible price (or no uncertainty) level of output implied by $(8)_{flex}$ (with Z_t set equal to 1) is $Y^*_t \equiv (1/\mu)^{1/(\gamma+\chi)}$. Is the $E_{t-1}Y_t$ implied by $(8)_{sticky}$ (with $\{Z_s\} = 1$) less than Y^*_t? If not, then we can increase average output by just creating monetary noise, a rather dubious proposition. In fact, the conventional wisdom seems to be that monetary noise is bad, and would lower average output.

For certain utility functions that have been used widely in the literature, we can answer this question by simply looking at $(13)_{sticky}$ (with $\{Z_s\} = 1$). The answer does not depend upon how the rest of the model is specified. We discuss three such cases below.

Utility functions for which monetary uncertainty would increase average output ($E_{t-1}Y_t > Y^*$) might be viewed as suspect, even if they have been used in the literature. A good student project would be to see which (if any) of the results that have been obtained depend on this fact.

Case 1 (used by Ireland (JPE, 1996)): $\gamma = 1$ and $\chi = 0 \Rightarrow E_{t-1}Y_t = Y^*$.

Proof: $Y^* = 1/\mu$ and $(8)_{sticky}$ (with $Z = 1$) $\Rightarrow E(Y) = 1/\mu = Y^*$

Case 2 (used by Obstfeld and Rogoff (book) and many others): $\gamma = 1$ and $\chi = 1 \Rightarrow E_{t-1}Y_t < Y^*$.

Proof: $Y^* \equiv (1/\mu)^{1/2}$ and $(8)_{sticky}$ (with $Z = 1$) $\Rightarrow E(Y^2) = 1/\mu \Rightarrow [E(Y)]^2 + VAR(Y) = 1/\mu$ $\Rightarrow E(Y) = [(1/\mu) - VAR(Y)]^{1/2} < (1/\mu)^{1/2} = Y^*$

Case 3 (used by Devereux and Engel (2000)): $\gamma = 2$ and $\chi = 0 \Rightarrow E_{t-1}Y_t > Y^*$.

Proof: $Y^* \equiv (1/\mu)^{1/2}$ and $(8)_{sticky}$ (with $Z = 1$) $\Rightarrow E(Y) = (1/\mu)E(1/Y) > (1/\mu)[1/E(Y)]$ $\Rightarrow [E(Y)]^2 > 1/\mu \Rightarrow E(Y) > (1/\mu)^{1/2} = Y^*$

Note: If the model is log-normal, this result can be generalized; see remark 4 on page 14 and the special cases in later sections.

Monetary Policy Question 1.2: In the Sticky Price Yeoman-Farmer Model, is there a monetary policy rule that will make $Y_t = Y^*_t$ and $P_t = P^T_t$, where P^T_t is an arbitrarily selected price level target (that is announced in advance of price setting)?

Discussion:

If the answer is yes, and if it can later be shown that the flex price solution in some sense "optimal", then there is no P (or inflation) -Y tradeoff in the Yeoman-Farmer Model.

If the answer is to be yes, then we will need $M_tV_t = P^T_tY^*_t = P^T_t(1/\mu)^{1/(\gamma+\chi)}Z_t^{(1+\chi)/(\gamma+\chi)}$; so, a good candidate for the rule we are seeking is:

(15) $M_t = \Omega_{t-1}Z_t^{(1+\chi)/(\gamma+\chi)}(1/V_t)$, where $\Omega_{t-1} = P^T_t(1/\mu)^{1/(\gamma+\chi)}$ is announced in period t-1 (or anytime before prices are set). This rule assumes that the disturbances are observed.

Monetary Policy Result 1.1: Let $\{P^T_s\}$ be an arbitrary target path. In the sticky price Yeoman-Farmer Model, the monetary policy rule (15) will make $Y_t = Y^*_t$ and $P_t = P^T_t$.

Proof:

In the sticky price model, we have to solve $\{(12), (13)_{sticky}$ and $(15)\}$ for $\{P_t, Y_t$ and $M_t\}$.

$Y^*_t \equiv (1/\mu)^{1/(\gamma+\chi)}Z_t^{(1+\chi)/(\gamma+\chi)}$ & (15) $\Rightarrow$ $M_tV_t = \Omega_{t-1}Z_t^{(1+\chi)/(\gamma+\chi)} = P^T_t(1/\mu)^{1/(\gamma+\chi)}Z_t^{(1+\chi)/(\gamma+\chi)} = P^T_tY^*_t$

So, (12) & (15) $\Rightarrow$ $M_tV_t = P_tY_t = P^T_tY^*_t$ $\Rightarrow$ $Y_t = (P^T_t/P_t)Y^*_t$

The final step is to use $(13)_{sticky}$ to show that $P_t = P^T_t$; it immediately follows that $Y_t = Y^*_{t.}$

Use $Y_t = (P^T_t/P_t)Y^*_t = (P^T_t/P_t)(1/\mu)^{1/(\gamma+\chi)}Z_t^{(1+\chi)/(\gamma+\chi)}$ to eliminate Y_t in both sides of $(13)_{sticky}$.

$$\mu E[(Y/Z)^{1+\chi}] = (P^T/P)^{1+\chi}\mu^{[1-(1+\chi)/(\gamma+\chi)]}E[Z^{[(1+\chi)/(\gamma+\chi)-1](1+\chi)}]$$

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note: $[(1+\chi)/(\gamma+\chi)-1](1+\chi) = (1+\chi-\gamma-\chi)[(1+\chi)/(\gamma+\chi)] = (1-\gamma)(1+\chi)/(\gamma+\chi)$

$[1-(1+\chi)/(\gamma+\chi)] = (\gamma+\chi-1-\chi)/(\gamma+\chi) = (\gamma-1)/(\gamma+\chi)$

we were able to bring P and P^T out of $E[\cdot]$ since they are determined in t-1

$$= (P^T/P)^{1+\chi}\mu^{(\gamma-1)/(\gamma+\chi)}E[Z^{(1-\gamma)(1+\chi)/(\gamma+\chi)}]$$

$$E[Y^{-(\gamma-1)}] = (P^T/P)^{-(\gamma-1)}\mu^{(\gamma-1)/(\gamma+\chi)}E[Z^{-(\gamma-1)(1+\chi)/(\gamma+\chi)}]$$

So, $(3)_{sticky}$ $\Rightarrow$ $(P^T/P)^{1+\chi}\mu^{(\gamma-1)/(\gamma+\chi)}E[Z^{(1-\gamma)(1+\chi)/(\gamma+\chi)}] = (P^T/P)^{-(\gamma-1)}\mu^{(\gamma-1)/(\gamma+\chi)}E[Z^{(1-\gamma)(1+\chi)/(\gamma+\chi)}]$

$\Rightarrow$ $(P^T/P)^{1+\chi} = (P^T/P)^{-(\gamma-1)}$ $\Rightarrow$ $P_t = P^T_t$ QED

Next, we want to ask if the flex price solution is somehow optimal.

Remarks: *Normative Analysis* –

1. The following lemma shows that, for a certain class of utility functions, we can aggregate the consumption and leisure terms in the utility function, and preserve log-linearity. This will obviously facilitate the normative analysis.
3. Many studies postulate money in the utility function (instead of our cash-in-advance). This adds a third term to the utility function. In the normative analysis, this real balances term is often ignored, on the grounds that seigniorage costs are relatively unimportant.

Lemma 1.1: If $u(\cdot)$ and $h(\cdot)$ have constant elasticity – $u(C) = (1-\gamma)^{-1}C^{1-\gamma}$ & $h(N) = (1+\chi)^{-1}(N)^{1+\chi}$ – then the expected disutility of work is proportional to the expected utility of consumption; moreover, if $\gamma = 1$, the expected disutility of work is constant, and welfare depends only

on the expected utility of consumption. Letting $\Lambda_t \equiv E_{t-1}[u(C_t) - h(N_t)]$, we have:

$$\Lambda_t = \begin{cases} -\,[(\gamma-1)^{-1} + \mu^{-1}(1+\chi)^{-1}]E_{t-1}[C_t^{-(\gamma-1)}] & \text{if } \gamma > 1 \\ E_{t-1}[\log(C_t)] - 1/\mu(1+\chi) & \text{if } \gamma = 1 \end{cases}$$

Proof:

If $\gamma > 1$, $(13)_{sticky}$ $\mu E_{t-1}[h'(N_t)N_t] = E_{t-1}[u'(C_t)C_t] \Rightarrow \mu E_{t-1}[N_t^{1+\chi}] = E_{t-1}[C_t^{1-\gamma}]$

$\Rightarrow \mu(1+\chi)E_{t-1}[h(N_t)] = (1-\gamma)E_{t-1}[u(C_t)] \Rightarrow E_{t-1}[h(N_t)] = [(1-\gamma)/\mu(1+\chi)]E_{t-1}[u(C_t)]$

If $u(C) = \log(C)$, $(13)_{sticky}$ $\mu E_{t-1}[h'(N_t)N_t] = E_{t-1}[u'(C_t)C_t] \Rightarrow \mu E_{t-1}[N_t^{1+\chi}] = E_{t-1}[1] = 1$

$\Rightarrow \mu(1+\chi)E_{t-1}[h(N_t)] = 1 \Rightarrow E_{t-1}[h(N_t)] = 1/\mu(1+\chi)$

Remarks:

1. Maximizing $\Lambda_t \Leftrightarrow$ minimizing $E_{t-1}[C_t^{-(\gamma-1)}]$ if $\gamma > 1 \Leftrightarrow$ maximizing $E_{t-1}[\log(C_t)]$ if $\gamma = 1$.
2. Lemma 1 will not limited to the Yeoman-Farmer Model; we will see that (13) (or something like it) is a generic result in these models.
3. Why is $E_{t-1}[h(N_t)]$ constant when $\gamma = 1$? Consider a monetary policy that increases N_t. Substitution and income effects cancel. Expand on this.

Monetary Policy Question 1.3: Is it "optimal" to achieve the flexible price solution in the Yeoman-Farmer Model?

Discussion:

If we set the subsidy, τ_t, equal to μI_t, both of the distortions in the model (seigniorage & monopoly) will be eliminated. Then, standard welfare economics tells us that the flex price equilibrium is Pareto Optimal. Henderson and Kim (1999) take this approach.

If we keep either distortion, then it would be Pareto Improving for the CB to try to increase Y_t above Y^*_t. This leaves us with a Barro-Gordon type of credibility/commitment problem (assuming some "cost" can be ascribed to inflation). The recent literature doesn't seem to want to revisit this issue – though some exceptions so exist: see Neiss (1996) or Lippi (1999). Most authors find some way of avoiding the time consistency problem.

One way of avoiding this issue is to assume that the CB is committed to a policy rule.

a. See for example Henderson and Kim (1999), who (like most in the recent literature) postulate an interest rate rule. With interest rate targeting, Y_t is determined by the dynamic Euler Equation (11). H&K are able, with the assumption of log-normality (and a lot of algebra), to use the method of undetermined coefficients to solve their model; see our Appendix 4. Much of the literature linearizes and resorts to computer simulation.

b. Here, we use money targeting rules, like (15), instead of interest rate targeting rules. This allows us to avoid the dynamics in (11), and get solutions more easily than H&K.

Here, we follow the literature in eliminating the seigniorage distortion (by setting $\tau_t = I_{t-1}$), but we will (in this section anyway) retain the monopoly distortion and eliminate the Barro-Gordon problem by assuming a pre-set rule (like (15)). *We want to see how the monopoly distortion might interact with the stabilization problem, absent considerations of time inconsistency. In particular, assuming that the CB is constrained to a pre-set rule and that fiscal policy does not eliminate the monopoly distortion, we want to ask if it is still optimal for the CB to make* $Y_t = Y^*_t$.

Note: Under monetary targeting, the model is stationary, and Lemma 1.1 implies that $\max E_{t-1}U_t = E_{t-1}\sum_{j=t}^{\infty}\beta^{j-t}\Lambda_j \Leftrightarrow \max \Lambda_t$.

Recall: Monetary Policy Result 2 says that if the answer to this question is "yes", then the CB can simultaneously achieve an independent price target P^*_t. There is no P-Y tradeoff.

Stochastic Environment:

Now, let the shocks V and Z be independent random variables with log normal distributions, and let small letters represent the logs of capital letters.

A quick review of the log-normality:

Let Q have a log-normal distribution; so, $q \equiv \ln Q \sim N(\overline{q},\sigma_q^2)$.

$\ln Q^k = kq \;\Rightarrow Q^k = \exp\{kq\}$

$\Rightarrow E(Q^k) = E(\exp\{kq\}) = \exp\{k\overline{q} + \tfrac{1}{2}k^2\sigma_q^2\}$

$\Rightarrow \ln E(Q^k) = k\overline{q} + \tfrac{1}{2}k^2\sigma_q^2$

Note:

If we assume $\overline{q} = 0$ (which seems natural) , then $E(Q) = \exp\{\tfrac{1}{2}\sigma_q^2\} \neq 1$ (which is not nice).

If we assume $E(Q) = 1$(which is nice), then $\overline{q} = -\tfrac{1}{2}\sigma_q^2$ (which can involve a lot of algebra).

Solving the Sticky Price Yeoman-Farmer Model for y_t ($\equiv$ log Y_t) and p_t ($\equiv$ log P_t):

Note: Instead of postulating a functional form – like (5) – for monetary policy, we will derive expressions for y_t and p_t in terms of the m_t and its various moments. These expressions will later on be used to answer the questions posed above.

Notation: We can often drop the time subscripts; all variables are dated t, and all expectations, variances and covariances are dated t-1. Let small letters denote logs of capital letters.

