

## CANZONERI'S HANDOUTS: Monetary Economics (Econ. 607)

### Hand out for MIU Lectures:

- (1)  $c_j + g_j = n_j$
- (2)  $x_j + n_j = 1$
- (3)  $U_t = E_t \sum_{j=t}^{\infty} \beta^{j-t} u(c_j, x_j, m_j)$

Household BC:

$$(4) P_j c_j + M_j + B_j = P_j(1-\tau_j)n_j - P_j \bar{\tau}_j + M_{j-1} + I_{j-1} B_{j-1}$$

Household maximization problem in period t –

Choose  $\{c_t, x_t, n_t, M_t, B_t\}$  to  $\max U_t$  s.t. (2) & (4).

$$\begin{aligned} \mathcal{L} = E_t \sum_{j=t}^{\infty} \beta^{j-t} \{ & u(c_j, 1-n_j, M_j/P_j) \\ & + \lambda_j [P_j(1-\tau_j)n_j - P_j \bar{\tau}_j + M_{j-1} + I_{j-1} B_{j-1} - (P_j c_j + M_j + B_j)] \} \end{aligned}$$

$$\mathcal{L}_c: u_c(c_t, x_t, m_t) - \lambda_t P_t = 0 \Rightarrow \lambda_t = u_c(c_t, x_t, m_t)/P_t$$

$$\mathcal{L}_n: -u_n(c_t, x_t, m_t) + \lambda_t P_t(1-\tau_t) = 0 \Rightarrow u_n(c_t, x_t, m_t) = (1-\tau_t)u_c(c_t, x_t, m_t) \quad (6)$$

$$\begin{aligned} \mathcal{L}_B: E_t[\beta \lambda_{t+1} I_t] - \lambda_t = 0 & \Rightarrow E_t[\beta I_t u_c(c_{t+1}, x_{t+1}, m_{t+1})/P_{t+1}] = u_c(c_t, x_t, m_t)/P_t \\ & \Rightarrow E_t[\beta I_t (P_t/P_{t+1}) u_c(c_{t+1}, x_{t+1}, m_{t+1})] = u_c(c_t, x_t, m_t) \quad (5) \end{aligned}$$

$$\begin{aligned} E_t[\beta \lambda_{t+1} I_t] - \lambda_t = 0 \text{ also} & \Rightarrow \lambda_t \{E_t[\beta (\lambda_{t+1}/\lambda_t) I_t] - 1\} = 0 \\ & \Rightarrow E_t[\beta (\lambda_{t+1}/\lambda_t) I_t] = 1 \Rightarrow E_t[\beta (\lambda_{t+1}/\lambda_t)] = I_t^{-1} \end{aligned}$$

(which will be helpful in a minute)

$$\begin{aligned} \mathcal{L}_M: u_m(c_t, x_t, m_t)/P_t + E_t[\beta \lambda_{t+1}] - \lambda_t = 0 & \Rightarrow u_m(c_t, x_t, m_t)/P_t + \lambda_t \{E_t[\beta \lambda_{t+1}/\lambda_t] - 1\} = 0 \\ & \Rightarrow u_m(c_t, x_t, m_t)/P_t + \lambda_t \{I_t^{-1} - 1\} = 0 \Rightarrow u_m(c_t, x_t, m_t)/P_t = [u_c(c_t, x_t, m_t)/P_t] (1 - I_t^{-1}) \end{aligned}$$

$$\text{(recall: } I_t = 1+i_t) \Rightarrow u_m(c_t, x_t, m_t) = (i_t/I_t)u_c(c_t, x_t, m_t) \text{ equation (7)}$$

### Deriving the Government PVBC –

The flow BC is:

$$(8) \quad P_t g_t + I_{t-1} B_{t-1} = P_t \tau_t n_t + P_t \bar{\tau}_t + B_t + (M_t - M_{t-1}) \Leftrightarrow M_{t-1} + I_{t-1} B_{t-1} = M_t + B_t + S_t$$

where  $S_t \equiv P_t \tau_t n_t + P_t \bar{\tau}_t - P_t g_t$  is the primary surplus

Next, we want to derive the Government's PVBC:

We have two state variables –  $M_t$  and  $B_t$ ; to solve (8) forward, we need to combine  $M_t$  and  $B_t$  into a single state variable.

We choose:  $W_{t+1} \equiv M_t + I_t B_t$ .  $W_{t+1}$  = total debt *at the beginning of*  $t+1$ .

We have to reformulate the flow budget constraint (8) in terms of  $W_t$ , and solve forward. In a stochastic framework, we must take care not do violence to the expectations operator.

$$(8) \Rightarrow W_t = M_{t-1} + I_{t-1} B_{t-1} = M_t + B_t + S_t = (M_t + I_t B_t) - i_t B_t + S_t = W_{t+1} - i_t B_t + S_t$$

Dividing by  $P_t$ , and letting  $R_t \equiv I_t(P_t/P_{t+1})$ :

$$\begin{aligned} w_t &= E_t[w_{t+1}(P_{t+1}/P_t)] - i_t b_t + s_t \\ &= E_t[(1/R_t)w_{t+1}] + E_t\{[(P_{t+1}/P_t) - (1/R_t)]w_{t+1}\} - i_t b_t + s_t \\ &= E_t[(1/R_t)w_{t+1}] + E_t\{(1/R_t)[(P_{t+1}/P_t)R_t - 1]w_{t+1}\} - i_t b_t + s_t \\ &= E_t[(1/R_t)w_{t+1}] + E_t\{(1/R_t)[I_t - 1]w_{t+1}\} - i_t b_t + s_t \end{aligned}$$

So, finally, the flow BC becomes:

$$\begin{aligned} w_t &= E_t[(1/R_t)w_{t+1}] + E_t[(1/R_t)i_t w_{t+1}] - i_t b_t + s_t \\ &= E_t[(1/R_t)w_{t+1}] + E_t\{(P_{t+1}/P_t)I_t^{-1}i_t[M_t + I_t B_t]/P_{t+1}\} - i_t b_t + s_t \\ &= E_t[(1/R_t)w_{t+1}] + I_t^{-1}i_t[m_t + I_t b_t] - i_t b_t + s_t \end{aligned}$$

or we can equivalently write (8) as

$$(8) \quad w_t = E_t[(1/R_t)w_{t+1}] + (i_t/I_t)m_t + s_t$$

Iterating (8) forward, letting  $\alpha_j \equiv (R_t \cdots R_{j-1})^{-1}$ , & imposing TC we have:

### The Government PVBC –

$$(8)' \quad w_t = (M_{t-1} + I_{t-1} B_{t-1})/P_t = E_t \sum_{j=t}^{\infty} \alpha_j [s_j + (i_t/I_j)m_j], \quad \text{where } \alpha_j \equiv (R_t \cdots R_{j-1})^{-1}$$

