

Question 1:

In this question, you will review the Sargent and Wallace “policy ineffectiveness” results, and you will show that they depend upon a rather peculiar assumption that S&W made about the dating of inflation expectations in their IS curve.

Consider the model

$$(1) m_t - p_t = - (p_{t+1|t} - p_t) + \epsilon_t \quad \text{"Cagan" money demand function}$$

$$(2) m_t = \gamma_0 \epsilon_t + \gamma_1 \epsilon_{t-1} \quad \text{Money supply rule}$$

$$(3) y_t = (p_t - p_{t|t-1}) \quad \text{"Fischer-Gray" supply function}$$

where ϵ_t is a serially uncorrelated money demand shock with zero mean. (1) uses the inflation rate instead of the interest rate as the "cost" of holding money; for simplicity y_t is left out of money demand. (2) is a money supply rule; it includes "contemporaneous" feedback if $\gamma_0 > 0$, and "lagged" feedback if $\gamma_1 > 0$.

Part A: Solve the model for p_t and then y_t . Use the method of undetermined coefficients.

Part B: Are "contemporaneous" feedback rules effective in this model? Can they force output to its natural rate (zero) each period? (Hint: set $\gamma_1 = 0$.)

Part C: Are "lagged" feedback rules effective in this model? Can they force output to its natural rate (zero) each period? (Hint: set $\gamma_0 = 0$.)

Part D: Which of your results in Parts B and C differ from the results reported in Sargent and Wallace (JPE, 1975)? Explain why. (Hint 1: How does the dating of inflation expectations in this model differ from the dating in S&W, and why does the difference matter? Hint 2: Forward the money supply rule one period and think of it as a promise about the way the Central Bank will set m_{t+1} .) Here, I want the economic intuition, and not just a restatement of the algebraic results in Parts B and C.

Part E: Which dating of expectations seem more realistic to you. Which results seem more realistic to you? (That is, are lagged feedback rules effective?)

In the next question, you will show how Poole's "instrument selection" problem (QJE, May 1970) can be extended to models with rational expectations.

Question 2: Consider the model:

- | | |
|---|--------------------------|
| (1) $y_t = - [i_t - (p_{t+1 t} - p_t)] + \delta(b_t - p_t) + u_t$ | IS-Curve |
| (2) $y_t = (p_t - p_{t t-1})$ | Output Supply Curve |
| (3) $m_t - p_t = - i_t + y_t - v_t$ | LM-Curve |
| (4a) $i_t = 0$ (m_t is endogenous) | Monetary Policy Rule "I" |
| (4b) $m_t = 0$ (i_t is endogenous) | Monetary Policy Rule "M" |

Most of the parameters have been set equal to unity to make the algebra simpler. The last term in the IS curve is (the log of) the real value of government bonds held by the private sector; government debt is a component of wealth that affects consumption decisions. Assume that the government holds the nominal debt constant over time; for simplicity, let $b_{t+j} = 0$ for all $j \geq 0$. u_t and v_t are serially uncorrelated shocks with zero mean.

Part A: Solve the model (using the method of undetermined coefficients) assuming the Fed is holding the money supply constant (Policy Rule M). In this case, we say that M is the instrument of monetary policy, and i_t is market determined. Calculate the variance of output.

Part B: Solve the model (using the method of undetermined coefficients) assuming the Fed varies M to hold the nominal interest rate constant (Policy Rule I). In this case, we say that i is the instrument of monetary policy, and M_t is market determined. Calculate the variance of output.

Part C: Which policy rule does a better job of stabilizing output? This was Poole's criterion for the Fed's choice of an instrument for implementing monetary policy.

Part D: Can you explain intuitively why you got the results you did on part C? What are the economic forces at play? It might be helpful to draw an IS-LM diagram.

Part E: Some economists assert that government bonds are not really a part of net private sector wealth, since they represent something that the households as taxpayers just owe themselves. Explain the difficulties that would arise if government bonds were eliminated from the IS curve. (Hint: now, solve the model using Sargent's method.)

M. Canzoneri**Due:**

In this exercise, you will show why the Fed's announcement of the "optimal" policy rule would not be credible, and you will explore the relationship between the Sargent and Wallace literature and the Barro and Gordon literature.

Question 1: Consider the model we studied in class.

Part A: Suppose the Fed announces the "optimal" rule

$$(11) \quad m_t^o = \pi^* - [u\alpha/(1 + u\alpha^2)]x_t,$$

and suppose further that the wage setters believe the Fed's announcement and form their price expectations based upon (11). Show that the Fed will then find it optimal to implement the "cheating" policy rule

$$(14) \quad m_t^c = \pi^* + n^*/(1 + u\alpha^2) - [u\alpha/(1 + u\alpha^2)]x_t.$$

Hint: Find the $m_{t|t-1}$ implied by (11). Substitute this expectation into the Fed's utility function, and then find the Fed's utility maximizing policy.

Part B: Derive the "cheating" solution:

$$n_t^c = n^*/(1 + u\alpha^2) - [u\alpha/(1 + u\alpha^2)]x_t$$

$$\pi_t^c = \pi^* + \alpha n^*/(1 + u\alpha^2) + x_t/(1 + u\alpha^2)$$

$$EU_t^c = EU_t^o + n^{*2}/(1 + u\alpha^2)$$

Part C: Is the Fed more expansionary under (14) than (11)? (That is, does the Fed renege on its promise not to inflate?) Can you explain why? Here I am looking for economic intuition, and not a verbal rehash of the algebra.

Question 2: Consider the following Fischer-Gray model:

$$(1) y_t = (p_t - p_{t-1})$$

Supply Curve

$$(2) m_t + v_t = p_t + y_t$$

Money Market Equilibrium

$$(3) v_t = v_{t-1} + \epsilon_t \quad (\epsilon_{t|t-j} = 0 \quad \forall j \geq 1)$$

Process for Velocity

Part A: Show that the reduced forms for p_t and y_t are

$$(4) p_t = m_{t|t-1} + v_{t-1} + \frac{1}{2}(m_t - m_{t|t-1}) + \frac{1}{2}\epsilon_t$$

$$(5) y_t = \frac{1}{2}(m_t - m_{t|t-1}) + \frac{1}{2}\epsilon_t$$

Part B: Show that contemporaneous feedback rules for m_t can reduce the variance of y_t to zero, while lagged feedback rules can not.

Part C: Is there a feedback rule (using contemporaneous and/or lagged feedback) that will reduce both the variance of y_t and the variance of p_t to zero?

Part D: Suppose in the model above the central bank chooses m_t to maximize

$$(6) U_t = - (y_t - \bar{y})^2 - \mu p_t^2, \quad \bar{y} > 0.$$

In the discretionary solution, how does the central bank's policy rule compare with the one you found in Part C? Does it stabilize p_t and y_t ? What is the Barro-Gordon inflation (or price) bias?

Part E: If Rogoff's approach to lowering the inflation bias (increasing μ) is applied in this model, is the stabilization effort distorted? Why do you think the literature on the "credibility-stabilization" tradeoff focuses on productivity shocks?

Part F: Walsh showed that the inflation bias would be eliminated if a linear inflation penalty was imposed on the central bank. In this model, would a linear penalty on the level of output accomplish the same goal? Derive your answer mathematically, then explain your result intuitively. Why do you think Walsh concentrated on inflation penalties instead of output penalties?

In this exercise, you will study a typical “optimal taxation” problem. Suppose the government fights the “war to end all wars” in period t ; then, in all subsequent periods, there is no more government spending: $g_t = g$ and $g_{t+j} = 0$ for all $j > 0$. (We get to make silly assumptions in macroeconomic theory.) Here you will show that it is optimal to spread the burden of financing this war across the taxes that are available and over time; a balanced budget amendment would be economically inefficient. There are high and low tax ways of financing the war, but you will show that the low tax way is best. You will also show that the optimal tax plan is not time consistent. Finally, you will show that the Laffer Curve disappears for some utility functions.