Solving for p and y:

Using (12) – $M_tV_t = P_tY_t$ – to eliminate Y in $(13)_{sticky}$, and noting that P is predetermined:

$E[\mu(MV/PZ)^{1+\chi}] = E[(MV/P)^{-(\gamma-1)}] \;\Rightarrow\; \mu(1/P)^{(1+\chi)}E[(MV/Z)^{1+\chi}] = (1/P)^{-(\gamma-1)}E[(MV)^{-(\gamma-1)}]$

$\mu(1/P)^{(1+\chi+\gamma-1)} = E[(MV)^{-(\gamma-1)}]/E[(MV/Z)^{1+\chi}]$

Taking logs, and letting $\kappa \equiv \log(\mu)$, and letting $V \equiv VAR_{t-1}$:

$\kappa - (\chi+\gamma)p = \log E[(MV)^{-(\gamma-1)}] - \log E[(MV/Z)^{1+\chi}]$

$= -(\gamma-1)E(m+v) - (1+\chi)E(m+v-z) + \tfrac{1}{2}(\gamma-1)^2V(m+v) - \tfrac{1}{2}(1+\chi)^2V(m+v-z)$

So, letting $\eta \equiv (\chi+\gamma)^{-1}$, $\kappa \equiv \log(\mu)$ and $C \equiv \mathrm{COV}_{t-1}$:

$y = m + v - p$ and $y^* = -(\gamma+\chi)^{-1}\log(\mu) + [(1+\chi)/(\gamma+\chi)]z = -\eta\kappa + \eta(1+\chi)z$

and

$$
\begin{aligned}
p &= \eta\kappa + \eta(\gamma-1)E(m+v) + \eta(1+\chi)E(m+v-z) + \tfrac{1}{2}\eta[(1+\chi)^2V(m+v-z) - (\gamma-1)^2V(m+v)] \\
&= [\eta\kappa - \eta(1+\chi)E(z)] + \eta[(\gamma-1) + (1+\chi)]E(m+v) + \tfrac{1}{2}\eta[(1+\chi)^2V(m+v-z) - (\gamma-1)^2V(m+v)] \\
&= -E(y^*) + E(m+v) + \tfrac{1}{2}\eta[(1+\chi)^2V(m+v-z) - (\gamma-1)^2V(m+v)] \quad \text{(see below for } E(y^*) \text{ term)} \\
&= -E(y^*) + E(m+v) + \tfrac{1}{2}\eta\{(1+\chi)^2[V(m+v) + V(z) - 2C(m+v, z)] - (\gamma-1)^2V(m+v)\} \\
&= -E(y^*) + E(m+v) + \tfrac{1}{2}\eta\{[(1+\chi)^2 - (\gamma-1)^2]V(m+v) + (1+\chi)^2[V(z) - 2C(m+v, z)]\}
\end{aligned}
$$

Summarizing, in the Sticky Price Yeoman-Farmer Model:

(16) $\log(P_t) = E_{t-1}[\log(M_tV_t)] - E_{t-1}[\log(Y^*_t)]$
$+ \tfrac{1}{2}(\gamma+\chi)^{-1}\{(1+\chi)^2\mathrm{VAR}_{t-1}[\log(M_tV_t/Z_t)] - (\gamma-1)^2\mathrm{VAR}_{t-1}[\log(M_tV_t)]\}$
$= E_{t-1}[\log(M_tV_t)] - E_{t-1}[\log(Y^*_t)] - (\gamma+\chi)^{-1}(1+\chi)^2\mathrm{COV}_{t-1}[M_tV_t, Z_t]$
$+ \tfrac{1}{2}(\gamma+\chi)^{-1}\{[(1+\chi)^2 - (\gamma-1)^2]\mathrm{VAR}_{t-1}[\log(M_tV_t)] + (1+\chi)^2\mathrm{VAR}_{t-1}[\log(Z_t)]\}$

where $E_{t-1}[\log(Y^*_t)] = -(\gamma+\chi)^{-1}\log(\mu) + [(1+\chi)/(\gamma+\chi)]E_{t-1}[\log(Z_t)]$

(17) $\log(Y_t) - E_{t-1}[\log(Y^*_t)] = \{\log(M_tV_t) - E_{t-1}[\log(M_tV_t)\}$
$- \tfrac{1}{2}(\gamma+\chi)^{-1}\{(1+\chi)^2\mathrm{VAR}_{t-1}[\log(M_tV_t/Z_t)] - (\gamma-1)^2\mathrm{VAR}_{t-1}[\log(M_tV_t)]\}$
$= \{\log(M_tV_t) - E_{t-1}[\log(M_tV_t)]\} + (\gamma+\chi)^{-1}(1+\chi)^2\mathrm{COV}_{t-1}[\log(M_tV_t), \log(Z_t)]$
$+ \tfrac{1}{2}(\gamma+\chi)^{-1}\{[(\gamma-1)^2-(1+\chi)^2]\mathrm{VAR}_{t-1}[\log(M_tV_t)] - (1+\chi)^2\mathrm{VAR}_{t-1}[\log(Z_t)]\}$

Remarks: In the sticky price Yeoman-Farmer Model –

1. A decrease in μ, or an increase in $E_{t-1}[\log(Z_t)]$, lowers $\log(P_t)$ raises $E_{t-1}[\log(Y^*_t)]$.
2. Since $\log(Y_t)$ is demand determined, it does not depend on the productivity shock, Z_t.
3. An increase in $E_{t-1}[\log(M_tV_t)]$ passes through 1-1 to an increase in $\log(P_t)$, as we would expect.
4. The output gap –
   $\log(Y_t) - \log(Y^*_t) = \log(Y_t) - E_{t-1}[\log(Y^*_t)] - \{\log(Y^*_t) - E_{t-1}[\log(Y^*_t)]\}$
   $= \log(Y_t) - E_{t-1}[\log(Y^*_t)] - [(1+\chi)/(\gamma+\chi)][\log(Z_t) - E_{t-1}(Z_t)]$ –
   depends on monetary prediction errors – $\log(M_tV_t) - E_{t-1}[\log(M_tV_t)]$ and $\log(Z_t) - E_{t-1}(Z_t)$ – as in the RE literature.
5. ***The effects of monetary uncertainty*** (once again, see page 9, and page 29):
   As all uncertainty goes away, $\log(Y_t) \rightarrow \log(Y^*_t)$ or $Y_t \rightarrow Y^*_t$. Let $\mathrm{COV}_{t-1}[M_tV_t, Z_t] = 0$.
   Monetary uncertainty – measured by $\mathrm{VAR}_{t-1}[\log(M_tV_t)]$ – decreases $\log(Y_t) \Leftrightarrow (1+\chi)^2 > (\gamma-1)^2$.
   This condition differs that on page 9 since we are looking at logs of Y, not levels.

Optimal Stabilization Policy:

From Lemma 1, we have –

With a preset rule for monetary policy, the CB wants to

$$\max E_{t-1}U_t = E_{t-1}\sum_{s=t}^{\infty}\beta^{s-t}[(1-\gamma)^{-1}Y_s^{1-\gamma} - (1+\chi)^{-1}(Y_s/Z_s)^{1+\chi}] = E_{t-1}\sum_{j=t}^{\infty}\beta^{j-t}\Lambda_j \Leftrightarrow \max \Lambda_t.$$

$$\Lambda_t \equiv E_{t-1}[(1-\gamma)^{-1}Y_t^{1-\gamma} - (1+\chi)^{-1}(Y_t/Z_t)^{1+\chi}] = \begin{cases} -(1/\mu)[\mu(\gamma-1)^{-1} + (1+\chi)^{-1}]E_{t-1}[Y_t^{-(\gamma-1)}] \text{ if } \gamma > 1 \\ E_{t-1}[\log(Y_t) - (1/\mu)(1+\chi)^{-1}] \quad \text{if } \gamma = 1 \end{cases}$$

$$\text{maximizing } \Lambda_t \Leftrightarrow \begin{cases} \text{minimizing } E_{t-1}[Y_t^{-(\gamma-1)}] \text{ if } \gamma > 1 \\ \text{maximizing } E_{t-1}[\log(Y_t)] \text{ if } \gamma = 1. \end{cases}$$

Monetary Policy Result 1.2: There is no P-Y tradeoff in the Yeoman-Farmer Model. Let $\{P^T_t\}$ be an arbitrarily selected target path, and Y^*_t be the flex-price output. The money policy rule (15) $M_t = \Omega_{t-1}Z_t^{(1+\chi)/(\gamma+\chi)}(1/V_t)$, where $\Omega_{t-1} \equiv P^T_t(1/\mu)^{1/(\gamma+\chi)}$ is announced before prices are set, will make $Y_t = Y^*_t$ and $P_t = P^T_t$, and the flexible price solution is optimal in the sense that it maximizes $E_{t-1}U_t$.

Proof:

The first part of Result 1.2 was already proven as Result 1.1; we need to prove the last assertion.

Start with the simple case: $\gamma = 1$ –

The solution for output becomes:

(17) $\log(Y_t) - E_{t-1}[\log(Y^*_t)] = \{\log(M_tV_t) - E_{t-1}[\log(M_tV_t)]\} - ½(1+\chi)VAR_{t-1}[\log(M_tV_t/Z_t)]$

where $Y^*_t = (1/\mu)^{1/(1+\chi)}Z_t$

Taking the expectation of (17):

(18) $E_{t-1}[\log(Y_t)] = E_{t-1}[\log(Y^*_t)] - ½(1+\chi)VAR_{t-1}[\log(M_tV_t/Z_t)]$

Maximizing $E_{t-1}[\log(Y_t)] \Leftrightarrow$ Minimizing $VAR_{t-1}[\log(M_tV_t/Z_t)]$

(15) $\Rightarrow$ $M_tV_t/Z_t = \Omega_{t-1}$ (when $\gamma = 1$) $\Rightarrow$ $VAR_{t-1}[\log(M_tV_t/Z_t)] = 0$.

The rule, (15), which makes $Y_t = Y^*_t$ maximizes Λ_t and $E_{t-1}U_t$.

Remarks:

1. The optimal thing for the CB to do is to achieve the flex price solution (even when $\mu > 1$)!
2. Can verify the price result directly: (16) $\Rightarrow \log(P_t) = E_{t-1}[\log(M_tV_t/Y^*_t)] = E_{t-1}[\log(\Omega_{t-1}Z_t/Y^*_t)] = E_{t-1}[\log(\Omega_{t-1}\mu^{1/(1+x)})] = E_{t-1}[\log(P^*_t)] = \log(P^T_t)$. But, this was already shown in Result I.
3. In lowering $VAR_{t-1}[\log(M_tV_t/Z_t)]$, the optimal monetary policy *raises* $E[\log(Y_t)]$ to $E[\log(Y^*_t)]$ in addition to stabilizing the gap.

The General Case –
maximizing Λ_t $\Leftrightarrow$ minimizing $E_{t-1}[Y_t^{-(\gamma-1)}]$ if $\gamma > 1$; so, what is $E_{t-1}[Y_t^{-(\gamma-1)}]$
$\log E(Y^{-(\gamma-1)}) = -(\gamma-1)E(y) + \tfrac{1}{2}(\gamma-1)^2V(y)$ and from page 11 –
$y = E(y^*) + [(m+v) - E(m+v)] - \tfrac{1}{2}\eta\{(1+\chi)^2[V(m+v) + V(z) - 2C(m+v, z)] - (\gamma-1)^2V(m+v)\}$
$= E(y^*) + [(m+v) - E(m+v)] - \tfrac{1}{2}\eta\{[(1+\chi)^2 - (\gamma-1)^2]V(m+v) + (1+\chi)^2[V(z) - 2C(m+v, z)]\}$
$E(y) = E(y^*) - \tfrac{1}{2}\eta\{[(1+\chi)^2 - (\gamma-1)^2]V(m+v) + (1+\chi)^2[V(z) - 2C(m+v, z)]\}$ and $V(y) = V(m+v)$
$\log E(Y^{-(\gamma-1)}) = -(\gamma-1)E(y) + \tfrac{1}{2}(\gamma-1)^2V(y)$
$= -(\gamma-1)E(y^*) + \tfrac{1}{2}(\gamma-1)\eta\{[(1+\chi)^2-(\gamma-1)^2]V(m+v) + (1+\chi)^2[V(z)-2C(m+v, z)]\}$
$+ \tfrac{1}{2}(\gamma-1)^2V(m+v)$
$= -(\gamma-1)E(y^*) + \tfrac{1}{2}(\gamma-1)\eta\{[(\gamma-1)(\chi+\gamma)+(1+\chi)^2-(\gamma-1)^2]V(m+v) + (1+\chi)^2[V(z)-2C(m+v, z)]\}$
(Note: $(\gamma-1)(\chi+\gamma) + (1+\chi)^2 - (\gamma-1)^2 = (\gamma-1)[(\chi+\gamma) - (\gamma-1)] + (1+\chi)^2$
$= (\gamma-1)(1+\chi) + (1+\chi)^2 = (1+\chi)[(\gamma-1) + (1+\chi)] = (1+\chi)(\gamma+\chi)$)
$= -(\gamma-1)E(y^*) + \tfrac{1}{2}(\gamma-1)\eta\{(1+\chi)(\gamma+\chi)V(m+v) + (1+\chi)^2[V(z) - 2C(m+v, z)]\}$
$= -(\gamma-1)E(y^*) + \tfrac{1}{2}(\gamma-1)(1+\chi)\{V(m+v) + [(1+\chi)/(\gamma+\chi)][V(z) - 2C(m+v, z)]\}$
(since $\eta \equiv 1/(\gamma+\chi)$; now add and subtract $[(1+\chi)/(\gamma+\chi)]^2V(z)$)
$= -(\gamma-1)E(y^*) + \tfrac{1}{2}(\gamma-1)(1+\chi)\{V(m+v) + [(1+\chi)/(\gamma+\chi)]^2V(z) - 2[(1+\chi)/(\gamma+\chi)]C(m+v, z)]\}$
$+ \tfrac{1}{2}(\gamma-1)(1+\chi)\{[(1+\chi)/(\gamma+\chi)]V(z) - [(1+\chi)/(\gamma+\chi)]^2V(z)\}$
$= -(\gamma-1)E(y^*) + \tfrac{1}{2}(\gamma-1)(1+\chi)V[(m+v) - [(1+\chi)/(\gamma+\chi)]z]$
$+ \tfrac{1}{2}(\gamma-1)(1+\chi)[(1+\chi)/(\gamma+\chi)]\{1-[(1+\chi)/(\gamma+\chi)]\}V(z)$
so, finally
$\log E_{t-1}[Y_t^{-(\gamma-1)}] = -(\gamma-1)E_{t-1}[\log(Y_t^*)] + \tfrac{1}{2}(\gamma-1)(1+\chi)\{VAR_{t-1}[\log(M_tV_t/Z_t^{(1+\chi)/(\gamma+\chi)})]$
$+ [(1+\chi)/(\gamma+\chi)][1-((1+\chi)/(\gamma+\chi))VAR_{t-1}[\log(Z_t)]\}$
(15) $\Rightarrow$ $M_tV_t/Z_t^{(1+\chi)/(\gamma+\chi)} = \Omega_{t-1}$ $\Rightarrow$ $VAR_{t-1}[\log(M_tV_t/Z_t^{(1+\chi)/(\gamma+\chi)})] = 0$ $\Rightarrow$ $E_{t-1}[Y_t^{-(\gamma-1)}]$ is minimized
QED