### Summarizing the MIU Model:

- (1)  $c_t + g_t = n_t$  Resource Constraint or Equil. Goods Market  
 (2)  $x_t + n_t = 1$  Time Constraint  
 (3)  $U_t = E_t \sum_{j=t}^{\infty} \beta^{j-t} u(c_j, x_j, m_j)$  Household Utility

### Household Budget Constraints:

- (4)  $P_t c_t + M_t + B_t = P_t(1-\tau_t)n_t - P_t \bar{\tau}_t + M_{t-1} + I_{t-1} B_{t-1}$  Flow BC  
 (4)'  $w_t + E_t \sum_{j=t}^{\infty} \alpha_j [(1-\tau_j)n_j - \bar{\tau}_j - (i_j/I_j)m_j] = E_t \sum_{j=t}^{\infty} \alpha_j c_j$  PVBC

### Household FOC are:

- (5)  $u_c(c_t, x_t, m_t) = E_t [\beta I_t (P_t/P_{t+1}) u_c(c_{t+1}, x_{t+1}, m_{t+1})]$  Consumption - Savings  
 (6)  $u_x(c_t, x_t, m_t) = (1-\tau_t) u_c(c_t, x_t, m_t)$  Labor - Leisure  
 (7)  $u_m(c_t, x_t, m_t) = (i_t/I_t) u_c(c_t, x_t, m_t)$  Money - Bonds

### Government Budget Constraints:

- (8)  $P_t g_t + I_{t-1} B_{t-1} = P_t \tau_t n_t + P_t \bar{\tau}_t + B_t + (M_t - M_{t-1})$  Flow BC  
 (8)'  $w_t = (M_{t-1} + I_{t-1} B_{t-1})/P_t = E_t \sum_{j=t}^{\infty} \alpha_j [s_j + (i_t/I_t) m_j]$  PVBC  
 where  $\alpha_j \equiv (R_t \cdots R_{j-1})^{-1}$ ;  $R_j \equiv I_j (P_j/P_{j+1})$ ;  $s_j \equiv \tau_t n_t + \bar{\tau}_t - g_t$

Def: An “equilibrium” is a sequence  $\{c_j, x_j, n_j, m_j, I_j, P_j, g_j, \tau_j, \bar{\tau}_j\}$  that satisfies:

- A. Equilibrium Condition: (1)
- B. Time Constraint: (2)
- C. FOC's: (5), (6) & (7)
- D. The Government BC or The Household BC

Example 1: A simple example where we can find closed form solutions for everything.

For simplicity, assume no uncertainty; can just drop the  $E_t$ 's.

let  $u(c_j, x_j, m_j) = c_j + x_j - \frac{1}{2}x_j^2 - \frac{1}{2}(m_j - m^*)^2$ ;  $m^*$  is satiation point in money.

$$\Rightarrow u_c(c_j, x_j, m_j) = 1 \text{ and } u_x(c_j, x_j, m_j) = 1 - x_j = n_j \text{ and } u_m(\cdot) = -(m_j - m^*)$$

$$(5) u_c(c_t, x_t, m_t) = E_t[\beta I_t(P_t/P_{t+1})u_c(c_{j+1}, x_{j+1}, m_{t+1})]$$

$$\Rightarrow 1 = \beta R_j \text{ or } R_j = 1/\beta \text{ or } \alpha_j \equiv 1/(R_t \cdots R_{j-1}) = \beta^{j-t}$$

$$(6) u_x(c_t, x_t, m_t) = (1 - \tau_t)u_c(c_t, x_t, m_t)$$

$$\Rightarrow n_j = 1 - \tau_j \text{ and } x_j = 1 - n_j = \tau_j$$

$$(7) u_m(c_t, x_t, m_t) = (i_t/I_t)u_c(c_t, x_t, m_t)$$

$$\Rightarrow -(m_j - m^*) = i_j/I_j \Rightarrow m_j = m^* - i_j/I_j$$

$$(1) c_t + g_t = n_t$$

$$\Rightarrow c_j + g_j = n_j = 1 - \tau_j \Rightarrow c_j = 1 - \tau_j - g_j$$

$$(8)' \Rightarrow w_t + \sum_{j=t}^{\infty} \beta^{j-t} g_j = \sum_{j=t}^{\infty} \beta^{j-t} [\tau_j n_j + \bar{\tau}_j + (i_j/I_j) m_j] \\ = \sum_{j=t}^{\infty} \beta^{j-t} [\tau_j (1 - \tau_j) + \bar{\tau}_j + (i_j/I_j) (m^* - (i_j/I_j))]$$

$$(3) \Rightarrow U_t = \sum_{j=t}^{\infty} \beta^{j-t} [c_j + x_j - \frac{1}{2}x_j^2 - \frac{1}{2}(m_j - m^*)^2] \\ = \sum_{j=t}^{\infty} \beta^{j-t} [1 - g_j - \frac{1}{2}\tau_j^2 - \frac{1}{2}(i_j/I_j)^2]$$

### Problems caused by the lack of lump sum taxes:

Proposition 1: If  $g_j > 0$  in any period  $j$ , then “efficiency” requires the use of lump sum taxes.

Corollary: It is optimal (Pareto Efficient/Utility Maximizing) to set the distortionary taxes (income and seigniorage) equal to zero and to use the lump-sum taxes to finance government spending,  $g_j$ . Setting  $i_j = 0$  is known as “Friedman’s Rule”.

Proposition 2: (The Laffer Curve) Using distortionary taxes, there are usually high and low tax ways of financing a given level of government spending; the low tax way is generally preferable.

### Hand outs for CIA Model Lectures:

#### Household's maximization problem –

Choose  $\{c_t, x_t, n_t, M_t^p, B_t\}$  to max  $U_t = \sum_{j=t}^{\infty} \beta^{j-t} u(c_j, x_j)$  s.t. (2), (4) and (7)

use (2) and (7) to eliminate  $x_j$  and  $M_t^p$ :

$$\mathcal{L} = E_t \sum_{j=t}^{\infty} \beta^{j-t} \{u(c_j, 1-n_j) + \lambda_j [(1-\tau_{j-1})P_{j-1}n_{j-1} + I_{j-1}B_{j-1} - (P_j c_j + B_j + P_j \bar{\tau}_j)]\}$$

$$\mathcal{L} = E_t \sum_{j=t}^{\infty} \beta^{j-t} \{u(c_j, 1-n_j) + \lambda_j [(1-\tau_{j-1})P_{j-1}n_{j-1} + I_{j-1}B_{j-1} - (P_j c_j + B_j + P_j \bar{\tau}_j)]\}$$

$$\mathcal{L}_{c_t} = u_c(c_t, x_t) - \lambda_t P_t = 0 \Rightarrow \lambda_t = u_c(c_t, x_t)/P_t$$

$$\mathcal{L}_{B_t} = E_t(\beta \lambda_{t+1} I_t) - \lambda_t = 0 \quad \left\{ \begin{array}{l} \Rightarrow E_t[\beta I_t u_c(c_{t+1}, x_{t+1})/P_{t+1}] = u_c(c_t, x_t)/P_t \\ \Rightarrow E_t(\beta \lambda_{t+1}) = \lambda_t/I_t = u_c(c_t, x_t)/P_t I_t \end{array} \right.$$

$$\Rightarrow E_t[\beta I_t (P_t/P_{t+1}) u_c(c_{t+1}, x_{t+1})] = u_c(c_t, x_t)$$

$$\mathcal{L}_{n_t} = -u_x(c_t, x_t) + E_t(\beta \lambda_{t+1})(1-\tau_j)P_t = 0$$

$$\Rightarrow u_x(c_t, x_t) = [u_c(c_t, x_t)/P_t I_t](1-\tau_j)P_t = u_c(c_t, x_t)[(1-\tau_j)/I_t]$$