Part A: Let $u(c_j, x_j) = c_j + g_j + x_j - \frac{1}{2}x_j^2 - \frac{1}{2}(m_j - m^*)^2$.

The household values public and private spending equally. (Another silly assumption that simplifies the algebra.) Show that in equilibrium:

- (i) $R_j = 1/\beta$, $\alpha_j = \beta^{j-t}$, $n_j = 1 - \tau_j$, $m_j = m^* - i_j/I_j$, $x_j = \tau_j$, $c_j = 1 - \tau_j - g_j$.
- (ii) $g = \sum_{j=t}^{\infty} \beta^{j-t} [\tau_j(1-\tau_j) + (i_j/I_j)(m^* - (i_j/I_j))]$ (assuming no lump sum taxes, and that $w_t = 0$)
- (iii) $U_t = \sum_{j=t}^{\infty} \beta^{j-t} [1 - \frac{1}{2}\tau_j^2 - \frac{1}{2}(i_j/I_j)^2]$

Part B: Suppose the government is “benevolent” in the sense that it chooses the taxes $\{\tau_j\}$ and $\{i_j/I_j\}$ to maximize household utility, U_t . Assume (to simplify the algebra) that $m^* = 1$.

- (i) Show that a balanced budget is not optimal in this case; show that it is optimal to set $\tau_j = \tau$ (a constant) and $i_j/I_j = i/I$ (a constant), and spread the “costs” of financing g over time.
- (ii) Show that the optimal τ and i/I must satisfy:

$$g = (1-\beta)^{-1}(\tau - \tau^2) + (1-\beta)^{-1}(i/I - (i/I)^2) \quad \text{and} \quad \tau = i/I = \frac{1}{2}[1 \pm \sqrt{1 - 2(1-\beta)g}]$$

This is the optimal way of spreading the tax burden between taxes.

- (iii) In (ii) above you found high and low tax ways of financing g . Which will the benevolent government choose. (Hint: Use Part A(iii) to show that $dU_t/d\tau < 0$.)

Part C: Show that the optimal tax plan is time inconsistent. That is, show that in period $t+1$, a benevolent government would choose to default on its debt.

Part D: How would the availability of lump sum taxes change your answers in Part B? You need not redo all of the mathematics; just give an intuitive, verbal answer. Relate your discussion to the notion of “Ricardian Equivalence.”

Part E: Seigniorage revenues are quite small in most OECD countries. Suppose you want to ignore the seigniorage consequences of a change in monetary policy (and eliminate all of the rather tedious algebra that goes along with it). Can you think of a modeling trick that will eliminate the budgetary consequences of a change in monetary policy? Hint: let money enter utility logarithmically: $u(c_j, x_j) = c_j + g_j + x_j - \frac{1}{2}x_j^2 + \log(m_j)$. What happens to the government’s PVBC (the equation (ii) above) with this utility function? Is there still a Laffer Curve for the seigniorage tax? If not, why not?

Consider Lucas & Fuerst's "limited participation model". Recall the equilibrium conditions for our version of this (which has a different CIA constraint for consumers):

We have four equations in: p_t ($= P_t/M_t$), h_t , w_t ($= W_t/M_t$), I_t , and x_t ($= X_t/M_t$)

- (a) $pf(h,z) = 1 + x$ (this is the cash in advance constraint, (6))
 (b) $n + x = wh$ (supply = demand for loans from fin. intermediary)
 (c) $f_1(h,z) = (w/p)I$ (labor demand curve)
 (d) $u_2(f,1-h) = (w/p)u_1(f,1-h)$ (labor supply curve)
 (e) $f(h,z) = zh^\alpha$ Fuerst's specification of the production and utility functions
 $u(c,1-l) = \gamma \log(c) + (1-l)$ and utility functions

Part A: A monetary injection produces a "liquidity effect" – $dI/dx < 0$ – in this model. Under what conditions, will a monetary injection raise the price level: $dp/dx > 0$? (Hint: evaluate the derivative at $x = 0$.)

Part B: The choice of specific functional forms can sometimes have surprising – and unintended – consequences.

- i. In the NNS models we study next, we generally assume a linear production function. Show that if we replace Fuerst's specification of the production function with $f(h,z) = zh$, we get the same qualitative results: a monetary injection still decreases I and increases h . (Keep Fuerst's specification of the utility function.)
- ii. It is puzzling that, in the model we developed in class, the productivity shock does not affect the level of employment, h . In the NNS models we study next, we see that this kind of result is generally associated with a log specification of the utility of consumption. Suppose we replace Fuerst's utility function with a constant elasticity function, $u(c,1-l) = (1-\gamma)^{-1}c^{1-\gamma} + (1-l)$. (Keep Fuerst's specification of the production function.) Does the productivity shock affect the level of employment? Can you explain the result intuitively? Does a monetary injection still lower I and raise h ?

Econ. 671

Exercise 6

M. Canzoneri

Due:

Question 1: Consider an economy with one flexible wage/price sector (denoted by $s = f$) and one sticky wages/flexible price sector (denoted by $s = w$); productivity shocks in the two sectors are perfectly correlated. To safeguard your mental health, limit discussion to the case where $\gamma = 1$.

Part A: Show that the policy rule $\Omega_t = A_t^{-1/(1+\chi)}$ would bring the entire economy to the flexible wage/price solution.

Part B: Show that if this policy were implemented, wages in the flexible wage/price sector would converge on the wage set in the fixed wage sector, stabilizing the “aggregate” wage rate.

Part C: Use the results of Part B to give an intuitive explanation for how the optimal policy rule brings about the flexible wage/price solution.

Question 2: With log utility of consumption, the Euler equation becomes: $I_t^{-1} = 1/(1+i_t) = \beta E_t[P_t C_t / P_{t+1} C_{t+1}]$. Assume the shocks are i.i.d. and distributed log-normally, and let small letters represent the logs of big letters. Show that this Euler equation can be expressed as an IS curve: $i_t - (E_t[p_{t+1}] - p_t) = r - c_t$ where r is a time invariant intercept term. What is r ?

Question 3: Consider an economy with one flexible wage/price sector, two fixed price sectors, and four fixed wage/flexible price sectors. (In this economy, there is more wage inertia than price inertia.) Productivity shocks in the fixed price sectors are perfectly correlated (and denoted by $z_{p,t}$), and productivity shocks in the fixed wage/flexible price sectors are perfectly correlated (and denoted by $z_{w,t}$); however, these shocks are not correlated with each other, or the shock in the flexible wage/price sector (denoted by $z_{f,t}$).

Part A: What is the optimal policy rule for this economy? Assume that the rule takes the form:

$$\omega_t = r_f z_{f,t} + r_p z_{p,t} + r_w z_{w,t} + r_a a_t \text{ and find the optimal values of the coefficients } r_f, r_p, r_w \text{ \& } r_a.$$

Part B: Can you give an intuitive explanation for size of the coefficients in the optimal policy rule?

Question 1:

Part A: Solve the model for p_t and then y_t . Use the method of undetermined coefficients.

$$(1) \text{ and } (2) \rightarrow p_t = \frac{1}{2}p_{t+1|t} - \frac{1}{2}\epsilon_t + \frac{1}{2}m_t = \frac{1}{2}p_{t+1|t} + \frac{1}{2}(\gamma_0 - 1)\epsilon_t + \frac{1}{2}\gamma_1\epsilon_{t-1}$$

try

$$p_t = r_0\epsilon_t + r_1\epsilon_{t-1} \rightarrow p_{t+1|t} = r_1\epsilon_t \text{ and}$$

$$\begin{aligned} p_t &= \frac{1}{2}p_{t+1|t} + \frac{1}{2}(\gamma_0 - 1)\epsilon_t + \frac{1}{2}\gamma_1\epsilon_{t-1} = \frac{1}{2}r_1\epsilon_t + \frac{1}{2}(\gamma_0 - 1)\epsilon_t + \frac{1}{2}\gamma_1\epsilon_{t-1} \\ &= \frac{1}{2}(r_1 + \gamma_0 - 1)\epsilon_t + \frac{1}{2}\gamma_1\epsilon_{t-1} \end{aligned}$$

so

$$r_0 = \frac{1}{2}(r_1 + \gamma_0 - 1) \text{ and } r_1 = \frac{1}{2}\gamma_1,$$

so finally

$$r_0 = \frac{1}{2}(\frac{1}{2}\gamma_1 + \gamma_0 - 1) \text{ and } r_1 = \frac{1}{2}\gamma_1$$

and

$$p_t = r_0\epsilon_t + r_1\epsilon_{t-1} = \frac{1}{2}(\frac{1}{2}\gamma_1 + \gamma_0 - 1)\epsilon_t + \frac{1}{2}\gamma_1\epsilon_{t-1}$$

$$y_t = (p_t - p_{t|t-1}) = \frac{1}{2}(\frac{1}{2}\gamma_1 + \gamma_0 - 1)\epsilon_t$$

Part B: Are "contemporaneous" feedback rules effective in this model? Can they force output to its natural rate (zero) each period?