Note 2: *The New Keynesian "Phillips Curve" –*

References: Calvo (JME, 1983), Yun (JME, 1996) and King and Wolman (1996, 1999), Erceg, Henderson and Levin (JME, 2000), Gali and Gertler (JME, 1999) –

General Framework:

1. Firms get to set a new price with probability $1-\alpha$. Yun and EHL allow the "contract" to be "indexed": if prices are not reset, the old price is adjusted by a steady state inflation factor, $\Omega = P/P_{-1}$. King and Wolman (1996, 1999) do not allow this indexing. Indexing (or the lack thereof) has important implications.
2. The expected length of the "contract" is: $(1-\alpha)\cdot 1 + (1-\alpha)\cdot\alpha\cdot 2 + \ldots + (1-\alpha)\cdot\alpha^{n-1}\cdot n + \ldots = (1-\alpha)^{-1}$. For example, if $1-\alpha = \frac{1}{4}$, then a quarter of the firms adjust each quarter, and the average length of "contracts" is a year; this is the benchmark value in King and Wolman (1996).
3. The fraction of firms with "contracts" set j periods ago is: $\omega_j = (1-\alpha)\alpha^j$.
4. Comparing "contracting"approaches: There is some probability that a Calvo contract will last an arbitrarily long period of time; Taylor contracts may therefore be more appealing. On the other hand, Calvo contracts pick up the randomness of price changes. Neither form of contracting allows the length of the contract to be affected by the state of the economy.

The "aggregate" price level:

Let $P^*_t(f)$ be the price that firm-f would set if it got to reset its price in period t, then:

$$P_t = [\textstyle\int_0^1 P_t(f)^{1-\theta}df]^{1/(1-\theta)} = [\textstyle\sum_{j=0}^{\infty}\omega_j(\Omega^j)^{1-\theta}(P^*_{t-j}(f))^{1-\theta}]^{1/(1-\theta)}$$

Note: $P^*_t(f)$ is "indexed" at the gross rate Ω, if indexing is not allowed, we just set $\Omega = 1$.

Lagging P_t in the equation above, it is straightforward to show that:

(A1) $P_t = [(1-\alpha)P^*_t(f)^{1-\theta} + \alpha(\Omega P_{t-1})^{1-\theta}]^{1/(1-\theta)}$

Derivation of the Phillips Curve proceeds in three steps –

Step 1: Derive $P^*_t(f)$ & log-linearize around a steady state in which nominal prices grow at rate Π.

Step 2: Log-linearize (A1) around the same steady state, and insert the results of Step 1.

Step 3: Replace marginal cost with an "output gap".

Step 1: Derivation and log-linearization of $P^*_t(f)$ –

Optimal price setting in period t –

Firm-f seeks to maximize it's market value:

$MV_t = E_t\sum_{s=t}^{\infty}\beta^{s-t}\lambda_s[P_s(f)Y_s(f) - TC_s(Y_s(f))]$

where TC is total cost, and it will be recalled that $Y^d_t(f) = (P_t(f)/P_t)^{-\theta}Y_t$

With probability α^{s-t}, price $\Omega^{s-t}P^*_t(f)$ will be in effect in period s; so, firm-f sets $P^*_t(f)$ to maximize:

$$MV_t = E_t\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\lambda_s[\Omega^{s-t}P^*_t(f)Y_s(f) - TC_s(Y_s(f))], \text{ where } Y_s(f) = Y^d_s(f) = (\Omega^{s-t}P^*_t(f)/P_s)^{-\theta}Y_s$$

$$= E_t\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\lambda_s[(\Omega^{s-t})^{1-\theta}P^*_t(f)^{1-\theta}(1/P_s)^{-\theta}Y_s - TC_s((\Omega^{s-t})^{-\theta}P^*_t(f)^{-\theta}(1/P_s)^{-\theta}Y_s)]$$

FOC is:

$$0 = E_t\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\lambda_s[(1-\theta)(\Omega^{s-t})^{1-\theta}P^*_t(f))^{-\theta}(1/P_s)^{-\theta}Y_s + \theta MC_s(\cdot)(\Omega^{s-t})^{-\theta}P^*_t(f)^{-\theta-1}(1/P_s)^{-\theta}Y_s]$$

$$= E_t\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\lambda_s[(1-\theta)\Omega^{s-t}(\Omega^{s-t}P^*_t(f)/P_s)^{-\theta}Y_s + \theta MC_s(\cdot)(\Omega^{s-t}P^*_t(f)/P_s)^{-\theta}Y_s/P^*_t(f)]$$

$$= E_t\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\lambda_s[(1-\theta)\Omega^{s-t}Y_s(f) + \theta MC_s(\cdot)Y_s(f)/P^*_t(f)]$$

$$= E_t\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}(\lambda_s/P^*_t(f))(1-\theta)Y_s(f)[\Omega^{s-t}P^*_t(f) - \mu_p MC_s(\cdot)] \quad \text{where } \mu_p \equiv \theta/(\theta-1)$$

Remarks:

1. This seems consistent with equation (8) in Erceg, Henderson and Levin (2000).
2. FOC says the firm sets expected price equal to a mark up, μ_p, over expected marginal cost.
3. "Taylor contracts" yield a similar formula. $P^*_t(f)$ lasts for, say, n periods. So, set $\alpha = 1$, and just take derivatives for n periods. But, this doesn't aggregate as nicely as what follows.

So, finally (going back to the next to the last expression above):

$$\text{(A2)}\quad P^*_t(f) = \mu_p \frac{E_t\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\lambda_s MC_s(\cdot)Y_s(f)}{E_t\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\lambda_s\Omega^{s-t}Y_s(f)}$$

Remarks:

1. Setting $\Omega = 1$, this seems consistent with equation (16) in King and Wolman (1996).
2. If $\alpha = 0$ (so that firms reset prices each period), then this reduces to $P^*_t(f) = \mu_p MC_t(\cdot)$, which is the same as our earlier model.

Log-linearizing $P_t^*(f)$ –

Log-linearize around a deterministic steady state in which real values are constant and nominal values are growing at the gross rate Π. (We initially allow for the possibility that $\Omega \neq \Pi$.)

So, let detrended nominal values be defined by:

$P_t^{*d} \equiv P_t^*/\Pi^t$, $MC_s^{\,d} = MC_s/\Pi^s$, and $\lambda_s^{\,d} \equiv \lambda_s \Pi^s$ (since $1/\lambda$ = \$/util is a nominal value)

Then, (A2) becomes:

$$P_t^{*d}(f)\Pi^t = \mu_p \frac{E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \lambda_s^d MC_s^d(\cdot) Y_s(f)}{E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \lambda_s^d \left(\Omega^{s-t}/\Pi^s\right) Y_s(f)}$$

And dividing by Π^t, this becomes:

$$P_t^{*d}(f) = \mu_p \frac{E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \lambda_s^d MC_s^d(\cdot) Y_s(f)}{E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \lambda_s^d \left(\Omega/\Pi\right)^{s-t} Y_s(f)}$$

Now, we log-linearize around the detrended nominal values:

$$P_t^{*d}(f) E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \lambda_s^d \left(\Omega/\Pi\right)^{s-t} Y_s(f) = \mu_p E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \lambda_s^d Y_s(f) MC_s^d(\cdot)$$

Take differential with respect to P_t^{*d} and $\lambda_s^d Y_s$ and $MC_s^d(\cdot)$ (and drop the "f" index):

$$(P_t^{*d} - \overline{P}^d)\overline{\lambda^d Y} \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left(\Omega/\Pi\right)^{s-t} + \overline{P}^d E_t \left[\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \lambda_s^d \left(\Omega/\Pi\right)^{s-t} Y_s - \overline{\lambda^d Y} \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left(\Omega/\Pi\right)^{s-t} \right]$$

$$= \mu_p \overline{MC}^d E_t \left[\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \lambda_s^d Y_s - \overline{\lambda^d Y} \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \right] + \mu_p \overline{\lambda^d Y} E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left(MC_s^d - \overline{MC}^d \right)$$

Dividing by $\lambda Y \sum (\alpha\beta)^{t-s} (\Omega/\Pi)^{t-s}$:

$$(P_t^{*d} - \overline{P}^d) + \overline{P}^d \left[\frac{E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \lambda_s^d \left(\Omega/\Pi\right)^{s-t} Y_s}{\overline{\lambda^d} Y \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left(\Omega/\Pi\right)^{s-t}} - 1 \right]$$

$$= \mu_p \overline{MC}^d \left[\frac{E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \lambda_s^d Y_s}{\overline{\lambda^d} Y \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left(\Omega/\Pi\right)^{s-t}} - \frac{\sum_{s=t}^{\infty} (\alpha\beta)^{s-t}}{\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left(\Omega/\Pi\right)^{s-t}} \right] + \mu_p \left[\frac{E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left(MC_s^d - \overline{MC}^d \right)}{\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left(\Omega/\Pi\right)^{s-t}} \right]$$

Dividing by $\bar{P}^d = \mu_p \overline{MC}^d \left(\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \Big/ \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} (\Omega/\Pi)^{s-t} \right)$:

$$\frac{P_t^{*d} - \bar{P}^d}{\bar{P}^d} = 1 - \frac{E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \lambda_s^d (\Omega/\Pi)^{s-t} Y_s}{\overline{\lambda^d Y} \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} (\Omega/\Pi)^{s-t}} + \frac{E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \lambda_s^d Y_s}{\overline{\lambda^d Y} \sum_{s=t}^{\infty} (\alpha\beta)^{s-t}} - 1$$

$$+ \left(\frac{1}{\sum_{s=t}^{\infty} (\alpha\beta)^{s-t}} \right) E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left(\frac{MC_s^d - \overline{MC}^d}{\overline{MC}^d} \right)$$

Or finally:

$$\text{(A3)} \quad \frac{P_t^{*d} - \bar{P}^d}{\bar{P}^d} = \left(\frac{E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \lambda_s^d Y_s}{\overline{\lambda^d Y} \sum_{s=t}^{\infty} (\alpha\beta)^{s-t}} - \frac{E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} (\Omega/\Pi)^{s-t} \lambda_s^d Y_s}{\overline{\lambda^d Y} \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} (\Omega/\Pi)^{s-t}} \right)$$

$$+ (1 - \alpha\beta) E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left(\frac{MC_s^d - \overline{MC}^d}{\overline{MC}^d} \right)$$

Remarks:

1. We get a good linearization iff $\Omega = \Pi$. Otherwise, we have $\lambda_t^d Y_t$ innovations (since the first bracketed term does not cancel out). If we just ignore these innovations (ie the bracketed term), then the errors build up over time in the Phillips Curve we derive below.
2. Our linearization requires that the "indexing" parameter (Ω) must match the inflation trend (Π) that is being caused by monetary policy. If this is not allowed (by assumption, or by a price setting error), then the Phillips Curve we derive below is not a good approximation.

Letting $p_t^* \equiv \log(P_t^{*d}) - \log(\bar{P}^d)$ and $mc_t^d \equiv \log(MC_t^{*d}) - \log(\overline{MC}^d)$, & assuming $\Omega = \Pi$:

$$\text{(A4)} \quad p_t^* = (1 - \alpha\beta) E_t \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} mc_s^d(\cdot) \Rightarrow p_t^* = (1 - \alpha\beta) mc_t^d(\cdot) + E_t(\alpha\beta) p_{t+1}^*$$

Finally, we make the pricing equation "real" (by subtracting p_t from both sides of (A4):

(A5) $p_t^* - p_t = (1 - \alpha\beta)[mc_t^d(\cdot) - p_t] + \alpha\beta[E_t(p_{t+1}^*) - p_t] = (1 - \alpha\beta) mc_t(\cdot) + \alpha\beta[E_t(p_{t+1}^*) - p_t]$

$mc_t(\cdot) \equiv mc_t^d(\cdot) - p_t$ is the deviation of real marginal cost from steady states

Step 2: Derivation of the "New Keynesian" Phillips Curve –

Recall the "aggregate" price level is:

(A1) $P_t = [(1\text{-}\alpha)P_t^{*1\text{-}\theta} + \alpha(\Omega P_{t\text{-}1})^{1\text{-}\theta}]^{1/(1\text{-}\theta)} \Leftrightarrow P_t^{1\text{-}\theta} = (1\text{-}\alpha)P_t^{*1\text{-}\theta} + \alpha(\Omega P_{t\text{-}1})^{1\text{-}\theta}$

or converting to "discounted" prices:

$(P_t^d\Pi^t)^{1\text{-}\theta} = (1\text{-}\alpha)(P_t^{*d}\Pi^t)^{1\text{-}\theta} + \alpha(\Omega P_{t\text{-}1}^d\Pi^{t\text{-}1})^{1\text{-}\theta} \Rightarrow (\text{if } \Omega = \Pi) \;\; (P_t^d)^{1\text{-}\theta} = (1\text{-}\alpha)(P_t^{*d})^{1\text{-}\theta} + \alpha(P_{t\text{-}1}^d)^{1\text{-}\theta}$

Remarks:

1. Here again, we need $\Omega = \Pi$ to get a valid linearization.
2. If indeed $\Omega \neq \Pi$, and we just ignore it, then the errors would build up over time.