Note:  $I^{-1} = 1 + I^{-1} - 1 = 1 + I^{-1}(1 - I) = 1 - I^{-1}i = 1 - (i/I)$

### Summarizing the CIA Model:

(1) $c_t + g_t = n_t$	Equilibrium in Goods Market
(9) $M_t = P_t n_t$ or $m_t = n_t$	Equilibrium in Money Market
(2) $x_t + n_t = 1$	Time Constraint
(3) $U_t = E_t \sum_{j=t}^{\infty} \beta^{j-t} u(c_j, x_j)$	Household Utility

### Household Budget Constraints:

(4) $P_t c_t + B_t = P_{t-1}(1-\tau_{t-1})n_{t-1} - P_t \bar{\tau}_t + I_{t-1} B_{t-1}$	Flow BC
(4)' $w_t + E_t \sum_{j=t}^{\infty} \alpha_j [(1-\tau_{j-1})n_{j-1} - \bar{\tau}_j - (i_j/I_j)m_j] = E_t \sum_{j=t}^{\infty} \alpha_j c_j$	PVBC

### Household FOC are:

(5) $u_c(c_t, x_t) = E_t [\beta I_t (P_t/P_{t+1}) u_c(c_{t+1}, x_{t+1})]$	Cons. - Savings
(6) $u_x(c_t, x_t) = [(1-\tau_t)/I_t] u_c(c_t, x_t) = [(1-\tau_t)(1-(i_t/I_t))] u_c(c_t, x_t)$	Labor - Leisure
(7) $M_t^p = P_t c_t$	Money Demand

### Government Budget Constraints:

(8) $P_t g_t + I_{t-1} B_{t-1} = P_t \tau_{t-1} n_{t-1} + P_t \bar{\tau}_t + B_t + (M_t - M_{t-1})$	Flow BC
(8)' $w_t = (M_{t-1} + I_{t-1} B_{t-1})/P_t = E_t \sum_{j=t}^{\infty} \alpha_j [s_j + (i_j/I_j)m_j]$	PVBC
or $w_t + E_t \sum_{j=t}^{\infty} \alpha_j g_t = E_t \sum_{j=t}^{\infty} \alpha_j [\tau_{j-1} n_{j-1} + \bar{\tau}_j + (i_j/I_j)m_j]$	
where $\alpha_j \equiv (R_t \cdots R_{j-1})^{-1}$ ; $R_j \equiv I_j (P_j/P_{j+1})$ ; $s_j \equiv \tau_{j-1} n_{j-1} + \bar{\tau}_j - g_j$	

Def: An “equilibrium” is a sequence  $\{c_j, x_j, n_j, m_j, I_j, P_j, g_j, \tau_j, \bar{\tau}_j\}$  that satisfies:

- Equilibrium Conditions: (1) & (9)
- Time Constraint: (2)
- FOC's: (5) & (6)
- A Government BC ((8) or (8)') or a Household BC ((4) or (4)')

Note: We have 6 equations and 9 variables. All four policy variables

$\{M_j \text{ (or } I_j), g_j, \tau_j, \bar{\tau}_j\}$  can not be chosen independently; only three.

Example 2: Same as Example 1, except we have CIA instead of MIU.

For simplicity, assume no uncertainty; can just drop the  $E_t$ 's.

$$\text{let } u(c_j, x_j) = c_j + x_j - \frac{1}{2}x_j^2$$

$$\Rightarrow u_c(c_j, x_j) = 1 \text{ and } u_x(c_j, x_j) = 1 - x_j = n_j$$

$$(5) \Rightarrow 1 = \beta R_j \text{ or } R_j = 1/\beta \text{ or } \alpha_j \equiv 1/(R_t \cdots R_{j-1}) = \beta^{j-t}$$

$$(6) \Rightarrow n_j = (1 - \tau_j)(1 - i_j/I_j) \approx 1 - \tau_j - (i_j/I_j) \text{ and } x_j = 1 - n_j \approx \tau_j + (i_j/I_j)$$

$$(9) \Rightarrow m_j = n_j = 1 - \tau_j - (i_j/I_j)$$

$$(1) \Rightarrow c_j + g_j = n_j = 1 - \tau_j - (i_j/I_j) \Rightarrow c_j = 1 - \tau_j - (i_j/I_j) - g_j$$

$$(8)' \Rightarrow w_t + \sum_{j=t}^{\infty} \beta^{j-t} g_j = \sum_{j=t}^{\infty} \beta^{j-t} [\tau_{j-1} n_{j-1} + \bar{\tau}_j + (i_j/I_j) m_j] \\ = \sum_{j=t}^{\infty} \beta^{j-t} [\tau_{j-1} (1 - \tau_{j-1} - (i_{j-1}/I_{j-1})) + \bar{\tau}_j + (i_j/I_j) (1 - \tau_j - (i_j/I_j))]$$

$$(3) \Rightarrow U_t = \sum_{j=t}^{\infty} \beta^{j-t} [c_j + x_j - \frac{1}{2}x_j^2] = \sum_{j=t}^{\infty} \beta^{j-t} \{1 - g_j - \frac{1}{2}[\tau_j + (i_j/I_j)]^2\}$$

$$\text{since } c_j + x_j = [1 - \tau_j - (i_j/I_j) - g_j] + [\tau_j + (i_j/I_j)] = 1 - g_j$$

Note: In the CIA Model, we again have Laffer curves for the seigniorage tax and income tax, but here the seigniorage tax rate gets into the income tax Laffer curve and vice versa.

**Hand out for: Inflation Targeting and the Price Indeterminacy Problem**

Consider the following model:

$$(1) m_t - p_t = -\lambda i_t + y_t,$$

$$(2) i_t - (p_{t+1|t} - p_t) = r - \delta y_t + u_t, \quad \text{where } u_t \text{ is a r.v. with zero mean.}$$

$$(3) y_t = y^*$$

(2) and (3)  $\Rightarrow$

$$(4) i_t - (p_{t+1|t} - p_t) = r - \delta y^* + u_t.$$

Remarks:

(1) and (4) are two equations in  $p_t$ ,  $i_t$  and  $m_t$

i. When CB sets  $m_t$ , (1) and (4) determine  $p_t$  and  $i_t$  simultaneously.

ii. When CB sets  $i_t$ , the system dichotomizes:

(4) must determine  $p_t$ , and then (1) must determine  $m_t$ .