Yes. $\gamma_0 = 1$ (with $\gamma_1 = 0$) makes $y_t = 0$.

Part C: Are "lagged" feedback rules effective in this model? Can they force output to its natural rate (zero) each period?

Yes. $\gamma_1 = 2$ (with $\gamma_0 = 0$) makes $y_t = 0$.

Part D: Which of your results differ from those in Sargent & Wallace (JPE, 1975) and why?

In Part B, contemporaneous feedback rules are effective, S&W did not consider contemporaneous feedback rules. Contemporaneous rules are effective because the Fed exploits information that the wage setters did not have when they set W_t . By offsetting the effects of ϵ_t , the Fed can eliminate the wage setters prediction errors.

In Part C, lagged feedback rules are effective; they were not in S&W. The reason for

this difference in results lies in the dating of expectations for next period's price level. Here, we have $p_{t+1|t}$, while S&W had $p_{t+1|t-1}$. To see why this makes a difference, forward the money supply rule one period and take expectations conditional on period t information: $m_{t+1|t} = \gamma_1 \epsilon_t$. The lagged feedback rule is, in effect, a promise to make next period's money supply move systematically in response to ϵ_t . With rational expectations, this links $p_{t+1|t}$ with ϵ_t . A lagged feedback rule can make $p_{t+1|t}$ offset ϵ_t in (1) before it ever affects p_t or y_t .

Part E: A variety of answers might make sense here. A major point to note is that the intuition of Part C goes through even if expectations of p_{t+1} only reflect partial current information.

Question 2:

Part A: Solve the model assuming the Fed is holding the money supply constant (Policy Rule M). In this case, we say that M is the instrument of monetary policy, and i_t is market determined. Calculate the variance of output.

(3) & (2) imply $i_t = p_t + y_t - v_t = p_t + (p_t - p_{t|t-1}) - v_t$

substituting into (1)

$$(p_t - p_{t|t-1}) = - [p_t + (p_t - p_{t|t-1}) - v_t - (p_{t+1|t} - p_t)] - \delta p_t + u_t$$

$$p_t = (2+\delta)^{-1} p_{t+1|t} - 2(2+\delta)^{-1} (p_t - p_{t|t-1}) + (2+\delta)^{-1} (u_t + v_t)$$

$$= c p_{t+1|t} - 2(2+\delta)^{-1} (p_t - p_{t|t-1}) + (2+\delta)^{-1} (u_t + v_t)$$

need an equation to iterate on; so, forward price equation j periods

$$p_{t+j} = c p_{t+j+1|t+j} - 2(2+\delta)^{-1} (p_{t+j} - p_{t+j|t+j-1}) + (2+\delta)^{-1} (u_{t+j} + v_{t+j})$$

$$p_{t+j|t} = c p_{t+j+1|t} - 2(2+\delta)^{-1} (p_{t+j|t} - p_{t+j|t}) + 0 = c p_{t+j+1|t} \text{ for } j \geq 1$$

use this to eliminate $p_{t+1|t}$ in the price equation

$$p_t = \lim_{T \rightarrow \infty} c^T p_{t+T|t} - 2(2+\delta)^{-1} (p_t - p_{t|t-1}) + (2+\delta)^{-1} (u_t + v_t)$$

$$= - 2(2+\delta)^{-1} (p_t - p_{t|t-1}) + (2+\delta)^{-1} (u_t + v_t)$$

(Since $c = (2+\delta)^{-1} < 1$, we can use the no "speculative bubbles" criterion to select this solution.)

now, eliminate the price prediction error in the usual way:

$$p_{t|t-1} = 0$$

$$p_t = [1/(1 + 2(2+\delta)^{-1})](2+\delta)^{-1}(u_t + v_t) = (u_t + v_t)/(4+\delta)$$

Then, letting " σ " denote variance and assuming u_t and v_t are uncorrelated,

$$y_t = (p_t - p_{t|t-1}) = (u_t + v_t)/(4+\delta)$$

$$\sigma_y = (\sigma_u + \sigma_v)/(4+\delta)^2$$

Part B: Solve the model assuming the Fed varies M to hold the nominal interest rate constant

(Policy Rule I). In this case, we say that i is the instrument of monetary policy, and M_t is market determined. Calculate the variance of output.

(1) & (2) imply

$$(p_t - p_{t|t-1}) = - [0 - (p_{t+1|t} - p_t)] - \delta p_t + u_t$$

$$p_t = (1+\delta)^{-1}p_{t+1|t} - (1+\delta)^{-1}(p_t - p_{t|t-1}) + (1+\delta)^{-1}u_t$$

$$= cp_{t+1|t} - (1+\delta)^{-1}(p_t - p_{t|t-1}) + (1+\delta)^{-1}u_t$$

need an equation to iterate on; so, forward price equation j periods

$$p_{t+j} = cp_{t+j+1|t+j} - (1+\delta)^{-1}(p_{t+j} - p_{t+j|t+j-1}) + (1+\delta)^{-1}u_{t+j}$$

$$p_{t+j|t} = cp_{t+j+1|t} - (1+\delta)^{-1}(p_{t+j|t} - p_{t+j|t}) + 0 = cp_{t+j+1|t} \text{ for } j \geq 1$$

use this to eliminate $p_{t+1|t}$ in the price equation

$$p_t = \lim_{T \rightarrow \infty} c^T p_{t+T|t} - (1+\delta)^{-1}(p_t - p_{t|t-1}) + (1+\delta)^{-1}u_t$$

$$= - (1+\delta)^{-1}(p_t - p_{t|t-1}) + (1+\delta)^{-1}u_t$$

(Since $c = (1+\delta)^{-1} < 1$, we can apply the no "speculative bubbles" criterion.)

now, eliminate the price prediction error in the usual way:

$$p_{t|t-1} = 0$$

$$p_t = [1/(1 + (1+\delta)^{-1})](1+\delta)^{-1}u_t = u_t/(2+\delta)$$

Then, letting " σ " denote variance and assuming u_t and v_t are uncorrelated,

$$y_t = (p_t - p_{t|t-1}) = u_t/(2+\delta)$$

$$\sigma_y = \sigma_u/(2+\delta)^2$$

Part C: Which policy rule does a better job of stabilizing output?

It depends on which shock is more important.

For v_t shocks, $i_t = 0$ (Rule I) works best; the shocks don't even show up.

For u_t shocks, $m_t = 0$ (Rule M) works best.

Part D: Can you explain intuitively why you got the results you did on part C? What are the economic forces at play? It might be helpful to draw an IS-LM diagram.

Rule I requires the Fed to automatically accommodate v_t before it passes to prices or output.

Rule M allows the real interest rate to move to absorb some of the u_t shock; Rule I keeps this from happening.

See for example the diagrammatic discussion in Dornbusch and Fischer, *Macroeconomics*, 5th Edition, 1990, page 410.

Part E: Some economists assert that government bonds are not really a part of net private sector wealth, since it represents something that the households as taxpayers just owe themselves. Explain the difficulties that would arise if government bonds were eliminated from IS curve.