Taking the differential of (A1) with respect to the three prices:

$$(1-\theta)\left(\overline{P}^d\right)^{-\theta}(P_t^d - \overline{P}^d) = (1-\alpha)(1-\theta)\left(\overline{P}^d\right)^{-\theta}(P_t^{*d} - \overline{P}^d) + \alpha(1-\theta)\left(\overline{P}^d\right)^{-\theta}(P_{t-1}^d - \overline{P}^d)$$

Dividing by $(1-\theta)\left(\overline{P}^d\right)^{1-\theta}$:

$$\left(\frac{P_t^d - \overline{P}^d}{\overline{P}^d}\right) = (1-\alpha)\left(\frac{P_t^{*d} - \overline{P}^d}{\overline{P}^d}\right) + \alpha\left(\frac{P_{t-1}^d - \overline{P}^d}{\overline{P}^d}\right)$$

Finally, letting small p's represent log deviations from the "discounted" price level, we have:

(A6) $p_t = (1\text{-}\alpha)p_t^* + \alpha p_{t\text{-}1}$

To get the Phillips curve, we use (A6) to eliminate the p^*'s in (A5):

(A6) $\Rightarrow p_t^* = (1\text{-}\alpha)^{\text{-}1}p_t - \alpha(1\text{-}\alpha)^{\text{-}1}p_{t\text{-}1}$

(A5) $\Rightarrow p_t^* - p_t = (1\text{-}\alpha\beta)mc_t(\cdot) + \alpha\beta[E_t(p_{t+1}^*) - p_t]$

$(1\text{-}\alpha)^{\text{-}1}p_t - \alpha(1\text{-}\alpha)^{\text{-}1}p_{t\text{-}1} - p_t = (1\text{-}\alpha\beta)mc_t(\cdot) + \alpha\beta[E_t((1\text{-}\alpha)^{\text{-}1}p_{t+1} - \alpha(1\text{-}\alpha)^{\text{-}1}p_t) - p_t]$

$\alpha(1\text{-}\alpha)^{\text{-}1}(p_t - p_{t\text{-}1}) = (1\text{-}\alpha\beta)mc_t(\cdot) + \alpha\beta(1\text{-}\alpha)^{\text{-}1}E_t(p_{t+1} - p_t)$

$\pi_t = (1\text{-}\alpha\beta)[(1\text{-}\alpha)/\alpha]mc_t(\cdot) + (\alpha\beta/\alpha)E_t(\pi_{t+1})$ where $\pi_t \equiv p_t - p_{t\text{-}1}$

$$\text{(A7)} \;\; \pi_t = \left[\tfrac{(1-\alpha)(1-\alpha\beta)}{\alpha}\right]mc_t(\cdot) + \beta E_t\pi_{t+1}$$

Step 3: Replacing marginal cost, $mc_t(\cdot)$, with an "output-gap" term –

This step makes the "New Keynesian" Phillips Curve look like the old Phillips Curves, but it is necessarily model specific (and, Gali and Gertler (1999) suggest, empirically unwise); our derivation requires two basic assumptions:

1. Constant elasticity functions: $U(C, N) = (1-\gamma)^{-1}C^{1-\gamma} - A(1+\chi)^{-1}N^{1+\chi}$ and $Y(N) = ZN^{\eta}$.
2. Flexible wage setting; so, employment is determined by the labor supply curve: $W/P = \mu_w U_N(\cdot)/U_C(\cdot)$, where μ_w is the wage setters' markup (if any).

Let MC(Y) be real marginal cost:

$$MC(Y) = \frac{W/P}{MPL} = \mu_w \frac{U_N}{U_C}\frac{1}{MPL} = \mu_w \frac{AN^{\chi}}{Y^{-\gamma}}\frac{1}{\eta ZN^{\eta-1}} = \mu_w \frac{AN^{\chi+1-\eta}}{\eta ZY^{-\gamma}}$$

$$= \mu_w \frac{A(Y/Z)^{(\chi+1-\eta)/\eta}}{\eta ZY^{-\gamma}} = \frac{\mu_w A}{\eta}\frac{Y^{[\chi+1+\eta(\gamma-1)]/\eta}}{Z^{(\chi+1)/\eta}}$$

In the flex-price solution (denoted by *'s), we also have $W/P = (1/\mu_p)MPL$. So, $MC(Y^*) = 1/\mu_p$ and

$$\frac{1}{\mu_p} Z^{(\chi+1)/\eta} = \frac{\mu_w A}{\eta} Y^{*[\chi+1+\eta(\gamma+1)]/\eta} \quad \Rightarrow \quad MC(Y) = \frac{1}{\mu_p}\left(Y/Y^*\right)^{[\chi+1+\eta(\gamma-1)]/\eta}$$

And finally:

(A8) $\log(MC) = [\gamma-1 + (1+\chi)/\eta][\log(Y) - \log(Y^*)] - \log(\mu_p)$

From Step 2: $mc_t(\cdot) = \log(MC) - \log(\overline{MC}) = \log(MC) + \log(\mu_p)$
and let $y_t \equiv \log(Y_t) - \log(\overline{Y})$ and $y_t^* \equiv \log(Y_t^*) - \log(\overline{Y})$

Then:

(A9) $mc_t(\cdot) = [\gamma-1 + (1+\chi)/\eta](y_t - y_t^*) = [\gamma-1 + (1+\chi)/\eta]g_t$

where $g_t \equiv y_t - y_t^*$ is the "output gap".

Putting it all together, we have:

$$\text{(A10)} \quad \pi_t = \beta\pi_{t+1} + \left[\frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\right]mc_t(\cdot)$$

$$= \beta E_t\pi_{t+1} + \left[\frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\right]\left[\frac{\chi+1+\eta(\gamma\text{-}1)}{\eta}\right]g_t$$

Remarks: **Important Caveats** —

1. The "New Keynesian" Phillips Curve is essentially a relationship between current inflation and current and expected future marginal costs. We can make this optimal pricing equation look like a traditional Phillips Curve (as shown in Step 3), but the exercise may be more misleading than it is worth, for reasons given in the next three comments.
2. There is no natural error term in (A10). Some economists have simply tacked one on, and Bob King has interpreted such a residual as a "systematic price setting error". But, to think of this manufactured "shock" as a supply shock, and to build a theory of macroeconomic stabilization around it (as some have done) seems to us to be the wrong thing to do.
3. Since $\beta < 1$, a casual interpretation of (A10) might suggest that there is a long-run tradeoff between inflation and the output gap, and that one can use (A10) to analyze the implications of this tradeoff for monetary policy. If $\Omega \neq \Pi$, there is indeed a long-run tradeoff in "New Keynesian" Models. For example, King and Wolman (1999, 1996), analyze the case in which there is steady state inflation ($\Pi > 1$) and no "indexing" ($\Omega = 1$). However, it would be inappropriate to use (A10) (or (A7)) to analyze this tradeoff. As explained in Steps 1 and 2, this linearization is not valid unless $\Omega = \Pi$. King and Wolman (1999), using two-period Taylor contacts, derived the long-run Phillips Curve:

$$\frac{C}{C^p} = \left(\frac{1}{2}\right)^{1/(\theta-1)} \frac{(1+\Pi^{\theta-1})^{\theta/(\theta-1)}}{1+\Pi^{\theta}},$$

   where C^p is the consumption level that would be obtained in a flex price setting.
4. Using (A10) for empirical work may be problematic, since a number of strong assumptions were needed to obtain this linearization of firms' pricing decisions: (1) "Calvo contracts" were assumed to obtain a nice aggregation of the firms' prices. "Taylor contracts" are the current alternative to "Calvo contracts", and they may be thought more realistic – "Calvo contracts" imply that some small fraction of the firms keep the same price for arbitrarily long periods of time. However, with "Taylor contracts"we can not aggregate prices as above to get the traditional Phillips Curve. (2) Constant elasticity utility functions and production functions were needed to replace the marginal cost term with an "output gap"; they may, or may not, work well empirically. (3) As noted in remark 2, there is no natural error term for (A10). See Gertler and Gali (1999) for empirical support.

Note 3: *Linearizing the Demand Side* –

We can log-linearize around either the "no-shock" steady state equilibrium or the "flexible price" equilibrium. Bars denote "no-shock" steady state values, and "*'s the flex price values.

The Consumption Euler Equation, or IS-curve:

In the non-stochastic steady state, there is no uncertainty; so –

$$\overline{C}_t^{-\gamma} = \beta(1 + \bar{r}_t)\overline{C}_{t+1}^{-\gamma}$$

Divide the Euler equation by this steady state equation –

$$\left(\frac{C_t}{\overline{C}_t}\right)^{-\gamma} = \frac{1 + r_t}{1 + \bar{r}_t} E_t\left(\frac{C_{t+1}}{\overline{C}_{t+1}}\right)^{-\gamma}$$

Take logs of both sides, and divide by (-γ) –

$$\begin{aligned}
\log C_t - \log \overline{C}_t &\approx -\tfrac{1}{\gamma}(r_t - \bar{r}_t) - \tfrac{1}{\gamma}\log E_t\left[1 + \left(\left(\frac{C_{t+1}}{\overline{C}_{t+1}}\right)^{-\gamma} - 1\right)\right] \\
&= -\tfrac{1}{\gamma}(r_t - \bar{r}_t) - \tfrac{1}{\gamma}\log\left[1 + E_t\left(\left(\frac{C_{t+1}}{\overline{C}_{t+1}}\right)^{-\gamma} - 1\right)\right] \\
&\approx -\tfrac{1}{\gamma}(r_t - \bar{r}_t) - \tfrac{1}{\gamma} E_t\left(\frac{C_{t+1}^{-\gamma} - \overline{C}_{t+1}^{-\gamma}}{\overline{C}_{t+1}^{-\gamma}}\right) \\
&\approx -\tfrac{1}{\gamma}(r_t - \bar{r}_t) - \tfrac{1}{\gamma} E_t\left[\log C_{t+1}^{-\gamma} - \log \overline{C}_{t+1}^{-\gamma}\right]
\end{aligned}$$

so finally,

$$(\log C_t - \log \overline{C}_t) \approx -\tfrac{1}{\gamma}(r_t - \bar{r}_t) + E_t(\log C_{t+1} - \log \overline{C}_{t+1})$$

and similarly, for the flex price solution,

$$(\log C_t^* - \log \overline{C}_t) \approx -\tfrac{1}{\gamma}(r_t^* - \bar{r}_t) + E_t(\log C_{t+1}^* - \log \overline{C}_{t+1})$$

and subtracting,

$$(\log C_t - \log C_t^*) \approx -\tfrac{1}{\gamma}(r_t - r_t^*) + E_t(\log C_{t+1} - \log C_{t+1}^*)$$

$$r_t - \bar{r}_t = (i_t - E_t\pi_t) - (\bar{i}_t - \bar{\pi}_t)$$

$$r_t - r_t^* = (i_t - E_t\pi_t) - (i_t^* - E_t\pi_t^*) = [(i_t - E_t\pi_t) - (\bar{i}_t - \bar{\pi}_t)] - [(i_t^* - E_t\pi_t^*) - (\bar{i}_t - \bar{\pi}_t)]$$

Now, let the "gap" be defined by $g_t \equiv \log C_t - \log C^*_t$. We can define the IS-curve in two ways –

A. Perhaps the most natural way is:

(1a) $g_t = -(1/\gamma)(i_t - \pi_{t|t}) + g_{t+1|t} + (1/\gamma)r^*_t$

where and i_t, π_t and r^*_t are actual values (and not deviations from any baseline).

B. Alternatively, some use:

(1b) $g_t = -(1/\gamma)(i_t - \pi_{t|t}) + g_{t+1|t} + (1/\gamma)(r^*_t - \bar{r}_t)$

where i_t and π_t are defined as deviations from the non-stochastic steady state.

Remarks: **important caveats** —

1. Some studies add an expenditure (or government spending) shock and think of the error term in the IS curve as a demand shock. However, as (1a) and (1b) clearly indicate, the error term also includes r^*_t, which moves with productivity and/or labor supply shocks. See for example Clarida, Gertler and Gali (JEL, 1999) or Erceg, Henderson and Levin (JME, 2000).
2. The main point is that care should be exercised in interpreting the shocks to the "New Keynesian" IS and Phillips Curves derived in this appendix and the last.

Note 4: *Unbundling the Bundler –*

In the notes, we used the popular artifice of a "bundler" to short circuit the algebra involved with the composite goods and their prices. In this appendix, we provide the missing algebra (which we hope is correct). Using the utility function in Obstfeld and Rogoff (1996), we separate the consumer's maximization into a "temporal" and an "intertemporal" problems. See also Frenkel and Razin (1987, pg 171).

Household-j's Utility –

(1) $U_t^j = \sum_{s=t}^{\infty}\beta^{s-t}[\log C_s^j + \mu\log(M_s^j/P_s) - \tfrac{1}{2}\kappa y_s^j(j)^2]$,

(2) $P_s \equiv [\int_0^1 p_s(z)^{1-\theta}dz]^{1/(1-\theta)}$ and $C_s^j \equiv [\int_0^1 c^j(z)^{(\theta-1)/\theta}dz]^{\theta/(\theta-1)}$ and $\theta > 1$

Remarks:

1. There is a continuum of households (j) producing differentiated products (z); C_s^j is a consumption index; $y_s^j(j)$ is supply of product z = j; and P_s is discussed below. It turns out that θ is the elasticity of demand, which must be > 1 for a monopolist to have an interior solution.
2. Interpretation of $\kappa y_s^j(j)^2$ and Obstfeld &Rogoff's modeling of productivity shocks:
   let $-\omega\ell$ represent the disutility of labor, and $y = A\ell^{\alpha}$ be the production function;
   inverting the production function, $\ell = (y/A)^{1/\alpha}$ and $-\omega\ell = -\kappa y^{1/\alpha}$ where $\kappa = (\omega/A^{1/\alpha})$;
   here we let $\alpha = \tfrac{1}{2}$.
   Note: a positive productivity shock can be interpreted as a fall in κ.
3. It helps to derive the aggregate demand curves, $y^d(z)$, and the price index, P_s, before doing the full household optimization.
4. This can be done by decomposing the household maximization problem into:
   (1) the optimal *temporal* allocation of a fixed expenditure, E, over the c(z) (or its dual: minimizing the cost, E, of purchasing $C_s^j = 1$); and
   (2) the optimal *intertemporal* allocation of expenditure, E.