Consider two versions of the “Taylor Rule” for monetary policy:

$$(5a) i_t = i^* + \beta[(p_t - p_{t-1}) - \pi^*] + \gamma(y_t - 0) \quad \text{Inflation targeting}$$

$$(5b) i_t = i^* + \alpha[p_t - p_t^*] + \gamma(y_t - 0), \quad \text{Price level (or path) targeting}$$

where  $\pi^*$  is the inflation target, and  $i^* = r + \pi^*$ .

Remarks:

1.  $i^*$  is the no-shock interest rate that makes  $\pi = \pi^e = \pi^*$ ; if  $i_t - \pi^e = r$ , then  $\pi^e = i_t - r = i^* - r = \pi^*$ ; let  $r = i^* = \pi^* = y^* = p_t^* = 0$ , for simplicity.
2.  $\beta = \gamma = 0 \Rightarrow$  an interest rate “peg” – Sargent and Wallace (JPE, 1975).
3. Inflation Targeting vs Price Path Targeting is an issue for the BoC.
4. Empirical estimates: (post 1980)  $\beta \approx 1.5$  or  $2.0$  &  $\gamma \approx .5$ ; (pre 1980)  $\beta \approx 0.8$ .
5. The Taylor Principle:  $\beta > 1 \Rightarrow \{\pi \uparrow \Rightarrow i \uparrow$  enough to raise real interest rate & lower demand, but note it is lagged inflation}

Substituting two Taylor Rules into (4),

$$(6a) \pi_t = (1/\beta)\pi_{t+1|t} + (1/\beta)u_t, \text{ where } \pi_t \equiv p_t - p_{t-1}$$

$$(6b) p_t = (1+\alpha)^{-1}p_{t+1|t} + (1+\alpha)^{-1}u_t$$

Both equations take the form:

$$(6c) x_t = ax_{t+1|t} + du_t$$

where  $x$  is either  $p$  or  $\pi$ , and  $a$  and  $b$  are constants defined above.

Can (6c) can be solved for  $x_t$ ? Use Sargent's method –

Forward (6c) by  $j$  periods and take expectations

$$x_{t+j} = ax_{t+j+1|t} + du_{t+j} \Rightarrow (7) x_{t+j|t} = ax_{t+j+1|t}$$

Iterate "forward" with this equation,

$$(8) x_{t+1|t} = ax_{t+2|t} = a^2x_{t+3|t} = \dots = \lim_{T \rightarrow \infty} a^T x_{t+T+1|t}$$

Substitute back into (6c)

$$(9) x_t = a \cdot \lim_{T \rightarrow \infty} a^T x_{t+T+1|t} + du_t$$

Remarks:

1. If the limit on the RHS can be pinned down, then (9) is a solution for  $x_t$ . We are solving a difference equation, and we need a boundary condition.

2. If  $a < 1$ , then Sargent's "no speculative bubbles" criterion can be used to set

$$\lim_{T \rightarrow \infty} a^T x_{t+T+1|t} = 0.$$

3. So, now we can tell *which interest rate rules provide a nominal anchor*:

a. Inflation Targeting, (5a): rule must obey the Taylor Principle;  $\beta > 1 \Rightarrow a > 1$

b. An interest rate peg ( $\beta = \gamma = 0$ ):  $a = 1$ ,  $x_t = \lim_{T \rightarrow \infty} x_{t+T+1|t} + du_t$

c. Price level targeting (5b):  $\alpha > 0 \Rightarrow a > 1$ .

## For Lectures on Fiscal Theory of Price Determination:

Recall our derivation of the Government's PVBC:

The Government PVBC (two equivalent statements) –

$$(1) w_t = (M_{t-1} + I_{t-1}B_{t-1})/P_t = E_t \sum_{j=t}^{\infty} \alpha_j [s_j + (i_j/I_j)m_j] \Leftrightarrow \lim_{k \rightarrow \infty} E_t[\alpha_{k+1} w_{k+1}] = 0$$

where  $\alpha_j \equiv (R_t \cdots R_{j-1})^{-1}$ ;  $R_j \equiv I_j(P_j/P_{j+1})$ ; and  $s_j \equiv \tau_j n_j + \bar{\tau}_j - g_j$ .

let  $\sigma_j \equiv s_j + (i_j/I_j)m_j$  be the “total” or “seigniorage inclusive” surplus.

Woodford's definitions of Fiscal Regimes:

- 1. R Regime:** An endogenous fiscal policy makes  $\{\sigma_j\}$  respond to  $\{w_j\}$  in a way that satisfies (1) for any values  $\{\alpha_j\}$  and the current  $w_t$  might take in equilibrium.
- 2. NR Regime:** The sequence  $\{\sigma_j\}$  is independent of  $\{w_j\}$ ; so, in equilibrium  $\{\alpha_j\}$  and/or current  $w_t = (M_{t-1} + i_{t-1}B_{t-1})/P_t$  has to “jump” to satisfy (1).

### Example 1:

Let  $y_j = \bar{y}$ , and let  $g_j = \gamma y_j$ ; note:  $c_j = y_j - g_j = (1-\gamma)\bar{y}$ . Let  $u(c) = \log(c)$ .

Euler equation  $\Rightarrow R\beta = u'(c_j)/u'(c_{j+1}) = y_{j+1}/y_j = 1 \Rightarrow 1/R_j = \beta$ .

- |   |                            |
|---|----------------------------|
| (I) $(M_{t-1} + i_{t-1}B_{t-1})/P_t = \sum_{j=t}^{\infty} \beta^{j-t} \sigma_j$ | Government PVBC            |
| (II) $M_t = P_t \bar{y}$  | Cash in Advance Constraint |
| (III) $I_t(P_t/P_{t+1}) = R_t = 1/\beta$  | Euler Equation             |

Control of  $P_t$  in NR Regimes: CB has to work through  $i_t$  and seigniorage.

$$\sigma_j \equiv \frac{T_j - G_j}{P_j} + \frac{M_j}{P_j} \left( \frac{i_j}{1+i_j} \right)$$

C&D show for Spain:

$$-.01 + (.13)\Delta i_t = 0 \Rightarrow \Delta i_t = .077$$

(No CB would do this!!)