If $\delta = 0$, then we get the familiar price level indeterminacy (that, for example, Sargent and Wallace wrote about in their JPE paper). In solving for p_t in part B, we would have $c = 1$, and we could not apply the no speculative bubbles criterion.

Question 1:

Part A:

Taking expectations of (1) conditional on the wage setters information,

$$m_{t|t-1} = \pi^*$$

As shown in class, the F.O.C for the Fed's maximization problem is

$$(1 + u\alpha^2)(m_t - m_{t|t-1}) - n^* + u\alpha m_{t|t-1} + u\alpha x_t - u\alpha\pi^* = 0$$

or using (11),

$$(1 + u\alpha^2)(m_t - \pi^*) - n^* + u\alpha\pi^* + u\alpha x_t - u\alpha\pi^* = 0$$

and so

$$m_t^c = \pi^* + n^*/(1 + u\alpha^2) - [u\alpha/(1 + u\alpha^2)]x_t$$

Part B:

Using the reduced forms and the utility function:

$$n_t^c = m_t - m_{t|t-1} = n^*/(1 + u\alpha^2) - [u\alpha/(1 + u\alpha^2)]x_t$$

$$\begin{aligned} \pi_t^c &= m_{t|t-1} + \alpha(m_t - m_{t|t-1}) + x_t \\ &= \pi^* + \alpha n^*/(1 + u\alpha^2) + x_t/(1 + u\alpha^2) \end{aligned}$$

So,

$$\begin{aligned} EU_t^c &= - [u\alpha^2/(1 + u\alpha^2)]^2 n^{*2} - u\alpha^2/(1 + u\alpha^2)^2 - [(u^2\alpha^2 + u)/(1 + u\alpha^2)^2] \sigma_x^2 \\ &= - [u\alpha^2/(1 + u\alpha^2)] n^{*2} - [(u^2\alpha^2 + u)/(1 + u\alpha^2)^2] \sigma_x^2 \\ &= EU_t^o + n^{*2}/(1 + u\alpha^2) \end{aligned}$$

Part C:

Yes. A new inflation bias has been introduced (the second term on the LHS of (14)). (The stabilization (or x_t) term is the same as before.) Since the wage setters do not build inflationary expectations (in excess of π^*) into their wages, the Fed has an incentive to inflate the real wage down and achieve a higher n_t . Only in the discretionary solution is this temptation eliminated. For this reason, the “optimal” solution is not credible.

Question 2:

Part A: Show that the reduced forms for p_t and y_t are

$$(4) p_t = m_{t|t-1} + v_{t-1} + \frac{1}{2}(m_t - m_{t|t-1}) + \frac{1}{2}\epsilon_t$$

$$(5) y_t = \frac{1}{2}(m_t - m_{t|t-1}) + \frac{1}{2}\epsilon_t$$

$$(1) \& (2) \Rightarrow (6) m_t + v_t = p_t + (p_t - p_{t|t-1})$$

$$\text{taking expectations: } (7) m_{t|t-1} + v_{t|t-1} = p_{t|t-1} + 0$$

$$\text{subtracting: } (m_t - m_{t|t-1}) + (v_t - v_{t|t-1}) = (p_t - p_{t|t-1}) + (p_t - p_{t|t-1}) = 2(p_t - p_{t|t-1})$$

$$\text{so: } (8) p_t - p_{t|t-1} = \frac{1}{2}[(m_t - m_{t|t-1}) + (v_t - v_{t|t-1})] = \frac{1}{2}[(m_t - m_{t|t-1}) + \epsilon_t]$$

$$\text{and: } (1) \& (8) \Rightarrow (5)$$

$$(7) \& (8) \Rightarrow (4), \text{ since } v_{t|t-1} = v_{t-1}.$$

Part B: Show that contemporaneous feedback rules for m_t can reduce the variance of y_t to zero, while lagged feedback rules can not.

let (9) $m_t = \alpha\epsilon_t + \beta v_{t-1}$ be the monetary policy rule.

$$(9) \Rightarrow m_{t|t-1} = \beta v_{t-1} \text{ and } m_t - m_{t|t-1} = \alpha\epsilon_t$$

so,

$$(4)' p_t = m_{t|t-1} + v_{t-1} + \frac{1}{2}(m_t - m_{t|t-1}) + \frac{1}{2}\epsilon_t = (\beta + 1)v_{t-1} + \frac{1}{2}(\alpha + 1)\epsilon_t$$

$$(5)' y_t = \frac{1}{2}(m_t - m_{t|t-1}) + \frac{1}{2}\epsilon_t = \frac{1}{2}(\alpha + 1)\epsilon_t$$

letting $\alpha = -1$, $y_t = 0$ and $\text{var}(y_t)$ (however defined) = 0.

Parameter β has no effect on y_t or $\text{var}(y_t)$, and v_{t-1} is a summary statistic for lagged information.

Part C: Is there a feedback rule (using contemporaneous and/or lagged feedback) that will reduce both the variance of y_t and the variance of p_t to zero?

yes. Letting $\alpha = \beta = -1$, (4)' & (5)' $\Rightarrow y_t = p_t = 0$.

Part D: Suppose in the model above the central bank chooses m_t to maximize

$$(6) U_t = - (y_t - \bar{y})^2 - \mu p_t^2, \quad \bar{y} > 0.$$

In the discretionary solution, how does the central bank's policy rule compare with the one you found in Part C? Does it stabilize p_t and y_t ? What is the Barro-Gordon inflation (or price) bias?

From (4) & (5)

$$U_t = - (y_t - \bar{y})^2 - \mu p_t^2 \\ = - [1/2(m_t - m_{t|t-1}) + 1/2\epsilon_t - \bar{y}]^2 - \mu[m_{t|t-1} + v_{t-1} + 1/2(m_t - m_{t|t-1}) + 1/2\epsilon_t]^2$$

the central bank's first order condition is

$$dU_t/dm_t = - 2[1/2(m_t - m_{t|t-1}) + 1/2\epsilon_t - \bar{y}]^{1/2} - 2\mu[m_{t|t-1} + v_{t-1} + 1/2(m_t - m_{t|t-1}) + 1/2\epsilon_t]^{1/2} = 0 \text{ or} \\ 1/2(1 + \mu)(m_t - m_{t|t-1}) + 1/2(1 + \mu)\epsilon_t - \bar{y} + \mu m_{t|t-1} + \mu v_{t-1} = 0$$

the private sector takes expectation of this (conditional on t-1 information) to find $m_{t|t-1}$

$$(10) m_{t|t-1} = (\bar{y}/\mu) - v_{t-1}$$

using (10) in the first order condition, we find the money supply rule

$$(11) m_t = m_{t|t-1} - \epsilon_t = (\bar{y}/\mu) - v_{t-1} - \epsilon_t$$

using (11) in (4) & (5), we find p_t and y_t

$$(4)'' p_t = m_{t|t-1} + v_{t-1} + 1/2(m_t - m_{t|t-1}) + 1/2\epsilon_t = (\bar{y}/\mu - v_{t-1}) + v_{t-1} - 1/2\epsilon_t + 1/2\epsilon_t = \bar{y}/\mu$$

$$(5)'' y_t = 1/2(m_t - m_{t|t-1}) + 1/2\epsilon_t = - 1/2\epsilon_t + 1/2\epsilon_t = 0$$

So, the money supply rule (11) stabilizes y_t and p_t perfectly; that is, they have zero variance. In this sense (11) is just like the policy rule in Part C; however, (11) has a Barro-Gordon inflation bias equal to \bar{y}/μ .

Part E: If Rogoff's approach to lowering the inflation bias (increasing μ) is applied in this model, is the stabilization effort distorted? Why do you think the literature on the "credibility-stabilization" tradeoff focus on productivity shocks?

No, we can raise μ (and lower the inflation bias) without any distortion of the stabilization effort. From, (4)'' and (5)'', we see that p_t and y_t are fully stabilized (in the sense that their variances are reduced to zero) for all values of μ .