The optimal *temporal* allocation problem –

We can drop subscripts/superscripts here; we are talking about household j at some point in time.

Solve the *dual* problem first to derive the price index, P:

Show that P (defined above) is the minimum cost (or price) of a unit of $C \equiv [\int_0^1 c(z)^{(\theta-1)/\theta}dz]^{\theta/(\theta-1)}$

Let $E \equiv \int_0^1 p(z)c(z)dz$ be total cost (or expenditure).

Min $E \equiv \int_0^1 p(z)c(z)dz$ over $c(z)$ subject to $C \equiv [\int_0^1 c(z)^{(\theta-1)/\theta}dz]^{\theta/(\theta-1)} = 1$

the FOC is

(a) $p(z) = \lambda c(z)^{[(\theta-1)/\theta - 1]}[\int_0^1 c(z)^{(\theta-1)/\theta}dz]^{\theta/(\theta-1)-1} = \lambda c(z)^{-1/\theta}[\int_0^1 c(z)^{(\theta-1)/\theta}dz]^{1/(\theta-1)}$

here, $C = 1$; so, $\int_0^1 c(z)^{(\theta-1)/\theta}dz = 1$, and (a) becomes

(a)' $p(z)c(z) = \lambda c^{(\theta-1)/\theta}$ or $\lambda = p(z)c(z)^{1/\theta}$

using (a)' repeatedly:

$P \equiv \text{Min } E = \int_0^1 p(z)c(z)dz = \lambda\int_0^1 c(z)^{(\theta-1)/\theta}dz = \lambda$ (since $\int_0^1 c(z)^{(\theta-1)/\theta}dz = 1$)

$\lambda = p(z)c(z)^{1/\theta} \Rightarrow \lambda^{1-\theta} = p(z)^{1-\theta}c(z)^{(1-\theta)/\theta} \Rightarrow \lambda^{1-\theta}c(z)^{(\theta-1)/\theta} = p(z)^{1-\theta}$

integrating over z:

$\lambda^{1-\theta}\int_0^1 c(z)^{(\theta-1)/\theta}dz = \int_0^1 p(z)^{1-\theta}dz$

so finally,

$\lambda = P = [\int_0^1 p(z)^{1-\theta}dz]^{1/(1-\theta)}$ (again, since $\int_0^1 c(z)^{(\theta-1)/\theta}dz = 1$)

Solve the *primal* problem to derive the individual and aggregate demand curves:

Max $C \equiv [\int_0^1 c(z)^{(\theta-1)/\theta}dz]^{\theta/(\theta-1)}$ over $c(z)$ subject to $\int_0^1 p(z)c(z)dz = E$,

where here, E is an arbitrary, but fixed, level of expenditure.

We get the same FOC (where Lagrangian multiplier $\phi = 1/\lambda = 1/P$, but here $C \neq 1$):

(a) $\phi p(z) = c(z)^{[(\theta-1)/\theta - 1]}[\int_0^1 c(z)^{(\theta-1)/\theta}dz]^{\theta/(\theta-1)-1} = c(z)^{-1/\theta}[\int_0^1 c(z)^{(\theta-1)/\theta}dz]^{1/(\theta-1)}$

So, since $\phi = 1/P$,

$p(z)/P = c(z)^{-1/\theta}[\int_0^1 c(z)^{(\theta-1)/\theta}dz]^{1/(\theta-1)} \Rightarrow [p(z)/P]^{-\theta} = c(z)[\int_0^1 c(z)^{(\theta-1)/\theta}dz]^{-\theta/(\theta-1)}$

And finally (reintroducing the household superscript, j, and time subscript, s),

(b) $c^j_s(z) = [p_s(z)/P_s]^{-\theta}[\int_0^1 c^j_s(z)^{(\theta-1)/\theta}dz]^{\theta/(\theta-1)} = [p_s(z)/P_s]^{-\theta}C^j_s = [p_s(z)/P_s]^{-\theta}(E^j_s/P_s)$

Remarks:

1. $C = E/P$ since $E = \$$ and $P = \$/C$.
2. (b) $\Rightarrow$ $p(z)c(z) = [P/p(z)]^{\theta-1}E$; $[P/p(z)]^{\theta-1}$ is the “share” of E going to c(z).

<u>Aggregate demand</u> for product z at time s is (using (b):

(3) $y^d_s(z) = \int^1_0 c^j_s(z)dj = \int^1_0 [p_s(z)/P_s]^{-\theta} C^j_s dj = [p_s(z)/P_s]^{-\theta} \int^1_0 C^j_s dj = [p_s(z)/P_s]^{-\theta} C_s$,

where $C_s \equiv \int^1_0 C^j_s dj$ is aggregate consumption of all households.

Digression: Suppose there are a finite (but large) number n of household/producers.

Let $C \equiv n^{1/(1-\theta)}[\sum^n_{z=1} c(z)^{(\theta-1)/\theta}]^{\theta/(\theta-1)}$ be the consumption aggregate, and show that

$P \equiv [(1/n)\sum^n_{z=1} p(z)^{1-\theta}]^{1/(1-\theta)}$ is the price index (or minimum cost of a unit of C).

(This specification is from Blanchard and Fischer, page 376; see also King and Wolman.)

Let $E \equiv \sum p(z)c(z)$ be total cost (or expenditure).

Min $E \equiv \sum p(z)c(z)$ over c(z) subject to $C \equiv n^{1/(1-\theta)}[\sum c(z)^{(\theta-1)/\theta}]^{\theta/(\theta-1)} = 1$

the FOC is

(a) $p(z) = \lambda n^{1/(1-\theta)} c(z)^{[(\theta-1)/\theta - 1]}[\sum c(z)^{(\theta-1)/\theta}]^{\theta/(\theta-1)-1} = \lambda n^{1/(1-\theta)} c(z)^{-1/\theta}[\sum c(z)^{(\theta-1)/\theta}]^{1/(\theta-1)}$

here, C = 1; so, $n^{1/(1-\theta)}[\sum c(z)^{(\theta-1)/\theta}]^{\theta/(\theta-1)-1} = [\sum c(z)^{(\theta-1)/\theta}]^{-1} = n^{-1/\theta}$

and (a) becomes

(a)' $p(z)c(z) = \lambda n^{-1/\theta} c(z)^{(\theta-1)/\theta}$ or $\lambda = n^{1/\theta} p(z)c(z)^{1/\theta}$

using (a)' repeatedly:

$P \equiv \text{Min } E = \sum p(z)c(z) = \lambda n^{-1/\theta}\sum c(z)^{(\theta-1)/\theta} = \lambda$ (since $\sum c(z)^{(\theta-1)/\theta} = n^{1/\theta}$)

$\lambda = n^{1/\theta} p(z)c(z)^{1/\theta} \Rightarrow \lambda^{1-\theta} = n^{(1-\theta)/\theta} p(z)^{1-\theta} c(z)^{(1-\theta)/\theta} \Rightarrow \lambda^{1-\theta} c(z)^{(\theta-1)/\theta} = n^{(1-\theta)/\theta} p(z)^{1-\theta}$

adding over the z:

$\lambda^{1-\theta}\sum c(z)^{(\theta-1)/\theta} = n^{(1-\theta)/\theta}\sum p(z)^{1-\theta}$ or $\lambda^{1-\theta} n^{1/\theta} = n^{(1-\theta)/\theta}\sum p(z)^{1-\theta}$ or $\lambda^{1-\theta} = n^{-1}\sum p(z)^{1-\theta}$

so finally,

$\lambda = P = [(1/n)\sum p(z)^{1-\theta}]^{1/(1-\theta)}$

The optimal *intertemporal* allocation problem --

First, invert the aggregate demand curve (3) for product z:

(c) $p_s(z) = P_s[y^d_s(z)/C_s]^{-1/\theta}$

So, equating supply and demand, the revenue for the supplier of product z = j is:

(d) $p_s(j)y^j_s(j) = P_s y^j_s(j)^{1-1/\theta} C_s^{\ 1/\theta} = P_s y^j_s(j)^{(\theta-1)/\theta} C_s^{\ 1/\theta}$

Household j's BC in period s --

(4) $P_s C^j_s + B^j_s + M^j_s + P_s\tau_s = p_s(j)y^j_s(j) + I_{s-1}B_{s-1} + M^j_{s-1} = P_s y^j_s(j)^{(\theta-1)/\theta} C_s^{\ 1/\theta} + I_{s-1}B_{s-1} + M^j_{s-1}$

Remarks:

1. Household exploits the demand curve for its product just as a monopolist would.
2. Household takes aggregate consumption, C_s, as given. However, this provides the externality.

Household j's maximization problem –

$\mathcal{L} = \sum_{s=t}^{\infty}\beta^{s-t}\{[\log C^j_s + \mu\log(M^j_s/P_s) - ½\kappa y^j_s(j)^2] + \lambda_s[P_s y^j_s(j)^{(\theta-1)/\theta} C_s^{\ 1/\theta} + I_{s-1}B_{s-1} + M^j_{s-1} - P_s C^j_s - B^j_s - M^j_s - P_s\tau_s]\}$

B^j_t: $\beta\lambda_{t+1}I_t - \lambda_t = 0 \quad \Rightarrow \quad \beta I_t = \lambda_t/\lambda_{t+1}$

C^j_t: $1/C^j_t - \lambda_t P_t = 0 \quad \Rightarrow \quad \lambda_t/\lambda_{t+1} = P_{t+1}C^j_{t+1}/P_t C^j_t$

M^j_t: $\mu/(M^j_t/P_t)P_t + \beta\lambda_{t+1} - \lambda_t = 0 \quad \Rightarrow \quad \mu/M^j_t = \lambda_t[1 - \beta(\lambda_{t+1}/\lambda_t)] = (1/C^j_t P_t)[1 - (1/I_t)]$

$y^j_t(j)$: $-\kappa y^j_t + \lambda_t P_t[(\theta-1)/\theta] y_t^{j[(\theta-1)/\theta - 1]} C_t^{\ 1/\theta} = 0 \quad \Rightarrow \quad \kappa y^j_t = [(\theta-1)/\theta] y_t^{j[(\theta-1)/\theta - 1]} C_t^{(1/\theta - 1)}$

FOC's for Household j are:

(5) $\beta I_t(P_t/P_{t+1}) = C^j_{t+1}/C^j_t$ or $\beta I_t(P_t/P_{t+1}) = C_{t+1}/C_t$ (see remark 2)

(6) $M^j_t/P_t = \mu C^j_t[(1 + i_t)/i_t]$ or $M_t/P_t = \mu C_t[(1 + i_t)/i_t]$ (see remark 2)

(7) $y_t^{j(\theta+1)/\theta} = [(\theta-1)/\theta\kappa] C_t^{(1-\theta)/\theta}$

Remarks on symmetry and aggregation:

1. The FOC for C^j_t, and M^j_t are the same for all households. So, $C_t \equiv \int_0^1 C^j_t dj = C^j_t \int_0^1 dj = C^j_t$ and $M_t \equiv \int_0^1 M^j_t dj = M^j_t \int_0^1 dj = M^j_t$; C_t and M_t simultaneously represent “aggregate”, “individual” and “representative” values of consumption and money.

2. Therefore, (5) and (6) can therefore be rewritten in terms of C_t and M_t.
3. From (c), $y_t^d(z) = [p_t(z)/P_t]^{-\theta}C_t$ and from (7), $y_t^{z\ (\theta+1)/\theta} = [(\theta-1)/\theta\kappa]C_t^{(1-\theta)/\theta}$. Neither supply nor demand for good z depends on the index z. Therefore, as Obstfeld and Rogoff note on page 663, all suppliers will choose to set the same price, $p_t(z)/P_t$, in equilibrium.
4. Since all $p_t(z)$ are the same in equilibrium, $P_t \equiv [\int_0^1 p_t(z)^{1-\theta}dz]^{1/(1-\theta)} = p_t(z)$ in equilibrium.
5. (7) $\Rightarrow$ y_t^j is independent of index j. So, $Y_t \equiv \int_0^1 [p_t(j)/P_t]y_t^j dj = [p_t(j)/P_t]y_t^j \int_0^1 dj = [p_t(j)/P_t]y_t^j = y_t^j$. All producers produce the same amount, and Y_t simultaneously represents "aggregate", "representative" and "individual" output (measured here in units of C).

Note 5: *Interest Rate Rules –*

In the notes, we assumed a cash-in-advance constraint and money targeting. This allowed us to solve the models without worrying about the dynamics inherent in the Euler equation. However, the literature has focused in interest rate rules instead of money targeting. Here, we show how the method of undetermined coefficients can be used to solve the Yeoman Farmer Model with interest rate targeting. Here, we have taken freely from Henderson and Kim (????). These notes are rather incomplete – a work in progress.

Household j's Utility –

(1) $\mathrm{U}^j_t = \mathrm{E}_t\sum_{s=t}^{\infty}\beta^{s-t}[(1-\gamma)^{-1}(\mathit{C}^{j\,1-\gamma}_s - 1) - ½(1/\mathrm{X}_s)\mathrm{y}^j_s(j)^2]\mathrm{U}_s$

(2) $\mathrm{P}_s \equiv [\int_0^1 \mathrm{p}_s(z)^{1-\theta}dz]^{1/(1-\theta)}$

where $\mathrm{C}^j_s \equiv [\int_0^1 c^j(z)^{(\theta-1)/\theta}dz]^{\theta/(\theta-1)}$, $\theta > 1$, and U_s is Henderson and Kim's "IS" shock.