## Exploring the NR Regime –

recall –

### Deriving the Government PVBC –

The flow BC is:

$$(8) P_t g_t + I_{t-1} B_{t-1} = P_t \tau_t n_t + P_t \bar{\tau}_t + B_t + (M_t - M_{t-1}) \Leftrightarrow M_{t-1} + I_{t-1} B_{t-1} = M_t + B_t + S_t$$

where  $S_t \equiv P_t \tau_t n_t + P_t \bar{\tau}_t - P_t g_t$  is the primary surplus

Next, we want to derive the Government's PVBC:

$$(8) \Rightarrow W_t = M_{t-1} + I_{t-1} B_{t-1} = M_t + B_t + S_t = (M_t + I_t B_t) - i_t B_t + S_t = W_{t+1} - i_t B_t + S_t$$

Dividing by  $P_t$ , and letting  $R_t \equiv I_t(P_t/P_{t+1})$  (8) becomes:

$$(8) w_t = E_t[w_{t+1}(P_{t+1}/P_t)] - i_t b_t + s_t = \dots$$

$$= E_t[(1/R_t)w_{t+1}] + (i_t/I_t)m_t + s_t$$

**the following is not in the webpage handouts!**

(8) can be re-written as:

$$(2) w_{t+1} = R_t w_t - R_t \sigma_t$$

In NR regime, this is a difference equation with  $\sigma_t$  as forcing variable.

Review of Solving Difference Equations:

Simon and Blume (systems of difference equations, pg 579-597):

Theorem 23.6: The general solution of  $z_{j+1} = Az_j$  is

$$z_t = c_1 r_1^t v_1 + \dots + c_k r_k^t v_k$$

where the  $r$ 's are eigenvalues, and the  $v$ 's are eigenvectors, of matrix  $A$ , and the  $c$ 's are determined by the initial conditions.

Relevance for interpretation of NR regimes:

PVBC is satisfied iff  $\lim_{k \rightarrow \infty} E_t[\alpha_{k+1} w_{k+1}] = 0$  A sufficient condition is that  $w_j$  be bounded; we need a jumping variable to take care of the unstable root.

## Exploring the R regime –

What kinds of fiscal policy rules assure a R regime? ( $\Rightarrow \lim_{k \rightarrow \infty} E_t[\alpha_{k+1} w_{k+1}] = 0$ )

How plausible is the R regime? Woodford says it's a “special case”.

Consider our unstable flow budget constraint once again:

$$(2) w_{t+1} = R_t w_t - R_t \sigma_t$$

For concreteness, go back to one of our examples where  $R_t = 1/\beta > 1$ .

$$(3) w_{t+1} = \beta^{-1} w_t - \beta^{-1} \sigma_t$$

Suppose  $\{\sigma_j\}$  is expected to follow

$$(4) \sigma_j = c w_j$$

and insert (4) into the flow budget constraint,

$$(5) w_{j+1} = \underbrace{[(1-c)/\beta]}_{\gamma} w_j$$

Remarks:

1. If  $1 - \beta < c < 1$ , then  $0 < \gamma < 1$  and (5) is a stable difference equation. We don't have a stability problem; equilibrium  $P_t$  doesn't have to jump.
2. This is stronger than necessary. PVBC is satisfied if  $\lim_{k \rightarrow \infty} E_t[\alpha_{k+1} w_{k+1}] = 0$

**Proposition A:** If  $0 < c < 1$ , then (1) is satisfied for any value of  $w_t$ , and  $P_t$  can be determined elsewhere in the model. We have a R regime.

Proof:

Solve (5) forward from the current period  $t$ :

$$w_{t+1} = \gamma w_t$$

$$w_{t+2} = \gamma w_{t+1} \Rightarrow w_{t+2} = \gamma^2 w_t$$

$$w_{t+3} = \gamma w_{t+2} \Rightarrow w_{t+3} = \gamma^3 w_t$$

$$\dots \Rightarrow w_{t+T} = \gamma^T w_t$$

$$\lim_{T \rightarrow \infty} \beta^T w_{t+T} = \lim_{T \rightarrow \infty} \beta^T \gamma^T w_t = \lim_{T \rightarrow \infty} \beta^T [(1-c)/\beta]^T w_t = \lim_{T \rightarrow \infty} (1-c)^T w_t = 0$$

What kinds of fiscal policy guarantee a R Regime?

Suppose  $\{\sigma_j\}$  is expected to follow

$$(6) \sigma_j = c_j w_j + \epsilon_j \Rightarrow (7) w_{j+1} = [(1-c_j)/\beta] w_j = \gamma_j w_j$$

where  $\epsilon_j$  is counter cyclical policy or political noise.

Proposition (Canzoneri, Cumby and Diba): Assume that  $\{c_j\}$ ,  $\{\rho_j\}$  and  $\{\epsilon_j\}$  are deterministic sequences, that  $\{\epsilon_j\}$  is bounded, and that the following conditions hold:

$$(C1) 0 \leq c_j < 1 \text{ and } \limsup c_j > c^* > 0$$

(C2) a regularity condition we don't need here

Then, the flow budget constraint and the fiscal rule imply that the present value constraint holds for any arbitrary value of  $w_t$ ; we have a R regime.

Proof: see Canzoneri, Cumby and Diba (1999); proof of stochastic case is in the appendix.

GU Consortium Papers:

C, C & D, AER paper.

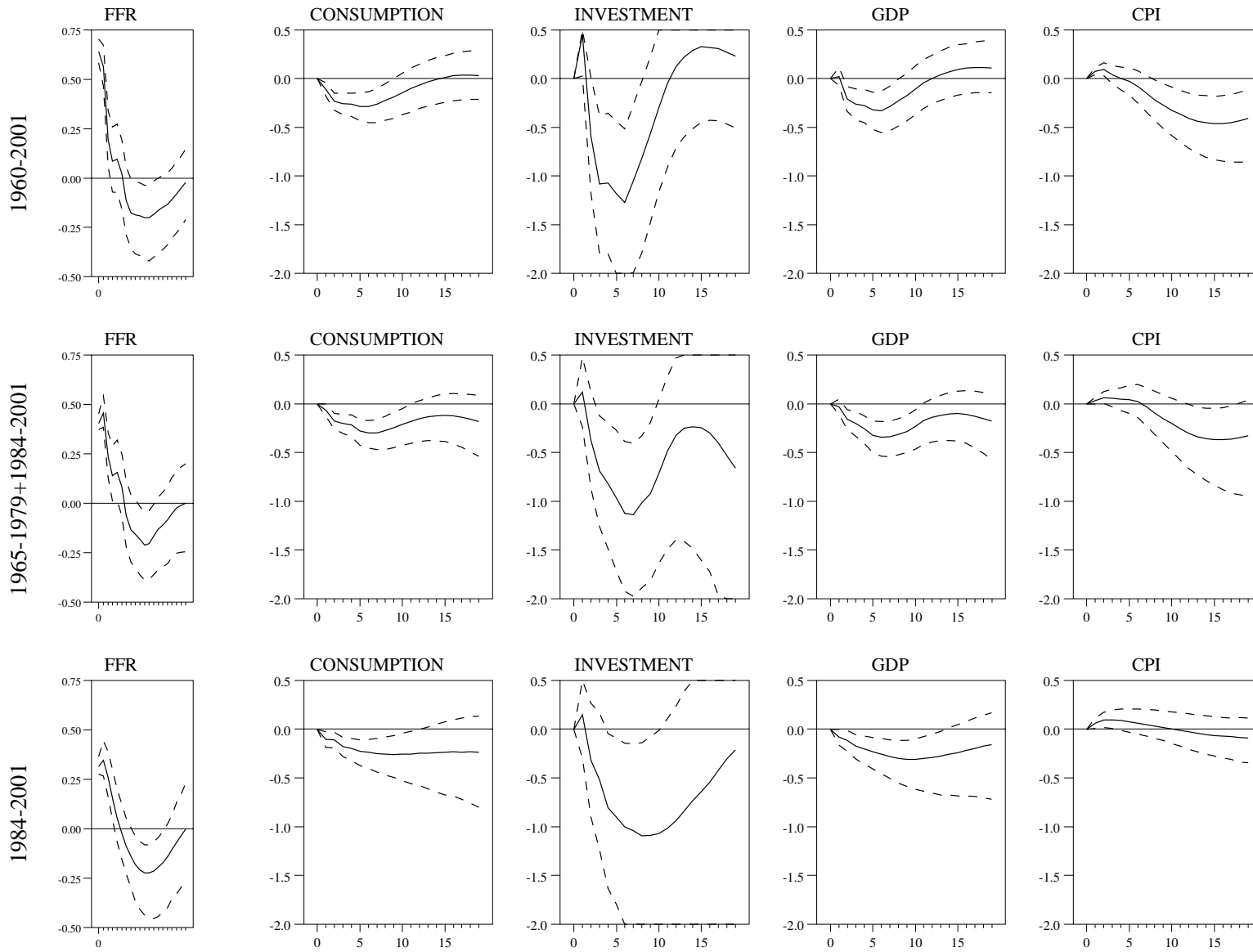
C, C & D, "Fiscal Discipline and Exchange Rate Regimes," The Economic Journal, 111, No. 474, October, 2001, p. 667 - 690.