This happens because we have only included velocity shocks. There is no tradeoff between inflation and output goals for velocity shocks: accommodating them stabilizes both inflation and output. Productivity shocks (the way I modeled them in class) are pure inflation shocks; curbing inflation means creating more unemployment. With productivity shocks, there is a tradeoff between inflation and output goals, and choosing

a central bank governor with perverse preferences (a μ that is too large) will cause him/her to favor inflation stabilization over output stabilization. Credibility (a lower inflation bias) comes at the expense of distorting the stabilization effort; there is a tradeoff (via Rogoff's solution) between credibility and stabilization.

Part F: Walsh showed that the inflation bias would be eliminated if a linear inflation penalty was imposed on the central bank. In this model, would a linear penalty on the level of output accomplish the same goal? Derive your answer mathematically, then explain your result intuitively. Why do you think Walsh concentrated on inflation penalties instead of output penalties?

Adding the penalty, ωy_t , and proceeding as above,

$$\begin{aligned} U_t &= - (y_t - \bar{y})^2 - \mu p_t^2 - \omega y_t \\ &= - [1/2(m_t - m_{t|t-1}) + 1/2\epsilon_t - \bar{y}]^2 - \mu [m_{t|t-1} + v_{t-1} + 1/2(m_t - m_{t|t-1}) + 1/2\epsilon_t]^2 - \omega [1/2(m_t - m_{t|t-1}) + 1/2\epsilon_t] \end{aligned}$$

the central bank's first order condition is

$$\begin{aligned} dU_t/dm_t &= -2[1/2(m_t - m_{t|t-1}) + 1/2\epsilon_t - \bar{y}] - 2\mu [m_{t|t-1} + v_{t-1} + 1/2(m_t - m_{t|t-1}) + 1/2\epsilon_t] - 1/2\omega \text{ or} \\ 1/2(1 + \mu)(m_t - m_{t|t-1}) + 1/2(1 + \mu)\epsilon_t - \bar{y} + \mu m_{t|t-1} + \mu v_{t-1} + 1/2\omega &= 0 \end{aligned}$$

the private sector takes expectation of this (conditional on t-1 information) to find $m_{t|t-1}$

$$(10) \quad m_{t|t-1} = (\bar{y}/\mu) - v_{t-1} - 1/2\omega/\mu$$

using (10) in the first order condition, we find the money supply rule

$$(11) \quad m_t = m_{t|t-1} - \epsilon_t = (\bar{y}/\mu) - 1/2\omega/\mu - v_{t-1} - \epsilon_t$$

using (11) in (4) & (5), we find p_t and y_t

$$(4)'' \quad p_t = m_{t|t-1} + v_{t-1} + 1/2(m_t - m_{t|t-1}) + 1/2\epsilon_t = \bar{y}/\mu - 1/2\omega/\mu$$

$$(5)'' \quad y_t = 1/2(m_t - m_{t|t-1}) + 1/2\epsilon_t = 0$$

Setting $\omega = 2\bar{y}$, we eliminate the inflation bias. The intuition is the same as for the inflation penalty: at the optimal solution, the Barro-Gordon model gives the central bank a constant (or state independent) incentive to inflate (or expand output). A linear penalty on output (or inflation) can just offset that marginal incentive to expand.

I suspect Walsh talked of inflation penalties (instead of output penalties) because that has a more natural interpretation when presenting the plan to the body politic for acceptance.

Why should the central bank be penalized for increasing output and employment?

Penalizing the central bank for inflation seems natural, like giving it the right incentives to do its institutional job.

In this exercise, you will study a typical “optimal taxation” problem. Suppose the government fights the “war to end all wars” in period t ; then, in all subsequent periods, there is no more government spending: $g_t = g$ and $g_{t+j} = 0$ for all $j > 0$. (We get to make silly assumptions in macroeconomic theory.) Here you will show that it is optimal to spread the burden of financing this war across the taxes that are available and over time; a balanced budget amendment would be economically inefficient. There are high and low tax ways of financing the war, but you will show that the low tax way is best. You will also show that the optimal tax plan is not time consistent. Finally, you will show that the Laffer Curve disappears for some utility functions.

Part A: Let $u(c_j, x_j) = c_j + g_j + x_j - \frac{1}{2}x_j^2 - \frac{1}{2}(m_j - m^*)^2$.

The household values public and private spending equally. (Another silly assumption that simplifies the algebra.) Show that in equilibrium:

- (i) $R_j = 1/\beta$, $\alpha_j = \beta^{j-t}$, $n_j = 1 - \tau_j$, $m_j = m^* - i_j/I_j$, $x_j = \tau_j$, $c_j = 1 - \tau_j - g_j$.
- (ii) $g = \sum_{j=t}^{\infty} \beta^{j-t} [\tau_j(1-\tau_j) + (i_j/I_j)(m^* - (i_j/I_j))]$ (assuming no lump sum taxes, and that $w_t = 0$)
- (iii) $U_t = \sum_{j=t}^{\infty} \beta^{j-t} [1 - \frac{1}{2}\tau_j^2 - \frac{1}{2}(i_j/I_j)^2]$

ANSWER –

This is very similar to Exercise 1, which we did in class.

Part B: Suppose the government is “benevolent” in the sense that it chooses the taxes $\{\tau_j\}$ and $\{i_j/I_j\}$ to maximize household utility, U_t . Assume (to simplify the algebra) that $m^* = 1$.

- (i) Show that a balanced budget is not optimal in this case; show that it is optimal to set $\tau_j = \tau$ (a constant) and $i_j/I_j = i/I$ (a constant), and spread the “costs” of financing g over time.

ANSWER –

The government maximizes U_t s.t. PVBC (ii) above:

$$L = \sum_{j=t}^{\infty} \beta^{j-t} [1 - \frac{1}{2}\tau_j^2 - \frac{1}{2}(i_j/I_j)^2] - \theta \{g - \sum_{j=t}^{\infty} \beta^{j-t} [\tau_j(1-\tau_j) + (i_j/I_j)(1 - (i_j/I_j))]\}$$

$$\partial L / \partial \tau_j = \beta^{j-t} [-\tau_j + \theta(1 - 2\tau_j)] = 0 \Rightarrow \tau_j = \theta / (1 + 2\theta) = \tau \text{ for all } j$$

$$\partial L / \partial (i_j/I_j) = \beta^{j-t} [-(i_j/I_j) + \theta(1 - 2(i_j/I_j))] = 0 \Rightarrow i_j/I_j = \theta / (1 + 2\theta) = i/I \text{ for all } j$$

(θ has to be > 0 if the budget constraint is binding, which we see it is.)

Note that $i/I = \tau!$ This is due to the extreme symmetry of our example. In general, it will not be optimal to equalize all of the tax rates.

(ii) Show that the optimal τ and i/I must satisfy:

$$g = (1-\beta)^{-1}(\tau - \tau^2) + (1-\beta)^{-1}(i/I - (i/I)^2) \quad \text{and} \quad \tau = i/I = \frac{1}{2}[1 \pm \sqrt{1 - 2(1-\beta)g}]$$

This is the optimal way of spreading the tax burden between taxes.

ANSWER –

The PVBC must be satisfied:

$$\begin{aligned} g &= \sum_{j=t}^{\infty} \beta^{j-t} [\tau_j(1-\tau_j) + (i_j/I_j)(1 - (i_j/I_j))] = \sum_{j=t}^{\infty} \beta^{j-t} [2\tau(1-\tau)] \quad (\text{since } i/I = \tau \text{ in this example}) \\ &= 2\tau(1-\tau) \sum_{j=t}^{\infty} \beta^{j-t} = 2\tau(1-\tau)/(1-\beta) \end{aligned}$$

$$\text{so, } \frac{1}{2}g = (1-\beta)^{-1}(\tau - \tau^2) \quad \text{and} \quad \tau = \frac{1}{2}[1 \pm \sqrt{1 - 2(1-\beta)g}]$$

(iii) In (ii) above you found high and low tax ways of financing g . Which will the benevolent government choose. (Hint: Use Part A(iii) to show that $dU_t/d\tau < 0$.)