Aggregate demand for good-z –

(3) $y^d_s(z) = \int_0^1 c^j_s(z)dj = \int_0^1[p_s(z)/P_s]^{-\theta}C^j_s dj = [p_s(z)/P_s]^{-\theta}C_s$ (see Appendix 3)

Household j's transactions technology –

(4) $\mathit{C}^j_s = \min(\mathrm{C}^j_s, \mathrm{M}^j_s/\mathrm{V}_s\mathrm{P}_s)$

Remark: It is optimal for the household to keep $\mathrm{M}^j_s = \mathrm{P}_s\mathrm{V}_s\mathrm{C}^j_s = \mathrm{P}_s\mathrm{V}_s\mathrm{E}^j_s = \mathrm{P}_s\mathrm{V}_s\int_0^1 p_t(z)c^j_s(z)dz$ and thus $\mathit{C}^j_s = \mathrm{C}^j_s$. V_s is proportion of expenditures requiring CIA.

Household j's BC in period-s –

(5) $P_sC^j_s + M^j_s + B^j_s + P_s\tau_s = \phi p(j)y^j_s(j) + M^j_{s-1} + I_{s-1}B_{s-1} = \phi p_s(j)^{1-\theta}(1/P_s)^{-\theta}C_s + M^j_{s-1} + I_{s-1}B^j_{s-1}$

Remarks:

1. Household-j maximizes (1) s.t. (4), (5) and $y^j_s(j) = y^d_s(j)$, where $y^d_s(j)$ is given by (3).
2. ϕ is a state dependent government subsidy to household receipts.

Household-j's maximization problem --

$$\mathcal{L} = E_t\textstyle\sum_{s=t}^{\infty}\beta^{s-t}\{[(1-\gamma)^{-1}(C_s^{j\,1-\gamma} - 1) - \tfrac{1}{2}(1/X_s)(p_s(j)/P_s)^{-2\theta}C_s^2]U_s$$

$$+ \lambda_s[\phi p_s(j)^{1-\theta}(1/P_s)^{-\theta}C_s + M_{s-1}^j + I_{s-1}B_{s-1}^j - P_sC_s^j - M_s^j - B_s^j - P_s\tau_s]\}$$

the following FOC's hold for any pricing assumption:

C_t^j: $U_t/(C_t^j)^\gamma - \lambda_tP_t = 0$

B_t^j: $- \beta I_tE_t\lambda_{t+1} + \lambda_t = 0$

M_t^j: $M_t = V_tP_tC_t^j$

these FOC's ⇒ the Euler Equation –

(6) $\beta I_tE_t\{U_{t+1}/C_{t+1}^\gamma P_{t+1}) = U_t/C_t^\gamma P_t$ (recalling that $C_t^j = C_t$)

if prices are flexible:

$p_t(j)$: $(\theta/X_t)(1/P_t)(p_t(j)/P_t)^{-2\theta-1}C_t^2U_t + \lambda_t\phi(1-\theta)p(j)^{-\theta}(1/P_t)^{-\theta}C_t = 0$

$\lambda_t = U_t/P_tC_t^\gamma$, and in equilibrium, $p_t(j) = P_t$ and $C_t = Y_t$

$(\theta/X_t)(1/P_t)Y_t^2U_t + \phi(1-\theta)(U_t/P_tY_t^\gamma)Y_t = 0$

canceling the $(1/P_t)U_t$

$(\theta/X_t)Y_t^2 + \phi(1-\theta)(1/Y_t^{\gamma-1}) = 0$ or $(\theta/X_t)Y_t^{\gamma+1} + \phi(1-\theta) = 0$

$(7)_{flex}$ $Y_t = \{[\phi(\theta-1)/\theta]X_t\}^{1/(\gamma+1)} = X_t^{1/(\gamma+1)}$

Remarks:

1. $(\theta-1)/\theta$ is the distortions created by monopolistic competition. Following Henderson and Kim, we set $\phi = \theta/(\theta-1)$ to eliminate it.
2. (4) eliminates the standard seigniorage distortion.

if prices are set one period ahead:

$p_{t+1}(j)$: $E_t[(1/X_{t+1})(1/P_{t+1})Y_{t+1}^2U_{t+1} - (U_{t+1}/P_{t+1}Y_{t+1}^\gamma)Y_{t+1}] = 0$

since $p_{t+1}(j) = P_{t+1}$ is known, can cancel the $1/P_{t+1}$

$(8)_{sticky}$ $E_t\{U_{t+1}[(1/X_{t+1})Y_{t+1}^2 - Y_{t+1}^{1-\gamma}]\} = 0$

Fiscal Policy – as in notes.

Monetary Policy –

(10) $I_t = \Pi\beta^{-1}U_t^{\lambda u}X_t^{\lambda x}P_t^{\lambda p}$

Remark:

1. This assumes (ala Henderson and Kim) a fully stationary solution.
2. $\lambda_p > 0$ will be needed to avoid the price indeterminacy problem.

Summarizing –

The Model:

(1) $\beta I_t E_t\{U_{t+1}/Y^{\gamma}_{t+1}P_{t+1}) = U_t/Y^{\gamma}_tP_t$	Euler Equation
(2) $I_t = \beta^{-1}U^{\lambda u}X^{\lambda x}P^{\lambda p}$	Monetary Policy
(3) $M_t = V_tP_tY_t$	Cambridge Equation
$(4)_{flex}$ $Y_t = X_t^{1/(\gamma+1)}$	Pricing Equation
$(4)_{sticky}$ $E_t\{U_{t+1}[(1/X_{t+1})Y^2_{t+1} - Y^{1-\gamma}_{t+1}]\} = 0$	Pricing Equation
(5) $\tau_t = (\phi-1)Y - (M_t - M_{t-1})/P_t$	Fiscal Policy

Stochastic Environment:

Let U, V and X be independent and have log normal distributions.

Let small letters represent the logs of capital letters.

Review of implications of our assumptions (see Henderson and Kim, Appendix A):

Let Q have a log normal distribution; so, $q \equiv \ln Q \sim N(\overline{q}, 2\sigma_q^2)$.

$\ln Q^k = kq \Rightarrow Q^k = \exp\{kq\} \Rightarrow E(Q^k) = E(\exp\{kq\}) = \exp\{k\overline{q} + k^2\sigma_q^2\} \Rightarrow \ln E(Q^k) = k\overline{q} + k^2\sigma_q^2$

Note:

If we assume $\overline{q} = 0$ (which seems natural) , then $E(Q) = \exp\{\sigma_q^2\} \neq 1$ (which is not nice).

If we assume $E(Q) = 1$(which is nice), then $\overline{q} = -\sigma_q^2$ (which will involve a lot of algebra).

Henderson and Kim assume $\overline{q} = 0$ (and implicitly $E(Q) = \exp\{\sigma_q^2\}$); we follow their lead.

Finding the Flexible Price Solution –

(1) $\beta I_t E_t\{U_{t+1}/Y^{\gamma}_{t+1}P_{t+1}) = U_t/Y^{\gamma}_t P_t$ Euler Equation

(2) $I_t = \Pi\beta^{-1}U^{\lambda u}_t X^{\lambda x}_t P^{\lambda p}_t$ Monetary Policy

(3) $M_t = V_t P_t Y_t$ Cambridge Equation

$(4)_{flex}$ $Y_t = X_t^{1/(\gamma+1)}$ Pricing Equation

letting $\alpha \equiv \gamma/(1+\gamma)$,

(1), (2) & (4) $\Rightarrow$ $U^{\lambda u}X^{\lambda x}P^{\lambda p}E(U_{+1}X_{+1}{}^{-\alpha}P_{+1}{}^{-1}) = UX^{-\alpha}P^{-1}$ $\Rightarrow$

(A) $\lambda_u u + \lambda_x x + \ln E[(U_{+1}X_{+1}{}^{-\alpha}P_{+1}{}^{-1})] = u - \alpha x - (1+\lambda_p)p$

conjecture that

(B) $P = \Omega U^{\omega u}X^{\omega x}$ (where Ω and the ω's are undetermined coefficients)

substitute (B) back into (A) and determine the coefficients

let $\lambda' = \lambda_u u + \lambda_x x$ and $\omega' = \omega_u u + \omega_x x$

$\lambda' + \ln E(U_{+1}X_{+1}{}^{-\alpha}\Omega^{-1}U_{+1}{}^{-\omega u}X_{+1}{}^{-\omega x}) = u - \alpha x - (1+\lambda_p)(\ln\Omega + \omega')$ $\Rightarrow$

(C) $\lambda' - \ln\Omega + \ln E(Z) = u - \alpha x - (1+\lambda_p)\ln\Omega - (1+\lambda_p)\omega'$ where $Z \equiv U_{+1}{}^{1-\omega u}X_{+1}{}^{-\alpha-\omega x}$

Note: if $\lambda_p = 0$, $\ln\Omega$ drops out at this point, and Ω would be a free parameter.

(D) $\ln E(Z) + \lambda_p\ln\Omega = u - \alpha x - \lambda' - (1+\lambda_p)\omega' = u - \alpha x - (\lambda_u u + \lambda_x x) - (1+\lambda_p)(\omega_u u + \omega_x x)$

$= [1 - \lambda_u - (1+\lambda_p)\omega_u]u - [\alpha + \lambda_x + (1+\lambda_p)\omega_x]x$

where $\ln E(Z) = (1 - \omega_u)^2\sigma^2_u + (\alpha + \omega_x)^2\sigma^2_x$

if the conjecture is to hold for all realizations, we have to let

(E) $\omega_u = (1-\lambda_u)/(1+\lambda_p)$, $\omega_x = -(\alpha + \lambda_x)/(1+\lambda_p)$, and

$\ln\Omega = -(1/\lambda_p)\ln E(Z) = -(1/\lambda_p)\{[(\lambda_u + \lambda_p)/(1+\lambda_p)]^2\sigma^2_u + [(\alpha\lambda_p-\lambda_x)/(1+\lambda_p)]^2\sigma^2_x\}$

Flex Price Solution:

(5a) $C_t = Y_t = X_t^{1/(\gamma+1)}$

(5b) $I_t = \Pi\beta^{-1}U_t^{\lambda u}X_t^{\lambda x}P_t^{\lambda p}$

(5c) $P = \Omega U^{(1-\lambda u)/(1+\lambda p)}X^{-(\alpha+\lambda x)/((1+\lambda p)}$, $\ln\Omega = -(1/\lambda_p)\{[(\lambda_u+\lambda_p)/(1+\lambda_p)]^2\sigma_u^2 + [(\alpha\lambda_p-\lambda_x)/(1+\lambda_p)]^2\sigma_x^2\}$

(5d) $M_t = V_tP_tY_t = V_tP_tX_t^{1/(\gamma+1)}$

Remarks:

1. Consumption and output (and therefore utility) depend only on the productivity shock. Policy parameters don't affect anything important.
2. We have the usual price indeterminacy problem:
   a. Is λ_p were = 0, Ω would drop out of (C) above, and it would be a free parameter in (5c). (5d) would be one equation in P_t and M_t.
   b. λ_p can be arbitrarily small. Doesn't look like you can take a limit and get anything useful.

Finding the Fixed Price Solution –

(1) $\beta I_tE_t(U_{t+1}/Y_{t+1}^{\gamma}P_{t+1}) = U_t/Y_t^{\gamma}P_t$ Euler Equation

(2) $I_t = \Pi\beta^{-1}U_t^{\lambda u}X_t^{\lambda x}P_t^{\lambda p}$ Monetary Policy

(3) $M_t = V_tP_tY_t$ Cambridge Equation

$(4)_{sticky}$ $E_{t-1}\{U_t[(1/X_t)Y_t^2 - Y_t^{1-\gamma}]\} = 0$ Pricing Equation

Here, P_t is preset (though we don't know its level yet), and Y_t is the unknown variable.

conjecture that

(A) $Y = \Omega Z$ where $Z \equiv U^{\omega u}X^{\omega x}$ (and where Ω and the ω's are undetermined coefficients)

substitute (A) into Pricing Equation (4) to determine Ω:

$$E\{U[X^{-1}\Omega^2Z^2 - \Omega^{1-\gamma}Z^{1-\gamma}] = 0 \;\Rightarrow\; \Omega^2E(UX^{-1}Z^2) = \Omega^{1-\gamma}E(UZ^{1-\gamma}) = 0 \;\Rightarrow$$

$$\Omega^{1+\gamma} = E(UZ^{1-\gamma})/E(UX^{-1}Z^2) = E[U(U^{\omega u}X^{\omega x})^{1-\gamma}]/E[UX^{-1}(U^{\omega u}X^{\omega x})^2] \;\Rightarrow$$

(B) $\Omega = [E(U^{1-\omega u(\gamma-1)}X^{-\omega x(\gamma-1)})/E(U^{1+2\omega u}X^{2\omega x-1})]^{1/(1+\gamma)}$

now substitute (A) into the Euler Equation (1) to determine the other coefficients:

(C) $U^{\lambda u}X^{\lambda x}P^{\lambda p}E(UY_{+1}{}^{-\gamma}P_{+1}{}^{-1}) = UY^{-\gamma}P^{-1}$

H&K (pg 519) assertion: in a stationary RE equilibrium with a levels reaction function, $P_{+1} = P$.