C, C & Diba, "Interest Rate Rules and Price Determinacy: the Role of Transactions Services of Bonds," June, 2000.

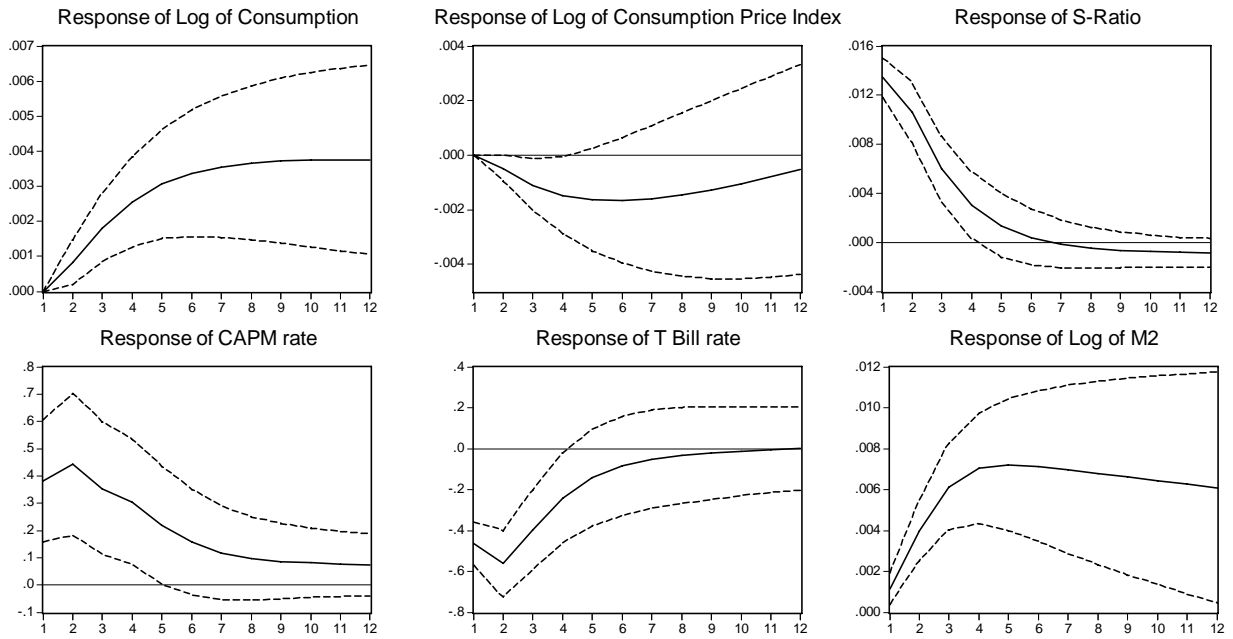
C, C & D, "The Spread Between the CCAPM Interest Rate and the Treasury-bill Rate Is Correlated with Monetary Policy," First Draft: Nov. 2000.

**For Lectures on "Limited Participation Models":**

Figure 1c: U.S., CEE identification



**FIGURE 3: Response to an S-Ratio Shock**



Lucas Fuerst model--

- (1)  $U = E_t^* \sum_{j=t} \beta^{j-t} u(c_j, 1-\ell_j)$  household utility
- (2)  $(M_j - N_j) + W_j \ell_j = P_j c_j$  household CIA constraint
- (3)  $B_j = W_j h_j$  firm's CIA constraint
- (4)  $[(M_{j-1} - N_{j-1}) + W_{j-1} \ell_{j-1} - P_{j-1} c_{j-1}] + [P_{j-1} f(h_{j-1}, z_{j-1}) - W_{j-1} h_{j-1} - B_{j-1} (I_{j-1} - 1)] + (N_{j-1} + X_{j-1}) I_{j-1} = (M_j - N_j) + N_j (= M_j)$  household BC

I use (2) and (3) to eliminate  $M_j$  and  $B_j$  in (4):

$$(4)' P_{j-1} f(h_{j-1}, z_{j-1}) - I_{j-1} W_{j-1} h_{j-1} + (N_{j-1} + X_{j-1}) I_{j-1} + W_j \ell_j = P_j c_j + N_j$$

looking ahead, in equilibrium:

- (5)  $c_j = f(h_j, z_j)$ ,  $N_j + X_j = B_j (= W_j h_j)$ ,  $h_j = \ell_j$
- (6)  $P_j f(h_j, z_j) = M_j + X_j (= \text{end of period balances})$

Recall the information structure implied by  $E_t^*$ :  $N_t$  set before  $X_t$  &  $z_t$  are known.

$$L = E_t^* \left\{ \sum_{j=t} \beta^{j-t} [u(c_j, 1-\ell_j) + \lambda_j [P_{j-1} f(h_{j-1}, z_{j-1}) - I_{j-1} W_{j-1} h_{j-1} + (N_{j-1} + X_{j-1}) I_{j-1} + W_j \ell_j - (P_j c_j + N_j)] \right\}$$

$$L_N: E_{t-1} \lambda_t = E_{t-1} (\beta I_t \lambda_{t+1})$$

$$L_c: u_1(t) = \lambda_t P_t \quad \left. \vphantom{L_c} \right\} \Rightarrow (7) \quad u_2(t) = (W_t/P_t) u_1(t) \quad \text{Labor Supply Curve}$$