ANSWER –

$$U_t = \sum_{j=t}^{\infty} \beta^{j-t} [1 - \frac{1}{2}\tau^2 - \frac{1}{2}(i/I)^2] \Rightarrow dU_t/d\tau = -(1-\beta)^{-1}\tau < 0 \quad \text{and} \quad dU_t/d(i/I) = -(1-\beta)^{-1}(i/I) < 0.$$

So, U_t decreases with the size of the tax rates, and the lower rates are utility maximizing.

Part C: Show that the optimal tax plan is time inconsistent. That is, show that in period $t+1$, a benevolent government would choose to default on its debt.

ANSWER –

$$U_{t+1} = \sum_{j=t+1}^{\infty} \beta^{j-(t+1)} 2(1 - \frac{1}{2}\tau^2) = 2(1 - \frac{1}{2}\tau^2)/(1-\beta) \quad (\text{since } \tau = i/I)$$

If the government defaults on the debt, and sets $\tau_j = i_j/I_j = 0$ for $j = t+1, t+2, \dots$, then

$$U_{t+1} = \sum_{j=t+1}^{\infty} \beta^{j-(t+1)} 2(1 - \frac{1}{2} \cdot 0^2) = 2/(1-\beta), \text{ which yields a higher utility than continuing with the plan.}$$

Of course, this incentive to renege on the debt creates credibility problems in period t .

Part D: How would the availability of lump sum taxes change your answers in Part B? You need not redo all of the mathematics; just give an intuitive, verbal answer. Relate your discussion to the notion of “Ricardian Equivalence.”

ANSWER –

With lump sum taxes, $\bar{\tau}_j$, we have Ricardian Equivalence, and it would not matter when the taxes were levied; that is, the values of the endogenous variables would be the same for any sequence $\{\bar{\tau}_j\}$ for which $g = \sum_{j=t}^{\infty} \beta^{j-t} \bar{\tau}_j$. If lump sum taxes were available, the government would want to use them to finance g , since $dU_t/d\tau < 0$ and $dU_t/d(i/I) < 0$.

Part E: Seigniorage revenues are quite small in most OECD countries. Suppose you want to ignore the seigniorage consequences of a change in monetary policy (and eliminate all of the rather tedious algebra that goes along with it). Can you think of a modeling trick that will eliminate the budgetary consequences of a change in monetary policy? Hint: let money enter utility logarithmically: $u(c_j, x_j) = c_j + g_j + x_j - \frac{1}{2}x_j^2 + \log(m_j)$. What happens to the government’s PVBC (the equation (ii) above) with this utility function? Is there still a Laffer Curve for the seigniorage tax? If not, why not?

ANSWER –

With this utility function, we have as before that

$$R_j = 1/\beta, \quad \alpha_j = \beta^{j-t}, \quad n_j = 1 - \tau_j, \quad x_j = \tau_j, \quad c_j = 1 - \tau_j - g_j.$$

But now, the first order condition for money balances implies: $m_j = I_j/i_j$

$$\text{So, the PVBC becomes: } g = \sum_{j=t}^{\infty} \beta^{j-t} [\tau_j n_j + (i_j/I_j) m_j] = \sum_{j=t}^{\infty} \beta^{j-t} [\tau_j (1 - \tau_j) + 1]$$

There is no Laffer Curve for the seigniorage tax. All values of i_j/I_j yield the same revenue.

Why is this? When the tax rate, i_j/I_j , goes up, the tax base, m_j , falls enough to leave tax revenues, $(i_j/I_j)m_j$, unaffected.

This is a good trick to know!

First, you will make the Fiscal Theory of Price Determination more operational by expressing the budget constraint in terms of variables that are normalized on GDP; then, you work a policy experiment; finally, you will show that the Maastricht Treaty would insure a Ricardian regime in Europe.

Consider the model:

- | | |
|---|----------------------------|
| (1) $U_t = \sum_{j=0}^{\infty} \beta^{j-t} \log(c_j)$ | Utility function |
| (2) $M_j^p = P_j c_j$ and $M_j^g = P_j g_j$ | Cash in Advance |
| (3) $P_{j-1} y_{j-1} + I_{j-1} B_{j-1} = M_j^p + B_j + T_j$ | Household B.C. |
| (4) $c_j + g_j = y_j$, $M_j = M_j^p + M_j^g$ | Equilibrium conditions |
| (5) $y_{j+1} = (1+n)y_j$ and $g_j = \eta y_j$
(where n and η are constants) | Exogenous growth in output |

Part A: Reformulate the Government Budget Constraints. Show that:

- | | |
|---|----------------------|
| (6) $w_j = s_j + \rho_j w_{j+1}$ | Flow Government B.C. |
| (7) $w_t = \sum_{j=t}^{\infty} \alpha_j s_j \quad \Leftrightarrow \quad \lim_{T \rightarrow \infty} \alpha_{t+T} w_{t+T} = 0$ | PV Government B.C. |

where $w_j = (M_j + I_{j-1} B_{j-1})/P_j y_j$, $s_j = (T_j - G_j)/P_j y_j + (M_j/P_j y_j)[i_j/(1+i_j)]$,

$$\rho_j = (1+n)/(1+r_j), \alpha_j = \rho_t \cdots \rho_{j-1}, \text{ and } \alpha_t = 1.$$

Note: s_j is the surplus to GDP ratio; w_j is the (total) government debt to GDP ratio.

Part B: Derive the household's first order conditions, and show that in equilibrium

- | |
|---|
| (8) $\rho_j = \beta$ |
| (9) $w_t = \sum_{j=t}^{\infty} \beta^{j-t} s_j$ |

Part C: The Maastricht Treaty's deficit criterion (which lives on in the Growth and Stability Pact) stipulates that the total deficit to GDP ratio can not exceed 3%. (Total deficit = primary deficit + interest on the debt.) Suppose this rule becomes binding throughout Euroland; that is, (ignoring the fact that our w includes M as well as B):

$$(10) -s_j + i_j w_j = .03$$

Suppose further that $\beta \equiv 1/(1+\delta) = 1/(1+.03) = \rho_j$, real growth is expected to be 1% (or .01), and inflation is also expected to be 1% (or .01).

i. Use these numbers to approximate any parameters you need; for example, show that the nominal interest rate can be approximated by $i_j = .05$.

ii. Show that, under all these assumptions, the fiscal policy rule (10) results in a R regime.

Part C: Suppose US fiscal policy is determined by (10), and the Fed sets $M_j = \bar{M}$, a constant.

i. Is the price level determined? If so, what is the reduced form for P_t ? And what is the equilibrium rate of inflation?

ii. Can the Fed determine the rate of inflation in this economy (assuming (10) is in force)?

What happened to Sargent & Wallace's "monetarist arithmetic".

Part A: Derive the household's first order conditions, and show that in equilibrium

$$(8) \rho_j = \beta$$

$$(9) w_t = \sum_{j=t}^{\infty} \beta^{j-t} s_j$$

ANSWER --

use the CIA constraint (which we assume is binding) to eliminate M_{j+1}^p

$$L = \sum_t \beta^{t-1} \{ \log(c_t) + \lambda_j [P_{j-1} y_{j-1} + B_j - (P_j c_j + B_{j+1}/I_t + T_j)] \}$$

$$\frac{\partial L}{\partial c_t} = (1/c_t) - \lambda_t P_t = 0 \quad \left. \vphantom{\frac{\partial L}{\partial c_t}} \right\} \Rightarrow I_t = \beta^{-1} (\lambda_t / \lambda_{t+1}) = \beta^{-1} (P_{t+1} c_{t+1} / P_t c_t)$$

$$\frac{\partial L}{\partial B_{t+1}} = (\lambda_t / I_t) - \beta \lambda_{t+1} = 0 \quad \left. \vphantom{\frac{\partial L}{\partial B_{t+1}}} \right\}$$

$$\text{so, } I_t (P_t / P_{t+1}) = \beta^{-1} (c_{t+1} / c_t) = \beta^{-1} (y_{t+1} / y_t) = \beta^{-1} (1+n) \Rightarrow \rho_j = (1+n)/(1+r_j) = \beta \text{ \& thus (9)}$$

Part B: The Maastricht Treaty's deficit criterion stipulates that the total deficit to GDP ratio can not exceed 3%. (Total deficit = primary deficit + interest on the debt.) Suppose that this rule becomes binding in say Italy; that is, (ignoring the fact that our w includes M as well as B):

$$(10) -s_j + i_j w_j = .03$$

Suppose further that $\beta \equiv 1/(1+\delta) = 1/(1+.03) = \rho_j$, real growth is expected to be 1% (or .01), and inflation is also expected to be 1% (or .01).

i. Use these numbers to approximate any parameters you need; for example, show that the nominal interest rate can be approximated by $i_j = .05$.