(D) $U^{\lambda u}X^{\lambda x}P^{\lambda p}E(UY_{+1}{}^{-\gamma}) = UY^{-\gamma} \Rightarrow U^{\lambda u}X^{\lambda x}P^{\lambda p}E(U\Omega^{-\gamma}Z_{+1}{}^{-\gamma}) = U\Omega^{-\gamma}Z^{-\gamma}$ (the Ω's cancel)

$\lambda_u u + \lambda_x + \lambda_p p + \ln E(U_{+1}{}^{1-\gamma\omega u}X_{+1}{}^{-\gamma\omega x}) = (1-\gamma\omega_u)u - \gamma\omega_x x$

(E) $\lambda_p p + \ln E(U_{+1}{}^{1-\gamma\omega u}X_{+1}{}^{-\gamma\omega x}) = (1 - \lambda_u - \gamma\omega_u)u - (\lambda_x + \gamma\omega_x)x$

Note: If we do not let $\lambda_p \neq 0$, there would be no way to make (E) work!!

if the conjecture is to hold for all realizations, we have to let

(F) $\omega_u = (1-\lambda_u)/\gamma$, $\omega_x = - \lambda_x/\gamma$, and

$p = - (1/\lambda_p)\ln E(U_{+1}{}^{1-\gamma\omega u}X_{+1}{}^{-\gamma\omega x}) = - (1/\lambda_p)\{(1-\gamma\omega_u)^2\sigma_u^2 + (\gamma\omega_x)^2\sigma_x^2\} = - (1/\lambda_p)(\lambda_u{}^2\sigma_u^2 + \lambda_x{}^2\sigma_x^2)$

$\ln\Omega = \ln[E(U^{1-\omega u(\gamma-1)}X^{-\omega x(\gamma-1)})/E(U^{1+2\omega u}X^{2\omega x-1})]^{1/(1+\gamma)}$

$= (1+\gamma)^{-1}\{\ln[E(U^{1-\omega u(\gamma-1)}X^{-\omega x(\gamma-1)}) - \ln E(U^{1+2\omega u}X^{2\omega x-1})]$

$= (1+\gamma)_{-1}\{[(1-(\gamma-1)\omega_u]^2 - [1+2\omega_u]^2\}\sigma_u^2 + (1+\gamma)_{-1}\{[(\gamma-1)\omega_x]^2 - [1-2\omega_x]^2\}\sigma_x^2$

Fixed Price Solution:

(6a) $C_t = Y_t = \Omega U_t^{(1-\lambda u)/\gamma}X_t^{-\lambda x/\gamma}$

where $(1+\gamma)\ln\Omega = \{[(1-(\gamma-1)\omega_u]^2 - [1+2\omega_u]^2\}\sigma_u^2 + \{[(\gamma-1)\omega_x]^2 - [1-2\omega_x]^2\}\sigma_x^2$

and $\omega_u = (1-\lambda_u)/\gamma$ and $\omega_x = - \lambda_x/\gamma$

(6b) $I_t = \Pi\beta^{-1}U_t^{\lambda u}X_t^{\lambda x}P_t^{\lambda p}$

(6c) $P_t = \exp\{-(1/\lambda_p)(\lambda_u{}^2\sigma_u^2 + \lambda_x{}^2\sigma_x^2)\} = \exp(\lambda_u{}^2\sigma_u^2 + \lambda_x{}^2\sigma_x^2)/\exp(1/\lambda_p)$

(5d) $M_t = V_tP_tY_t$

Remarks:

1. We don't know how to interpret the fact (see note on last page) that this solution method will not work if we do not have $\lambda_p > 0$.
2. The flex price output level is: $Y_t = X_t^{1/(\gamma+1)}$. It can be achieved by setting $\lambda_u = 1$ & $\lambda_x = -\alpha$. The policy rule that does this is: $I_t = \beta^{-1}UX^{-\alpha}P^{\lambda p}$.

3. How does this work? In period t, $C_t = Y_t$ is "demand determined" by the Euler Equation $\beta I_t E_t(U_{t+1}/Y^{\gamma}_{t+1}P_{t+1}) = U_t/Y^{\gamma}_t P_t$. $E_t(U_{t+1}/Y^{\gamma}_{t+1}P_{t+1})$ is just a fixed number. "Demand shocks", U_t, would shift $C_t = Y_t$ up; the policy rule just accommodates them. "Productivity shocks", X_t, would not affect $C_t = Y_t$; the policy rule stimulates the economy in response to them to get Y_t up to "potential".
4. Expected level of output (assuming $\lambda_u = \lambda_x = 0$):
   a. Flex Price: $\ln E(Y) = \ln E(X_t^{1/(\gamma+1)}) = (\gamma+1)^{-2}\sigma^2_x$
   b. Fixed Price: $\ln E(Y) = \ln E(\Omega U^{(1-\lambda u)/\gamma} X^{-\lambda x/\gamma}) = \ln\Omega + \ln E(U^{1/\gamma}) = \ln\Omega + (1/\gamma)^2\sigma^2_u = \ldots$
   $= - (1/\gamma)^2 (\gamma+2)\sigma^2_u - \sigma^2_x$ (Hope I did this right!)
   c. $E(Y)_{\textit{flex price}} > E(Y)_{\textit{fixed price}}$. So, policy that brings about flex price solution raises E(Y)!

Remarks:

1. We're not very happy with the way the price indeterminacy problem is resolved above. Also, we've had trouble getting trend inflation into the setup above.
2. In what follows, we follow Canzoneri, Henderson and Rogoff (QJE, 198?) on the price indeterminacy problem, and this allows us to model trend inflation.

Modeling Trend Inflation:

Change the policy rule to introduce trend inflation (and take out λ_p) –

The Model:

(1) $\beta I_t E_t\{U_{t+1}/Y^{\gamma}_{t+1}P_{t+1}) = U_t/Y^{\gamma}_t P_t$	Euler Equation
(2) $I_t = \Pi\beta^{-1}U^{\lambda u}X^{\lambda x}$	Monetary Policy
(3) $M_t = V_t P_t Y_t$	Cambridge Equation
$(4)_{\textit{flex}}$ $Y_t = X_t^{1/(\gamma+1)}$	Pricing Equation
$(4)_{\textit{sticky}}$ $E_t\{U_{t+1}[(1/X_{t+1})Y^2_{t+1} - Y^{1-\gamma}_{t+1}]\} = 0$	Pricing Equation
(5) $\tau_t = (\phi-1)Y - (M_t - M_{t-1})/P_t$	Fiscal Policy

Finding the Flexible Price Solution –

(1) $\beta I_t E_t\{U_{t+1}/Y^{\gamma}_{t+1}P_{t+1}) = U_t/Y^{\gamma}_t P_t$ Euler Equation

(2) $I_t = \Pi\beta^{-1}U_t^{\lambda u}X_t^{\lambda x}$ Monetary Policy

(3) $M_t = V_t P_t Y_t$ Cambridge Equation

$(4)_{flex}$ $Y_t = X_t^{1/(\gamma+1)}$ Pricing Equation

letting $\alpha \equiv \gamma/(1+\gamma)$,

(1), (2) & (4) ⇒ $\Pi U^{\lambda u}X^{\lambda x}E(U_{+1}X_{+1}^{-\alpha}P_{+1}^{-1}) = UX^{-\alpha}P^{-1}$ ⇒

(A) $\lambda_u u + \lambda_x x + \ln\Pi + \ln E[(U_{+1}X_{+1}^{-\alpha}P_{+1}^{-1})] = u - \alpha x - p$

conjecture that

(B) $P = \Omega\Lambda^t U^{\omega u}X^{\omega x}$ (where Ω, Λ and the ω's are undetermined coefficients)

substitute (B) back into (A) and determine the coefficients

$$\lambda_u u + \lambda_x x + \ln\Pi + \ln E(U_{+1}X_{+1}^{-\alpha}\Omega^{-1}\Lambda^{-(t+1)}U_{+1}^{-\omega u}X_{+1}^{-\omega x}) = u - \alpha x - (\ln\Omega + t\ln\Lambda + \omega_u u + \omega_x x) \Rightarrow$$

(C) $\lambda_u u + \lambda_x x + \ln\Pi - \ln\Omega - (t+1)\ln\Lambda + \ln E(Z) = u - \alpha x - \ln\Omega - t\ln\Lambda - \omega_u u - \omega_x x$

where $Z \equiv U_{+1}^{1-\omega u}X_{+1}^{-\alpha-\omega x}$ and $\ln E(Z) = (1-\omega_u)^2\sigma_u^2 + (\alpha+\omega_x)^2\sigma_x^2$

Note: $\ln\Omega$ drops out at this point; so and Ω will have to be determined some other way.

(D) $\ln\Pi + \ln E(Z) - \ln\Lambda = u - \alpha x - \lambda_u u - \lambda_x x - \omega_u u - \omega_x x = (1-\lambda_u-\omega_u)u - (\alpha+\lambda_x+\omega_x)x$

if the conjecture is to hold for all realizations, we have to let

(E) $\omega_u = 1-\lambda_u$, $\omega_x = -\alpha-\lambda_x$, and $\ln\Lambda = \ln\Pi + \ln E(Z) = \ln\Pi + \lambda_u^2\sigma_u^2 + \lambda_x^2\sigma_x^2$

To pin down Ω:

The policy rule (2) is implemented in period t = 0; in that period the money supply, M_0, is also announced. Substituting (B) into (3),

$$M_0 = V_0P_0X_0^{1/(\gamma+1)} = V_0X_0^{1/(\gamma+1)}\Omega U_0^{1-\lambda u}X_0^{-\alpha-\lambda x} = \Omega U_0^{1-\lambda u}V_0X_0^{-[(1-\gamma)/(1+\gamma)]-\lambda x} \Rightarrow$$

(F) $\Omega = M_0/U_0^{1-\lambda u}V_0X_0^{-[(1-\gamma)/(1+\gamma)]-\lambda x}$

Flex Price Solution:

(5a) $C_t = Y_t = X_t^{1/(\gamma+1)}$

(5b) $I_t = \Pi\beta^{-1}U_t^{\lambda u}X_t^{\lambda x}$

(5c) $P_t = \Omega\Lambda^t U_t^{(1-\lambda u)}X_t^{-(\alpha+\lambda x)}$, where $\ln\Lambda = \ln\Pi + \lambda_u^2\sigma_u^2 + \lambda_x^2\sigma_x^2$ and $\Omega = M_0/U_0^{1-\lambda u}V_0X_0^{-[(1-\gamma)/(1+\gamma)] - \lambda x}$

(5d) $M_t = V_tP_tY_t = V_tP_tX_t^{1/(\gamma+1)}$

Remarks:

1. Consumption and output (and therefore utility) depend only on the productivity shock. Policy parameters don't affect anything important.
2. We have the usual price indeterminacy problem. We solved it by letting one money supply be announced; see CHR(QJE, 198?)
3. Since the flex price equilibrium is optimal, we can set $\lambda_u = \lambda_x = 0$. Then, $\Lambda = \Pi$ is the trend rate of inflation in the flex price equilibrium.

Finding the Fixed Price Solution –

(1) $\beta I_tE_t(U_{t+1}/Y^{\gamma}_{t+1}P_{t+1}) = U_t/Y^{\gamma}_tP_t$	Euler Equation
(2) $I_t = \Pi\beta^{-1}U_t^{\lambda u}X_t^{\lambda x}$	Monetary Policy
(3) $M_t = V_tP_tY_t$	Cambridge Equation
$(4)_{sticky}$ $E_{t-1}\{U_t[(1/X_t)Y_t^2 - Y_t^{1-\gamma}]\} = 0$	Pricing Equation

Here, P_t is preset (though we don't know its level yet), and Y_t is the unknown variable.

conjecture that

(A) $Y = \Omega Z$ where $Z \equiv U^{\omega u}X^{\omega x}$ (and where Ω and the ω's are undetermined coefficients)

substitute (A) into Pricing Equation (4) to determine Ω:

$$E\{U[X^{-1}\Omega^2Z^2 - \Omega^{1-\gamma}Z^{1-\gamma}] = 0 \Rightarrow \Omega^2E(UX^{-1}Z^2) = \Omega^{1-\gamma}E(UZ^{1-\gamma}) = 0 \Rightarrow$$

$$\Omega^{1+\gamma} = E(UZ^{1-\gamma})/E(UX^{-1}Z^2) = E[U(U^{\omega u}X^{\omega x})^{1-\gamma}]/E[UX^{-1}(U^{\omega u}X^{\omega x})^2] \Rightarrow$$

(B) $\Omega = [E(U^{1-\omega u(\gamma-1)}X^{-\omega x(\gamma-1)})/E(U^{1+2\omega u}X^{2\omega x-1})]^{1/(1+\gamma)}$

now substitute (A) into the Euler Equation (1) to determine the other coefficients:

(C) $\Pi U^{\lambda u}X^{\lambda x}E(UY_{+1}{}^{-\gamma}P_{+1}{}^{-1}) = UY^{-\gamma}P^{-1}$

Replace H&K (pg 519) assertion with conjecture:

(D) $P_t = \Lambda^t P_0$ (where P_0 is determined as above)

$\Pi U^{\lambda u}X^{\lambda x}E(UY_{+1}{}^{-\gamma}\Lambda^{-(t+1)}) = UY^{-\gamma}\Lambda^{-t} \Rightarrow \Pi\Lambda^{-1}U^{\lambda u}X^{\lambda x}E(UZ_{+1}{}^{-\gamma}) = UZ^{-\gamma}$ (the Ω's & P_0's cancel)

$\lambda_u u + \lambda_x + \ln\Pi - \ln\Lambda + \ln E(U_{+1}{}^{1-\gamma\omega u}X_{+1}{}^{-\gamma\omega x}) = (1-\gamma\omega_u)u - \gamma\omega_x x$

(E) $\ln\Pi - \ln\Lambda + \ln E(U_{+1}{}^{1-\gamma\omega u}X_{+1}{}^{-\gamma\omega x}) = (1 - \lambda_u - \gamma\omega_u)u - (\lambda_x + \gamma\omega_x)x$

Note: We do need to let $\lambda_p \neq 0$ to make (E) work!!

if the conjectures are to hold for all realizations, we have to let

(F) $\omega_u = (1-\lambda_u)/\gamma$, $\omega_x = -\lambda_x/\gamma$, and

$\ln\Lambda = \ln\Pi + \ln E(U_{+1}{}^{1-\gamma\omega u}X_{+1}{}^{-\gamma\omega x}) = \ln\Pi + (1-\gamma\omega_u)^2\sigma_u^2 + (\gamma\omega_x)^2\sigma_x^2 = \ln\Pi + (1-\lambda_u)^2\sigma_u^2 + \lambda_x^2\sigma_x^2$

$\ln\Omega = \ln[E(U^{1-\omega u(\gamma-1)}X^{-\omega x(\gamma-1)})/E(U^{1+2\omega u}X^{2\omega x-1})]^{1/(1+\gamma)}$

$= (1+\gamma)^{-1}\{\ln[E(U^{1-\omega u(\gamma-1)}X^{-\omega x(\gamma-1)}) - \ln E(U^{1+2\omega u}X^{2\omega x-1})]$

$= (1+\gamma)_{-1}\{[(1-(\gamma-1)\omega_u]^2 - [1+2\omega_u]^2\}\sigma_u^2 + (1+\gamma)_{-1}\{[(\gamma-1)\omega_x]^2 - [1-2\omega_x]^2\}\sigma_x^2$

Fixed Price Solution:

(6a) $C_t = Y_t = \Omega U_t^{(1-\lambda u)/\gamma}X_t^{-\lambda x/\gamma}$

where $(1+\gamma)\ln\Omega = \{[(1-(\gamma-1)\omega_u]^2 - [1+2\omega_u]^2\}\sigma_u^2 + \{[(\gamma-1)\omega_x]^2 - [1-2\omega_x]^2\}\sigma_x^2$

and $\omega_u = (1-\lambda_u)/\gamma$ and $\omega_x = -\lambda_x/\gamma$

(6b) $I_t = \Pi\beta^{-1}U_t^{\lambda u}X_t^{\lambda x}$

(6c) $P_t = \Lambda^t P_0$ where $\ln\Lambda = \ln\Pi + (1-\lambda_u)^2\sigma_u^2 + \lambda_x^2\sigma_x^2$

(6d) $M_t = V_t P_t Y_t$

Remarks:

1. Here, the solution method does not require $\lambda_p > 0$, but we're not sure how to interpret the extra terms in Λ.
2. The flex price output level is: $Y_t = X_t^{1/(\gamma+1)}$. It can be achieved by setting $\lambda_u = 1$ & $\lambda_x = -\alpha$. The policy rule that does this is: $I_t = \beta^{-1}UX^{-\alpha}$.

