

**Question 1:**

In this question, you will review the Sargent and Wallace “policy ineffectiveness” results, and you will show that they depend upon a rather peculiar assumption that S&W made about the dating of inflation expectations in their IS curve.

Consider the model

$$(1) m_t - p_t = - (p_{t+1|t} - p_t) + \epsilon_t \quad \text{"Cagan" money demand function}$$

$$(2) m_t = \gamma_0 \epsilon_t + \gamma_1 \epsilon_{t-1} \quad \text{Money supply rule}$$

$$(3) y_t = (p_t - p_{t|t-1}) \quad \text{"Fischer-Gray" supply function}$$

where  $\epsilon_t$  is a serially uncorrelated money demand shock with zero mean. (1) uses the inflation rate instead of the interest rate as the "cost" of holding money; for simplicity  $y_t$  is left out of money demand. (2) is a money supply rule; it includes "contemporaneous" feedback if  $\gamma_0 > 0$ , and "lagged" feedback if  $\gamma_1 > 0$ .

Part A: Solve the model for  $p_t$  and then  $y_t$ . Use the method of undetermined coefficients.

Part B: Are "contemporaneous" feedback rules effective in this model? Can they force output to its natural rate (zero) each period? (Hint: set  $\gamma_1 = 0$ .)

Part C: Are "lagged" feedback rules effective in this model? Can they force output to its natural rate (zero) each period? (Hint: set  $\gamma_0 = 0$ .)

Part D: Which of your results in Parts B and C differ from the results reported in Sargent and Wallace (JPE, 1975)? Explain why. (Hint 1: How does the dating of inflation expectations in this model differ from the dating in S&W, and why does the difference matter? Hint 2: Forward the money supply rule one period and think of it as a promise about the way the Central Bank will set  $m_{t+1}$ .) Here, I want the economic intuition, and not just a restatement of the algebraic results in Parts B and C.

Part E: Which dating of expectations seem more realistic to you. Which results seem more realistic to you? (That is, are lagged feedback rules effective?)

In the next question, you will show how Poole's "instrument selection" problem (QJE, May 1970) can be extended to models with rational expectations.

**Question 2:** Consider the model:

- |   |                          |
|---|--------------------------|
| (1) $y_t = - [i_t - (p_{t+1 t} - p_t)] + \delta(b_t - p_t) + u_t$ | IS-Curve                 |
| (2) $y_t = (p_t - p_{t t-1})$                                     | Output Supply Curve      |
| (3) $m_t - p_t = -i_t + y_t - v_t$                                | LM-Curve                 |
| (4a) $i_t = 0$ ( $m_t$ is endogenous)                             | Monetary Policy Rule "I" |
| (4b) $m_t = 0$ ( $i_t$ is endogenous)                             | Monetary Policy Rule "M" |

Most of the parameters have been set equal to unity to make the algebra simpler. The last term in the IS curve is (the log of) the real value of government bonds held by the private sector; government debt is a component of wealth that affects consumption decisions. Assume that the government holds the nominal debt constant over time; for simplicity, let  $b_{t+j} = 0$  for all  $j \geq 0$ .  $u_t$  and  $v_t$  are serially uncorrelated shocks with zero mean.

Part A: Solve the model (using the method of undetermined coefficients) assuming the Fed is holding the money supply constant (Policy Rule M). In this case, we say that M is the instrument of monetary policy, and  $i_t$  is market determined. Calculate the variance of output.

Part B: Solve the model (using the method of undetermined coefficients) assuming the Fed varies M to hold the nominal interest rate constant (Policy Rule I). In this case, we say that  $i$  is the instrument of monetary policy, and  $M_t$  is market determined. Calculate the variance of output.

Part C: Which policy rule does a better job of stabilizing output? This was Poole's criterion for the Fed's choice of an instrument for implementing monetary policy.

Part D: Can you explain intuitively why you got the results you did on part C? What are the economic forces at play? It might be helpful to draw an IS-LM diagram.

Part E: Some economists assert that government bonds are not really a part of net private sector wealth, since they represent something that the households as taxpayers just owe themselves. Explain the difficulties that would arise if government bonds were eliminated from the IS curve. (Hint: now, solve the model using Sargent's method.)

**M. Canzoneri****Due:**

In this exercise, you will show why the Fed's announcement of the "optimal" policy rule would not be credible, and you will explore the relationship between the Sargent and Wallace literature and the Barro and Gordon literature.

**Question 1:** Consider the model we studied in class.

Part A: Suppose the Fed announces the "optimal" rule

$$(11) \quad m_t^o = \pi^* - [u\alpha/(1 + u\alpha^2)]x_t,$$

and suppose further that the wage setters believe the Fed's announcement and form their price expectations based upon (11). Show that the Fed will then find it optimal to implement the "cheating" policy rule

$$(14) \quad m_t^c = \pi^* + n^*/(1 + u\alpha^2) - [u\alpha/(1 + u\alpha^2)]x_t.$$

Hint: Find the  $m_{t|t-1}$  implied by (11). Substitute this expectation into the Fed's utility function, and then find the Fed's utility maximizing policy.

Part B: Derive the "cheating" solution:

$$n_t^c = n^*/(1 + u\alpha^2) - [u\alpha/(1 + u\alpha^2)]x_t$$

$$\pi_t^c = \pi^* + \alpha n^*/(1 + u\alpha^2) + x_t/(1 + u\alpha^2)$$

$$EU_t^c = EU_t^o + n^{*2}/(1 + u\alpha^2)$$

Part C: Is the Fed more expansionary under (14) than (11)? (That is, does the Fed renege on its promise not to inflate?) Can you explain why? Here I am looking for economic intuition, and not a verbal rehash of the algebra.