ANSWER

$$1/(1+.03) = \beta = \rho_j = (1+n)/(1+r_j) = (1+.01)/(1+r_j) \Rightarrow 1+r_j = 1.01 \cdot 1.03 = 1.0403 \approx 1.04$$

$$\text{So, } r_j \approx .04$$

$$i_j \approx r_j + \pi_j^e = .04 + .01 = .05$$

ii. Show that, under all these assumptions, the fiscal policy rule (10) results in an R regime.

ANSWER

(10) can be written as

$$(10)' \quad s_j = i_j w_j - .03 = .05 w_j - .03$$

We will show that the second statement of (6) is satisfied; that is, we will show that

$$\lim_{T \rightarrow \infty} \alpha_{t+T} w_{t+T} = \lim_{T \rightarrow \infty} \beta^T w_{t+T} = 0$$

Substituting (10)' into (6)

$$\begin{aligned} w_{j+1} &= (1/\rho_j) w_j - (1/\rho_j) s_j = (1/\beta) w_j - (1/\beta) s_j = (1-0.05)/\beta w_j + .03/\beta \\ &= (.95) \cdot 1.03 w_j + (.03 \cdot 1.03) \approx .98 w_j + .03 \end{aligned}$$

so, starting at the current period t:

$$w_{t+1} = .98 w_t + .03$$

$$\begin{aligned} w_{t+2} &= .98 w_{t+1} + .03 = .98(.98 w_t + .03) + .03 \\ &= .98^2 w_t + .03(1 + .98) \end{aligned}$$

....

$$w_{t+T} = .98^T w_t + .03(1 + .98 + .98^2 + \dots + .98^{T-1})$$

and,

$$\begin{aligned} \lim_{T \rightarrow \infty} \beta^T w_{t+T} &= \lim_{T \rightarrow \infty} \beta^T \cdot \lim_{T \rightarrow \infty} w_{t+T} \\ &= \lim_{T \rightarrow \infty} .97^T \cdot \lim_{T \rightarrow \infty} [.98^T w_t + .03(1 + .98 + \dots + .98^{T-1})] \\ &= \lim_{T \rightarrow \infty} .97^T \cdot \lim_{T \rightarrow \infty} (.98^T) \cdot w_t + \lim_{T \rightarrow \infty} .97^T \cdot \lim_{T \rightarrow \infty} (1 + .98 + \dots + .98^{T-1}) \cdot .03 \\ &= \lim_{T \rightarrow \infty} .97^T \cdot \lim_{T \rightarrow \infty} (.98^T) \cdot w_t + \lim_{T \rightarrow \infty} .97^T \cdot [.03/(1 - .98)] = 0 \text{ for any initial } w_t \end{aligned}$$

Part C: Suppose fiscal policy is determined by (9), and the Fed sets $M_j = \bar{M}$, a constant.

i. Is the price level determined? If so, what is the reduced form for P_t ? And what is the equilibrium rate of inflation?

ii. Can the Fed determine the rate of inflation in this economy (assuming (10) is in force)?
What happened to Sargent & Wallace's "monetarist arithmetic".

ANSWER --

i. Yes. (2) & (4) $\Rightarrow M_{t+1} = M_{t+1}^p + M_{t+1}^g = P_t(c_t + g_t) = P_t y_t$

So, $P_t = \bar{M}/y_t$ is the reduced form. (5) & this reduced form for $P_t \Rightarrow \pi = -n$

ii. Yes. $\pi = (\Delta M/M) - n$. Sargent and Wallace assumed $\{s_j\}$ is an exogenous process.
(10) makes the sum always converge, what ever seignorage is; the S&W analysis no longer applies.

Consider Lucas & Fuerst's "limited participation model". Recall the equilibrium conditions for our version of this (which has a different CIA constraint for consumers):

We have four equations in: p_t ($= P_t/M_t$), h_t , w_t ($= W_t/M_t$), I_t , and x_t ($= X_t/M_t$)

- (a) $pf(h,z) = 1 + x$ (this is the cash in advance constraint, (6))
 - (b) $n + x = wh$ (supply = demand for loans from fin. intermediary)
 - (c) $f_1(h,z) = (w/p)I$ (labor demand curve)
 - (d) $u_2(f,1-h) = (w/p)u_1(f,1-h)$ (labor supply curve)
 - (e) $f(h,z) = zh^\alpha$ Fuerst's specification of the production and utility functions
- $u(c,1-l) = \gamma \log(c) + (1-l)$ and utility functions

Part A: A monetary injection produces a "liquidity effect" – $dI/dx < 0$ – in this model. Under what conditions, will a monetary injection raise the price level: $dp/dx > 0$? (Hint: evaluate the derivative at $x = 0$.)

ANSWER –

From Prop. 3, $p(\gamma^\alpha z) = (1 + x)^{1+\alpha}(n + x)^{-\alpha}$

$$d[(1 + x)^{1+\alpha}(n + x)^{-\alpha}]/dx = (1+\alpha)(1+x)^\alpha(n+x)^{-\alpha} - \alpha(1+x)^{1+\alpha}(n+x)^{-(1+\alpha)}$$

and evaluating at $x = 0$

$$= (1+\alpha)n^{-\alpha} - \alpha n^{-(1+\alpha)} = (1+\alpha)n^{-(1+\alpha)}\{n - [\alpha/(1+\alpha)]\}$$

So, $dp/dx > 0$ if $n > \alpha/(1+\alpha)$. I can't really think of any intuition for this result.

Part B: The choice of specific functional forms can sometimes have surprising – and unintended – consequences.

i. In the NNS models we study next, we generally assume a linear production function.

Show that if we replace Fuerst's specification of the production function with $f(h,z) = zh$, we get the same qualitative results: a monetary injection still decreases I and increases h . (Keep Fuerst's specification of the utility function.)

ANSWER –

Essentially, all the proofs are the same as before, with α set equal to 1.

ii. It is puzzling that, in the model we developed in class, the productivity shock does not affect the level of employment, h . In the NNS models we study next, we see that this kind of result is generally associated with a log specification of the utility of consumption. Suppose we replace Fuerst's utility function with a constant elasticity function, $u(c, 1-l) = (1-\gamma)^{-1}c^{1-\gamma} + (1-l)$. (Keep Fuerst's specification of the production function.) Does the productivity shock affect the level of employment? Can you explain the result intuitively? Does a monetary injection still lower I and raise h ?

ANSWER –

$$(c) \ \& \ (d) \ \Rightarrow \ u_2/u_1 = (w/p) = f_1(h,z)/I = (\alpha f/h)/I$$

$$\text{so, } h = (u_1/u_2)(\alpha f/I) = (1/f')(\alpha f/I) = \alpha f^{1-\gamma}/I = \alpha(zh^\alpha)^{1-\gamma}/I \Rightarrow h^{1+\alpha(\gamma-1)} = \alpha z^{-\alpha(\gamma-1)}/I$$

$$h = [\alpha z^{-\alpha(\gamma-1)}/I]^{1/[1+\alpha(\gamma-1)]}$$

Prop. 1 (which did not depend on the form of $u(\cdot, \cdot)$) says $I = \alpha(1+x)/(n+x)$

$$\text{so, } h = [z^{-\alpha(\gamma-1)}(n+x)/(1+x)]^{1/[1+\alpha(\gamma-1)]}$$

If $\gamma > 1$, then $z \uparrow \Rightarrow h \downarrow$. Since households can produce and consume more with the same work effort, they choose to work a little less and consume a little more (instead of taking the whole productivity increase in consumption).