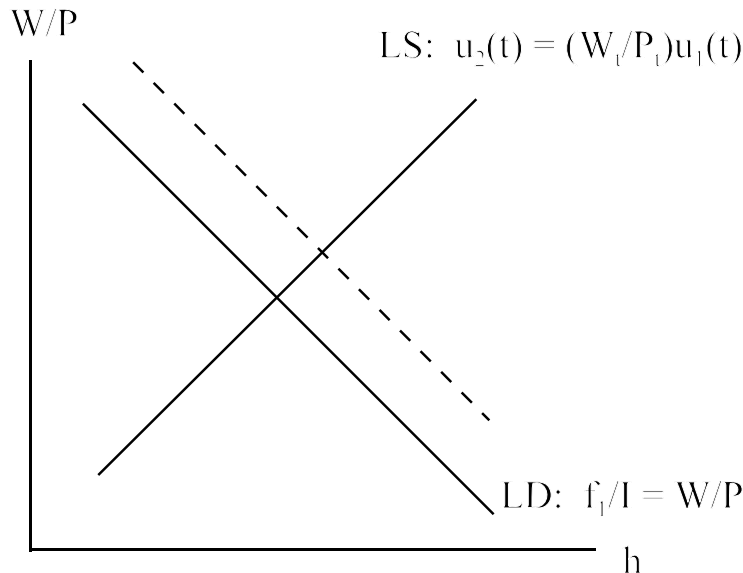
$$L_\ell: u_2(t) = \lambda_t W_t \quad \left. \vphantom{L_\ell} \right\} \quad (\text{marg disutility of work} = \text{marg utility of precedes})$$

$$(8) E_{t-1} [u_1(t)/P_t] = E_{t-1} [(\beta I_t/P_{t+1}) u_1(t+1)] \quad \text{Euler Equation}$$

Firm's profit max:

Choose  $h_t$  to max profits =  $P_t f(h_t, z_t) - W_t h_t + i_t W_t h_t = P_t f(h_t, z_t) - I_t W_t h_t$

(9)  $P_t f_1(t) = I_t W_t$  Labor Demand Curve



Intuition:

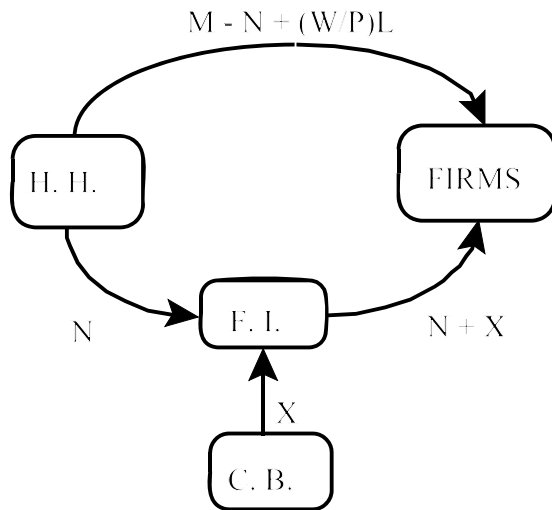
$X_t > X_{t|t-1} \Rightarrow$  supply of loans  $\uparrow$   
 $\Rightarrow I \downarrow \Rightarrow$  LD shifts out

Note similarity to  $Z_t$  shock

Summarizing: The Lucas Fuerst model –

- (1)  $U = E_t^* \sum_{j=t} \beta^{j-t} u(c_j, 1-\ell_j)$  household utility
- (2)  $(M_j - N_j) + W_j \ell_j = P_j c_j$  household CIA constraint
- (3)  $B_j = W_j h_j$  firm's CIA constraint
- (4)  $[(M_{j-1} - N_{j-1}) + W_{j-1} \ell_{j-1} - P_{j-1} c_{j-1}]$   
 $+ [P_{j-1} f(h_{j-1}, z_{j-1}) - W_{j-1} h_{j-1} - B_{j-1}(I_{j-1} - 1)] + (N_{j-1} + X_{j-1}) I_{j-1}$   
 $= (M_j - N_j) + N_j (= M_j)$  household BC
- (5)  $c_j = f(h_j, z_j)$ ,  $N_j + X_j = B_j (= W_j h_j)$ ,  $h_j = \ell_j$  equilibrium conditions
- (6)  $P_j f(h_j, z_j) = M_j + X_j (=$  end of period balances) CIA const. in equil.
- (7)  $u_2(t) = (W_t/P_t)u_1(t)$  household labor supply curve
- (8)  $E_{t-1}[u_1(t)/P_t] = E_{t-1}[(\beta I_t/P_{t+1})u_1(t+1)]$  Euler Equation
- (9)  $P_t f_1(t) = I_t W_t$  firm labor Demand Curve

## Review of Lucas-Fuerst Model –



1. Financial Intermediaries
2. 1<sup>st</sup> HH make deposit N at F.I.
3. 2<sup>nd</sup> X and Z are drawn
4. F.I. loans N+X to Firms against their wage bill
5. HH buy goods.
6. HH get profits from Firms & F.I.

The Equations:

- (1)  $U = E_t^* \sum_{j=t} \beta^{j-t} u(c_j, 1-\ell_j)$  household utility
- (2)  $(M_j - N_j) + W_j \ell_j = P_j c_j$  household CIA constraint
- (3)  $B_j = W_j h_j$  firm's CIA constraint
- (4)  $[(M_{j-1} - N_{j-1}) + W_{j-1} \ell_{j-1} - P_{j-1} c_{j-1}]$   
 $+ [P_{j-1} f(h_{j-1}, z_{j-1}) - W_{j-1} h_{j-1} - B_{j-1} (I_{j-1} - 1)] + (N_{j-1} + X_{j-1}) I_{j-1}$   
 $= (M_j - N_j) + N_j (= M_j)$  household BC
- (5)  $c_j = f(h_j, z_j)$ ,  $N_j + X_j = B_j (= W_j h_j)$ ,  $h_j = \ell_j$  equilibrium conditions
- (6)  $P_j f(h_j, z_j) = M_j + X_j (= \text{end of period balances})$  equilibrium CIA const
- (7)  $u_2(t) = (W_t/P_t) u_1(t)$  household labor supply curve
- (8)  $E_{t-1}[u_1(t)/P_t] = E_{t-1}[(\beta I_t/P_{t+1}) u_1(t+1)]$  Euler Equation
- (9)  $P_t f_1(t) = I_t W_t$  firm's labor Demand Curve

### Solution strategy (a typical Lucas trick) –

Measure nominal variables relative to  $M_t$  (using small letters), and

take  $n_t (= N_t/M_t)$  as given by  $L_N = 0$ .

then, we have four equations in:  $p_t (= P_t/M_t)$ ,  $h_t$ ,  $w_t (= W_t/M_t)$ , and  $I_t$

(a)  $pf(h,z) = 1 + x$  (this is the cash in advance constraint, (6))

(b)  $n + x = wh$  (supply = demand for loans)

(c)  $f_1(h,z) = (w/p)I$  (labor demand curve)

(d)  $u_2(f,1-h) = (w/p)u_1(f,1-h)$  (labor supply curve)

Need to make assumptions about  $f(h,z)$  and  $u(c,1-l)$  to proceed:

- $f(h,z) = zh^\alpha \Rightarrow f_1(h,z) = \alpha f/h$  (marginal prod. is prop. to average prod.)
- $u(c,1-l) = \gamma \log(c) + (1-l) \Rightarrow u_1/u_2 = \gamma/c$  (Fuerst's own assumption.)