**Question 2:** Consider the following Fischer-Gray model:

$$(1) y_t = (p_t - p_{t-1})$$

Supply Curve

$$(2) m_t + v_t = p_t + y_t$$

Money Market Equilibrium

$$(3) v_t = v_{t-1} + \epsilon_t \quad (\epsilon_{t|t-j} = 0 \quad \forall j \geq 1)$$

Process for Velocity

Part A: Show that the reduced forms for  $p_t$  and  $y_t$  are

$$(4) p_t = m_{t|t-1} + v_{t-1} + \frac{1}{2}(m_t - m_{t|t-1}) + \frac{1}{2}\epsilon_t$$

$$(5) y_t = \frac{1}{2}(m_t - m_{t|t-1}) + \frac{1}{2}\epsilon_t$$

Part B: Show that contemporaneous feedback rules for  $m_t$  can reduce the variance of  $y_t$  to zero, while lagged feedback rules can not.

Part C: Is there a feedback rule (using contemporaneous and/or lagged feedback) that will reduce both the variance of  $y_t$  and the variance of  $p_t$  to zero?

Part D: Suppose in the model above the central bank chooses  $m_t$  to maximize

$$(6) U_t = - (y_t - \bar{y})^2 - \mu p_t^2, \quad \bar{y} > 0.$$

In the discretionary solution, how does the central bank's policy rule compare with the one you found in Part C? Does it stabilize  $p_t$  and  $y_t$ ? What is the Barro-Gordon inflation (or price) bias?

Part E: If Rogoff's approach to lowering the inflation bias (increasing  $\mu$ ) is applied in this model, is the stabilization effort distorted? Why do you think the literature on the "credibility-stabilization" tradeoff focuses on productivity shocks?

Part F: Walsh showed that the inflation bias would be eliminated if a linear inflation penalty was imposed on the central bank. In this model, would a linear penalty on the level of output accomplish the same goal? Derive your answer mathematically, then explain your result intuitively. Why do you think Walsh concentrated on inflation penalties instead of output penalties?

In this exercise, you will study a typical “optimal taxation” problem. Suppose the government fights the “war to end all wars” in period  $t$ ; then, in all subsequent periods, there is no more government spending:  $g_t = g$  and  $g_{t+j} = 0$  for all  $j > 0$ . (We get to make silly assumptions in macroeconomic theory.) Here you will show that it is optimal to spread the burden of financing this war across the taxes that are available and over time; a balanced budget amendment would be economically inefficient. There are high and low tax ways of financing the war, but you will show that the low tax way is best. You will also show that the optimal tax plan is not time consistent. Finally, you will show that the Laffer Curve disappears for some utility functions.

**Part A:** Let  $u(c_j, x_j) = c_j + g_j + x_j - \frac{1}{2}x_j^2 - \frac{1}{2}(m_j - m^*)^2$ .

The household values public and private spending equally. (Another silly assumption that simplifies the algebra.) Show that in equilibrium:

- (i)  $R_j = 1/\beta$ ,  $\alpha_j = \beta^{j-t}$ ,  $n_j = 1 - \tau_j$ ,  $m_j = m^* - i_j/I_j$ ,  $x_j = \tau_j$ ,  $c_j = 1 - \tau_j - g_j$ .
- (ii)  $g = \sum_{j=t}^{\infty} \beta^{j-t} [\tau_j(1-\tau_j) + (i_j/I_j)(m^* - (i_j/I_j))]$  (assuming no lump sum taxes, and that  $w_t = 0$ )
- (iii)  $U_t = \sum_{j=t}^{\infty} \beta^{j-t} [1 - \frac{1}{2}\tau_j^2 - \frac{1}{2}(i_j/I_j)^2]$

**Part B:** Suppose the government is “benevolent” in the sense that it chooses the taxes  $\{\tau_j\}$  and  $\{i_j/I_j\}$  to maximize household utility,  $U_t$ . Assume (to simplify the algebra) that  $m^* = 1$ .

- (i) Show that a balanced budget is not optimal in this case; show that it is optimal to set  $\tau_j = \tau$  (a constant) and  $i_j/I_j = i/I$  (a constant), and spread the “costs” of financing  $g$  over time.
- (ii) Show that the optimal  $\tau$  and  $i/I$  must satisfy:

$$g = (1-\beta)^{-1}(\tau - \tau^2) + (1-\beta)^{-1}(i/I - (i/I)^2) \quad \text{and} \quad \tau = i/I = \frac{1}{2}[1 \pm \sqrt{1 - 2(1-\beta)g}]$$

This is the optimal way of spreading the tax burden between taxes.

- (iii) In (ii) above you found high and low tax ways of financing  $g$ . Which will the benevolent government choose. (Hint: Use Part A(iii) to show that  $dU_t/d\tau < 0$ .)

**Part C:** Show that the optimal tax plan is time inconsistent. That is, show that in period  $t+1$ , a benevolent government would choose to default on its debt.

**Part D:** How would the availability of lump sum taxes change your answers in Part B? You need not redo all of the mathematics; just give an intuitive, verbal answer. Relate your discussion to the notion of “Ricardian Equivalence.”

**Part E:** Seigniorage revenues are quite small in most OECD countries. Suppose you want to ignore the seigniorage consequences of a change in monetary policy (and eliminate all of the rather tedious algebra that goes along with it). Can you think of a modeling trick that will eliminate the budgetary consequences of a change in monetary policy? Hint: let money enter utility logarithmically:  $u(c_j, x_j) = c_j + g_j + x_j - \frac{1}{2}x_j^2 + \log(m_j)$ . What happens to the government’s PVBC (the equation (ii) above) with this utility function? Is there still a Laffer Curve for the seigniorage tax? If not, why not?