As before, $dI/dx > 0$; the proof is the same as before since Prop. 1 did not depend on the form $u(\cdot, \cdot)$. And since $h = [\alpha z^{-\alpha(\gamma-1)}/I]^{1/[1+\alpha(\gamma-1)]}$, $dh/dx = (dh/dI) \cdot (dI/dx) > 0$.

Question 1: Consider an economy with one flexible wage/price sector (denoted by $s = f$) and one sticky wages/flexible price sector (denoted by $s = w$); productivity shocks in the two sectors are perfectly correlated. To safeguard your mental health, limit discussion to the case where $\gamma = 1$.

Part A: Show that the policy rule $\Omega_t = A_t^{-1/(1+\chi)}$ would bring the entire economy to the flexible wage/price solution.

ANSWER –

The flexible wage/price sector takes care of itself; it is always at the flexible solution.

$$\Omega_t = A_t^{-1/(1+\chi)} \Rightarrow \omega_t = - (1/\psi)a_t \Rightarrow (\text{via Lemma 3, B}) y_{w,t} = y_{w,t}^*$$

Part B: Show that if this policy were implemented, wages in the flexible wage/price sector would converge on the wage set in the fixed wage sector, stabilizing the “aggregate” wage rate.

ANSWER –

$\omega_t = - (1/\psi)a_t \Rightarrow (\text{via Lemma 3,C}) w_{w,t}^* = w_{w,t}$. The policy brings the “notional” wage, $w_{w,t}^*$ in the fixed wage sector equal to the preset wage, $w_{w,t}$. Since both sectors have the same productivity shock, $w_{f,t}^* = w_{w,t}^*$. Wages in both sectors converge on the preset wage in the fixed wage sector; so, this policy stabilizes the aggregate wage (a weighted average of the sectoral wages) at the value set in the fixed wage sector, $w_{w,t}$.

Part C: Use the results of Part B to give an intuitive explanation for how the optimal policy rule brings about the flexible wage/price solution.

ANSWER –

By changing the demand for output, the optimal monetary policy brings what would be the flexible wage in the fixed wage sector ($w_{w,t}^*$) to the actual value that was set at the end of last period ($w_{w,t}$). This makes wage flexibility unnecessary or redundant. The sector is always at its flexible wage/price solution.

Question 2: With log utility of consumption, the Euler equation becomes: $I_t^{-1} = 1/(1+i_t) =$

$\beta E_t[P_t C_t / P_{t+1} C_{t+1}]$. Assume the shocks are i.i.d. and distributed log-normally, and let small letters represent the logs of big letters. Show that this Euler equation can be expressed as an IS curve: $i_t - (E_t[p_{t+1}] - p_t) = r - c_t$ where r is a time invariant intercept term. What is r ?

ANSWER –

$$\begin{aligned} -i_t &= \log(\beta) + (p_t + c_t) - \log E_t[1/P_{t+1} C_{t+1}] \\ &= \log(\beta) + (p_t + c_t) - E_t[p_{t+1}] - E_t[c_{t+1}] + \frac{1}{2} \text{VAR}_t[p_{t+1} + c_{t+1}] \end{aligned}$$

or

$$i_t = -\log(\beta) - (p_t + c_t) + E_t[p_{t+1}] + E_t[c_{t+1}] - \frac{1}{2} \text{VAR}_t[p_{t+1} + c_{t+1}]$$

rearranging, we get:

$$i_t - (E_t[p_{t+1}] - p_t) = r - c_t$$

$$\text{where } r \equiv \log(1/\beta) + E_t[c_{t+1}] - \frac{1}{2} \text{VAR}_t[p_{t+1} + c_{t+1}]$$

Question 3: Consider an economy with one flexible wage/price sector, two fixed price sectors, and four fixed wage/flexible price sectors. (In this economy, there is more wage inertia than price inertia.) Productivity shocks in the fixed price sectors are perfectly correlated (and denoted by $z_{p,t}$), and productivity shocks in the fixed wage/flexible price sectors are perfectly correlated (and denoted by $z_{w,t}$); however, these shocks are not correlated with each other, or the shock in the flexible wage/price sector (denoted by $z_{f,t}$).

Part A: What is the optimal policy rule for this economy? Assume that the rule takes the form:

$$\omega_t = r_f z_{f,t} + r_p z_{p,t} + r_w z_{w,t} + r_a a_t \text{ and find the optimal values of the coefficients } r_f, r_p, r_w \text{ \& } r_a.$$

ANSWER –

Proposition 1A \Rightarrow we want to choose the coefficients to maximize:

$$\begin{aligned} \sum_{s=1}^S E_{t-1} c_{s,t} &= -\text{Slog}(S) + \sum_{s=1}^S E_{t-1} y_{s,t}^* - \frac{1}{2} \sum_{s \in W} \text{VAR}_{t-1}[\omega_t + a_t/\psi] - \frac{1}{2} \sum_{s \in P} \text{VAR}_{t-1}[\omega_t - z_{s,t} + a_t/\psi] \\ &= -\text{Slog}(S) + \sum_{s=1}^S E_{t-1} y_{s,t}^* - \frac{1}{2}(4) \text{VAR}_{t-1}[\omega_t + a_t/\psi] - \frac{1}{2}(2) \text{VAR}_{t-1}[\omega_t - z_{p,t} + a_t/\psi] \end{aligned}$$

Clearly, we want to set $r_f = r_w = 0$; letting ω_t vibrate with these shocks would just increase the two variance terms. So, we want to choose r_p and r_a to minimize:

$$2\text{VAR}[\omega_t + a_t/\psi] + 1\text{VAR}[\omega_t - z_{p,t} + a_t/\psi] =$$

$$\begin{aligned}
2\text{VAR}[\omega_t + a_t/\psi] + \text{VAR}[\omega_t - z_{p,t} + a_t/\psi] &= \\
2\text{VAR}[r_p z_{p,t} + (r_a + \psi^{-1})a_t] + \text{VAR}[(r_p - 1)z_{p,t} + (r_a + \psi^{-1})a_t] &= \\
2r_p^2\text{VAR}[z_{p,t}] + 2(r_a + \psi^{-1})^2\text{VAR}[a_t] + (r_p - 1)^2\text{VAR}[z_{p,t}] + (r_a + \psi^{-1})^2\text{VAR}[a_t] &= \\
\{2r_p^2 + (r_p - 1)^2\}\text{VAR}[z_{p,t}] + 3(r_a + \psi^{-1})^2\text{VAR}[a_t] &
\end{aligned}$$

Clearly, we should set $r_a = -1/\psi$; this eliminates the second variance.

r_p should be set to minimize $\{2r_p^2 + (r_p - 1)^2\}$:

$$d\{2r_p^2 + (r_p - 1)^2\}/dr_p = 4r_p + 2(r_p - 1) = 6r_p - 2 = 0 \Rightarrow r_p = 2/6 = 1/3.$$

So, the optimal monetary policy rule for this economy is: $\omega_t = (1/3)z_{p,t} - a_t/\psi$.

Part B: Can you give an intuitive explanation for size of the coefficients in the optimal policy rule?

ANSWER –

$r_f = 0$ since the flexible wage/price sector takes care of itself; monetary policy doesn't have do anything. $r_w = 0$ since monetary policy should not respond to shocks in fixed wage/flexible price sectors. $r_p = 1$ would get things right in the fixed price sector, but it would mess things up in the fixed wage/flexible price sector. So, a smaller weight is put on the coefficient, a weight reflecting the relative importance of wage and price inertia in the economy.