Prop. 1:  $I = \alpha(1+x)/(n+x)$ , and we have the liquidity effect.

proof: from (c),  $\alpha f/h = f_1(h,z) = (w/p)I$  or  $\alpha pf = whI$

using (b),  $\alpha pf = (wh)I = (n+x)I$

then from CIA,  $\alpha(1+x) = \alpha pf = (n+x)I$

(See usefulness of  $\alpha f/h = f_1(h,z)$ : allows us to replace  $f_1$  and use CIA constraint.)

$$\frac{dI}{dx} = \alpha \left[ \frac{1}{(n+x)} - \frac{1+x}{(n+x)^2} \right] = \left[ \frac{\alpha}{(n+x)^2} \right] [n+x - (1+x)] = \left[ \frac{\alpha}{(n+x)^2} \right] (n-1) < 0$$

since  $n = N/M < 1$ .

Prop. 2:  $h = \gamma\alpha/I = \gamma(n+x)/(1+x)$ , and we have the liquidity effect for  $h$ .

proof: (c) & (d)  $\Rightarrow u_2/u_1 = (w/p) = f_1(h,z)/I = (\alpha f/h)/I$

so,  $h = (u_1/u_2)(\alpha f/I) = (\gamma/c)(\alpha f/I) = \gamma\alpha/I$

then, Prop. 1  $\Rightarrow h = \gamma(n+x)/(1+x)$  and  $dh/dx > 0$

Prop. 3: Some interesting results.

$$(A) \quad c = f = zh^\alpha = z[\gamma(n + x)/(1 + x)]^\alpha$$

$$(B) \quad w/p (= u_2/u_1 = c/\gamma) = z[(n + x)/(1 + x)]^\alpha/\gamma^{1-\alpha}$$

$$(C) \quad p = (1 + x)/f = (1/z)(1 + x)^{1+\alpha}/(n + x)^\alpha\gamma^\alpha$$

Remarks:

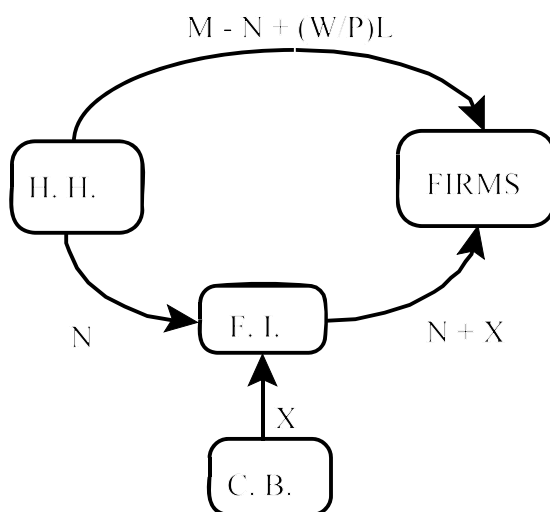
- $p$  may jump in response to  $x$ . We have not been able to say much about when this does or does not happen.
- $z$  has no effect on  $I$  or  $h$ ; seems to get absorbed by  $w/p$ , and  $c = zh^\alpha$ . Nobody (Christiano) seems to know just why. Has caused some problems for RBC modelers. Collateral damage of some simplifying assumptions?
- Real wages are procyclical! Another stylized fact, for US anyway. An old controversy.

## Review of Liquidity Effect –

$M \uparrow \Rightarrow I \downarrow \ \& \ R \downarrow \Rightarrow P \uparrow \ \& \ Y \uparrow$  (at least temporarily)

## The Lucas-Fuerst Model –

The basic ingredients:



1. Financial Intermediaries

2. HH make deposit  $N$  at F.I.

(note:  $N$  is not employment)

3.  $X$  and  $Z$  are drawn

4. F.I. loans  $N+X$  to Firms against their wage bill

5. HH buy goods.

6. HH get profits from Firms & F.I.

**Sargent & Wallace's “monetarist arithmetic”:** (slide, but not in handout)

Shows imitations of CB independence when fiscal authority is “leader”.

Consider either the MIU Model or the CIA Model

Illustrate with Example 1 (or 2):

Convert seigniorage tax,  $(i_t/I_t)m_t$ , into something in  $\pi_t$  –

Recall:  $m_t = m^* - (i_t/I_t)$ , and  $I_t(P_t/P_{t+1}) = 1/\beta \alpha_j \equiv 1/(R_t \cdots R_{j-1}) = \beta^{j-t}$

So,  $I_t^{-1} = \beta(P_t/P_{t+1}) = \beta(1-\pi_{t+1})$

And,  $I_t^{-1} = (1 + i_t - i_t)I_t^{-1} = 1 - (i_t/I_t)$

so,  $(i_t/I_t) = 1 - I_t^{-1} = 1 - \beta(1-\pi_{t+1}) = 1 - \beta + \beta\pi_{t+1}$

So,  $m_t = m^* - (i_t/I_t) = m^* - 1 + \beta - \beta\pi_{t+1} \Rightarrow m^d$  is a decreasing fn of  $\pi_{t+1}$

$(i_t/I_t)m_t = [1 - \beta + \beta\pi_{t+1}] [m^* - 1 + \beta - \beta\pi_{t+1}] \equiv \sigma(\pi_{t+1})$

$\Rightarrow$  Laffer curve in  $\pi_{t+1}$

PVBC:  $(M_{t-1} + I_{t-1}B_{t-1})/P_t = \sum_{j=t}^{\infty} \alpha_j [s_j + (i_j/I_j)m_j] = \sum_{j=t}^{\infty} \beta^{j-1} [s_j + \sigma(\pi_{j+1})]$

Or finally:  $\underbrace{\sum_{j=t}^{\infty} \beta^{j-1} (-s_j)}_C = \sum_{j=t}^{\infty} \beta^{j-1} \sigma(\pi_{j+1}) - (M_{t-1} + I_{t-1}B_{t-1})/P_t$

C = present value of the primary deficit.