3. How does this work? In period t, $C_t = Y_t$ is "demand determined" by the Euler Equation $\beta I_t E_t(U_{t+1}/Y^{\gamma}_{t+1}P_{t+1}) = U_t/Y^{\gamma}_t P_t$. $E_t(U_{t+1}/Y^{\gamma}_{t+1}P_{t+1})$ is just a fixed number. "Demand shocks", U_t, would shift $C_t = Y_t$ up; the policy rule just accommodates them. "Productivity shocks", X_t, would not affect $C_t = Y_t$; the policy rule stimulates the economy in response to them to get Y_t up to "potential".
4. Expected level of output (assuming $\lambda_u = \lambda_x = 0$):
   a. Flex Price: $\ln E(Y) = \ln E(X_t^{1/(\gamma+1)}) = (\gamma+1)^{-2}\sigma^2_x$
   b. Fixed Price: $\ln E(Y) = \ln E(\Omega U^{(1-\lambda u)/\gamma} X^{-\lambda x/\gamma}) = \ln\Omega + \ln E(U^{1/\gamma}) = \ln\Omega + (1/\gamma)^2\sigma^2_u = \ldots$
   $= - (1/\gamma)^2 (\gamma+2)\sigma^2_u - \sigma^2_x$ (Hope I did this right!)
   c. $E(Y)_{\textit{flex price}} > E(Y)_{\textit{fixed price}}$. So, policy that brings about flex price solution raises E(Y)!

Note 6: *The Sticky Wage/Flexible Price Model (with non-linear production)* –

Remarks: the basic setup –

1. The supply side: Wages are set one period in advance. Production is non-linear.
2. Market structure: We retain the monopolistic setting of wages, but allow firms to be competitive; this lightens the algebra considerably.

Households: (same as in the "Basic Model", except there is only one consumption good)

Household j's Utility –

(1) $U_t^j = E_t\sum_{s=t}^{\infty}\beta^{s-t}[u(C_s^j) - h(N_s^j)] = E_t\sum_{s=t}^{\infty}\beta^{s-t}[(1-\gamma)^{-1}C_s^{j\,1-\gamma} - (1+\chi)^{-1}A_s(N_s^j)^{1+\chi}]$

where $N_s^j \equiv \int_0^1 N_s^j(f)df$

Bundler of the Composite labor input N –

(2) $N_t = [\int_0^1 N_s^{j(\phi-1)/\phi}dj]^{\phi/(\phi-1)}$ where $\phi > 1$

(3) $W_t = [\int_0^1 W_t(j)^{1-\phi}dj]^{1/(1-\phi)}$ Price (of the bundler) for the composite labor input

(4) $N_t^{jd} = (W_t(j)/W_t)^{-\phi}N_t$ Demand (of the bundler) for the labor of household-j

Household j's cash in advance (CIA) constraint in period s –

(5) $M_s^j + \alpha_s P_s Y_s = P_s C_s^j$

Household j's BC in period s –

(6) $M_s^j + B_s^j + P_s\bar{\tau}_s = \tau_s W_{s-1}(j)N_{s-1}^j + I_{s-1}B_{s-1} = \tau_s W_{s-1}(j)(W_{s-1}(j)/W_{s-1})^{-\phi}N_{s-1} + I_{s-1}B_{s-1}^j$

Household-j's intertemporal maximization problem –

$$\mathcal{L} = E_t\sum_{s=t}^{\infty}\beta^{s-t}\{[u(C_s^j) - h((W_s(j)/W_s)^{-\phi}N_s)] + \lambda_s^j[\tau_s W_{s-1}(j)^{1-\phi}(1/W_{s-1})^{-\phi}N_{s-1} + I_{s-1}B_{s-1}^j$$
$$- (P_s C_s^j - \alpha_s P_s Y_s) - B_s^j - P_s\bar{\tau}_s)]\}$$

The following FOC's hold for any wage/price assumptions:

(7) C_t^j: $u'(C_t^j) = \lambda_t^j P_t$ or $1/C_t^{j\gamma} - \lambda_t^j P_t = 0$

(8) B_t^j: $- \beta I_t E_t \lambda_{t+1}^j + \lambda_t^j = 0$

if wages are flexible: (the same as in the "Basic Model")

$W_t(j)$: $\phi h'(\cdot)W_t(j)^{-1-\phi}(1/W_t)^{-\phi}N_t + \beta(E_t\lambda^j_{t+1})\tau_{t+1}(1-\phi)W_t(j)^{-\phi}(1/W_t)^{-\phi}N_t = 0$

$\phi h'(\cdot)W_t(j)^{-1-\phi}(1/W_t)^{-\phi}N_t + \lambda_t\beta E_t(\lambda^j_{t+1}/\lambda^j_t)\tau_{t+1}(1-\phi)W_t(j)^{-\phi}(1/W_t)^{-\phi}N_t = 0$

$\phi h'(\cdot)W_t(j)^{-1-\phi}(1/W_t)^{-\phi}N_t = (u'(C^j_t)/P_t)I_t^{-1}\tau_{t+1}(\phi-1)W_t(j)^{-\phi}(1/W_t)^{-\phi}N_t$

$h'(\cdot)(N_t/W_t) = \tau_{t+1}(1/I_t\mu)[u'(\cdot)/P_t]N_t$ where $\mu \equiv \phi/(1-\phi)$

and canceling the N_t & multiplying by W_t (which we can't do in the fixed wage case)

and letting $\tau_{t+1} = I_t$ and $\mu \equiv \phi/(1-\phi)$:

$(9)_{flex}$ $W_t/P_t = \mu[h_t'(\cdot)/u'(\cdot)]$

if wages are set 1 period ahead (and letting $\tau_{t+1} = I_t$ and $\mu \equiv \phi/(1-\phi)$):

$(9)_{sticky}$ $\mu E_{t-1}[h'(\cdot)(N_t/W_t)] = E_{t-n}[(u'(\cdot)/P_t)N_t]$ or $W_t = \mu E_{t-1}[h'(\cdot)N_t]/E_{t-1}[(u'(\cdot)/P_t)N_t]$

Remarks: We use the subsidy τ to eliminate the seigniorage tax distortion.

Firms:

Generalize the production function to :

(10) $Y_t(f) = Z_tN(f)^{\alpha}_t$, where Z_t is an aggregate productivity shock and $0 < \alpha \leq 1$.

Remarks:

1. When $\alpha = 1$, this reverts to the linear case.
2. This kind of "fixed factor" model has been criticized: merely breaking up the firms would increase output. Think of this as short-hand for a model in which capital is present (but fixed in supply) and the production function is constant returns to scale over both factors.
3. As noted earlier, firms are competitive wage and price takers.

Firm-f chooses $N(f)_t$ to maximize profits:

$\text{Profits}_t = P_tY_t - W_tN(f)_t = P_tZ_tN(f)^{\alpha}_t - W_tN(f)_t$

FOC is

(11) $\alpha P_tZ_tN(f)^{\alpha-1}_t - W_t = 0$

In a symmetric equilibrium:

Flexible Wage/Price Solution: or with constant elasticity functions

(12) $\beta I^*_t E_t[u'(Y^*_{t+1})P^*_t/u'(Y^*_t)P^*_{t+1}] = 1$ or $\beta I^*_t E_t[(Y^*_t/Y^*_{t+1})^\gamma (P^*_t/P^*_{t+1})] = 1$

(13) $M_t V_t = P_t Y^*_t$, where $V_t \equiv 1/(1-\alpha_t)$ or $M_t V_t = P^*_t Y^*_t$

(14) $MPL_t = W^*_t/P^*_t$ or $W^*_t/P^*_t = \alpha Z_t(1/N^*_t)^{1-\alpha}$

(15) $W^*_t/P^*_t = \mu[h_t'(\cdot)/u'(\cdot)]$ or $W^*_t/P^*_t = \mu A_t N_t^{*\chi} Y_t^{*\gamma} = \mu A_t Z_t^\gamma N_t^{*(\chi+\alpha\gamma)}$

(16) $N^*_t = \{(\alpha/\mu)(1/A_t)(1/Z_t)^{(\gamma-1)}\}^{1/[\chi+1+\alpha(\gamma-1)]}$ & $Y^*_t = Z_t N_t^{*\alpha}$

Sticky Wage Solution: or with constant elasticity functions

(17) $\beta I_t E_t[u'(Y_{t+1})P_t/u'(Y_t)P_{t+1}] = 1$ or $\beta I_t E_t[(Y_t/Y_{t+1})^\gamma (P_t/P_{t+1})] = 1$

(18) $M_t V_t = P_t Y_t$, where $V_t \equiv 1/(1-\alpha_t)$ or $M_t V_t = P_t Y_t$

(19) $MPL_t = W_t/P_t$ or $W_t/P_t = \alpha Z_t(1/N_t)^{1-\alpha}$

(20) $W_t = \mu E_{t-1}[h'(\cdot)N_t]/E_{t-1}[(u'(\cdot)/P_t)N_t]$ or $W_t = \mu E_{t-1}[A_t N_t^{1+\chi}]/E_{t-1}[(1/P_t)Y_t^{-\gamma}N_t]$

$= \mu E_{t-1}[A_t N_t^{1+\chi}]/E_{t-1}[(1/P_t Z_t^\gamma)N_t^{(1-\alpha\gamma)}]$

Remarks:

1. Model is easily log-linearized. Then, the responses to A_t and Z_t shocks are easy to see in a labor market diagram.
2. The labor supply curve it "too high" by the factor μ, and N^*_t is correspondingly "too low". This is one of the ingredients for a Barro-Gordon problem, but we would have to ascribed some cost to inflation. There isn't anything in the model so far.
3. Many studies let $\gamma = \chi = 1$, giving log utility for consumption and quadratic costs for labor. Letting $\gamma = 1$ lightens the algebra considerably, but it has strong implications for the effects of Z_t shocks and the optimal monetary response to them. Letting $\chi = 1$ simplifies the algebra somewhat, and does not seem to bias the results in any particular way.

CASE 1: $\gamma = \chi = 1$; $u(C) = \log(C)$ and $h(N) = \frac{1}{2}N^2$.

Flexible Wage/Price Solution: or with constant elasticity functions

(12) $\beta I^*_t E_t[u'(Y^*_{t+1})P^*_t/u'(Y^*_t)P^*_{t+1}] = 1$ or $\beta I^*_t E_t[(Y^*_t/Y^*_{t+1})(P^*_t/P^*_{t+1})] = 1$

(13) $M_t V_t = P_t Y^*_t$, where $V_t \equiv 1/(1-\alpha_t)$ or $M_t V_t = P^*_t Y^*_t$

(14) $MPL_t = W^*_t/P^*_t$ or $W^*_t/P^*_t = \alpha Z_t(1/N^*_t)^{1-\alpha}$

(15) $W^*_t/P^*_t = \mu[h_t'(\cdot)/u'(\cdot)]$ or $W^*_t/P^*_t = \mu A_t N^*_t Y^*_t = \mu A_t Z_t N_t^{*(1+\alpha)}$

(16) $N^*_t = (\alpha/\mu)^{\frac{1}{2}}(1/A_t)^{\frac{1}{2}}$, $W^*_t/P^*_t = \omega A_t^{\frac{1}{2}(1-\alpha)} Z_t$ & $Y^*_t = Z_t N_t^{*\alpha}$ where $\omega \equiv \alpha^{1-\frac{1}{2}(1-\alpha)}\mu^{\frac{1}{2}(1-\alpha)}$

Note: if $A_t = Z_t = 1$, then $W^*_t/P^*_t = \omega$; so, ω is the "expected"flex-price real wage.

Sticky Wage Solution: or with constant elasticity functions

(17) $\beta I_t E_t[u'(Y_{t+1})P_t/u'(Y_t)P_{t+1}] = 1$ or $\beta I_t E_t[(Y_t/Y_{t+1})(P_t/P_{t+1})] = 1$

(18) $M_t V_t = P_t Y_t$, where $V_t \equiv 1/(1-\alpha_t)$ or $M_t V_t = P_t Y_t$

(19) $MPL_t = W_t/P_t$ or $W_t/P_t = \alpha Z_t(1/N_t)^{1-\alpha}$

(20) $W_t = \mu E_{t-1}[h'(\cdot)N_t]/E_{t-1}[(u'(\cdot)/P_t)N_t]$ or $W_t = \mu E_{t-1}[A_t N_t^2]/E_{t-1}[(1/P_t Y_t)N_t]$

$= \mu E_{t-1}[A_t N_t^2]/E_{t-1}[(1/M_t V_t)N_t^{(1-\alpha)}]$

$= \mu E_{t-1}[A_t N_t^2]/E_{t-1}[(1/P_t Z_t)N_t^{(1-\alpha)}]$

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