

Interest Rate Rules and Price Determinacy: the Role of Transactions Services of Bonds*

by

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Abstract

Interest rate rules have been associated with price indeterminacy when they do not respond aggressively enough to inflation. Price indeterminacy is typically associated with indeterminacy of real bond balances, suggesting that the missing element is a meaningful role for government bonds. We assume that government bonds provide arbitrarily small transactions services and show that this can dramatically change the local and global determinacy conditions. In particular, the specification of fiscal policy affects the aggressiveness with which monetary policy must respond to inflation to deliver local determinacy – a range of passive monetary policies, even an interest-rate peg, may yield determinacy.

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I. INTRODUCTION

In standard macroeconomic models, the use of interest rate rules to characterize monetary policy can result in nominal indeterminacy; that is, the model determines the real variables of interest, but fails to pin down the price level. The problem typically arises when the interest rate does not respond aggressively enough to an increase in observed inflation; the classic example is an interest rate peg.¹ In this paper, we show that price indeterminacy is generally associated with an indeterminacy in real bond balances (making the label “nominal indeterminacy” a bit of a misnomer). This fact often goes unnoticed, since most discussions of the price indeterminacy problem assume a Ricardian model; bonds do not “matter”, and the real variables of interest can be determined without any reference to the real value of bonds. This observation suggests that the problem might be resolved by giving bonds a meaningful role in the equilibrium adjustment process.

There are, of course, different ways of introducing non-Ricardian elements – or making bonds matter – in a model. Woodford (1995), for example, considers a model with a non-Ricardian fiscal policy regime.² In this paper, we assume that the fiscal policy regime is Ricardian in the sense of Woodford – our specifications of fiscal policy ensure fiscal solvency, and the “fiscal theory of the price level” does not apply. We make bonds “matter” by assuming that government bonds (and possibly other money market assets as well) provide transactions services. We are certainly not the first to do so.³

Our arguments do not require that bonds be a particularly good substitute for money. We show that a much wider class of interest rate rules – even interest rate pegs – can achieve price determinacy if bonds provide an arbitrarily small amount of transactions services. For this reason, we do not think our assumption will be too controversial.

If government bonds provide transactions services, then fiscal policy – as a supplier of transactions balances – will play a role in price determination. It is the interaction between monetary and fiscal policy that determines whether or not the price level is determined. In Section III, we show how fiscal policy can set a nominal anchor when monetary policy does not. We combine the classic interest rate peg with Schmitt-Grohe and Uribe's (2000) balanced budget rule. Schmitt-Grohe and Uribe show that the price level is not pinned down in standard models. We show that our model has a unique bounded equilibrium (with the price level determined). Our arguments are similar to Calvo and Vegh's (1995), although their model was designed to capture Latin American banking institutions, and not what we usually call fiscal policy.⁴

In Section IV, we assume that fiscal policy is formulated in real terms; it does not set a nominal aggregate. Following Leeper (1991) and others, we combine a monetary policy rule (in which the interest rate responds to observed inflation) with a fiscal rule (in which the real deficit responds to the real level of public-sector debt), and we study the local determinacy of equilibrium. In the standard model (where bonds do not provide transactions services), inflation dynamics are decoupled from government debt dynamics; the interest rate has to respond to changes in observed inflation with an elasticity greater than one to achieve price determinacy. When government bonds provide transactions services, inflation dynamics interact with debt dynamics in a complex way. Using numerical examples, we show that monetary policy can respond less aggressively to inflation and still achieve price determinacy.

In Sections III and IV, fiscal policy plays a very prominent role in determining steady-state inflation. But, this is due to the fact that we have focused on solutions that converge to a

steady state; it is not an implication of our assumption that bonds provide transactions services. In Section V, we consider global determinacy of equilibrium without restricting ourselves to paths that converge to a steady state. In these equilibria, monetary policy can play a much stronger role in determining inflation. In particular, assuming that fiscal policy is disciplined enough to ensure solvency, monetary policy can select the interest rate as its instrument of monetary policy and “target” an inflation rate. In standard models, the price level would not be determined if the central bank set an interest rate and announced an inflation target (or a growth rate of the money supply). In our model, the price level is pinned down.

In Section VI, we summarize our results. We also discuss some of the broader implications of models in which bonds provide transactions services.

II. MODELING TRANSACTIONS SERVICES FOR BONDS

Here, we extend a standard cash-in-advance model to let bonds provide transactions services. By varying a parameter in the cash-in-advance constraint, we can make bonds a very good substitute, or a very poor substitute, for money.

The utility of the representative household is given by:

$$U_t = \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau), \quad (1)$$

where c_τ is consumption, and $u(\cdot)$ satisfies standard assumptions implying an interior solution to the household’s optimization problem. The household produces y units of the consumption good each period, and its consumption purchases are subject to a cash-in-advance constraint:

$$M_\tau + k \left(\frac{B_\tau}{P_\tau y} \right) P_\tau y \geq P_\tau c_\tau, \quad (2)$$

where M_τ and B_τ are the household's money and government bond holdings. We can interpret

$k(\cdot)$ as the fraction of current income, $P_\tau y$, that can be spent in the current period; the fraction depends on the amount of bonds that the household holds.

We make the following assumptions about the function $k(\cdot)$:

$$k(x) = 0 \text{ for } x \leq 0. \quad (\text{A1})$$

$$\text{for } x > 0, k(x) \text{ is twice differentiable with } k'(\cdot) > 0 \text{ and } k''(\cdot) < 0. \quad (\text{A2})$$

$$\lim_{x \downarrow 0} k'(x) \equiv k'(0^+) \leq 1, \quad \lim_{x \rightarrow +\infty} k'(x) = 0, \quad \text{and} \quad \lim_{x \rightarrow +\infty} k(x) < 1. \quad (\text{A2})$$

(A1) says that negative bond holdings provide no transactions services. (A2) and (A3) say that positive bond holdings provide diminishing marginal transactions services, but that these services are never sufficient to finance all purchases. If $k'(\cdot)$ is close to unity, bonds provide almost the same transactions services as money. If $k'(\cdot)$ is close to zero, bonds are a very poor substitute for money.

The household maximizes (1) subject to (2) and the budget constraint:

$$M_{\tau-1} + P_{\tau-1}(y - c_{\tau-1}) + (1+i_{\tau-1})B_{\tau-1} + (1+i_{\tau-1}^*)B_{\tau-1}^* \geq M_\tau + B_\tau + B_\tau^* + T_\tau, \quad (3)$$

where T_τ is a lump-sum tax, and B_τ^* is a bond that does not provide transactions services. (As explained below, we model B^* for reasons of exposition; we do not think that it corresponds to any bond that is traded in the real world.) The household's first-order conditions imply:

$$u'(c_t) = \beta(1 + i_t^*)u'(c_{t+1}) \quad (4a)$$

$$i_t^* - i_t = i_t^* k' \left(\frac{B_t}{P_t y} \right). \quad (4b)$$

Note that i_t^* – the interest rate on B^* – is the interest rate that appears in the Euler Equation (4a). Equation (4b) and the cash-in-advance constraint, (2), govern the allocation of wealth between M_t , B_t and B_t^* . If the household holds one more B_t , then it can hold $k'(\cdot)$ less M_t

and still satisfy the cash-in-advance constraint. $i_t^* - i_t$ is the marginal cost of holding one more B_t (instead of a B_t^*), while $i_t^*k'(\cdot)$ is the marginal benefit of holding $k'(\cdot)$ more B_t^* (instead of M_t). The household will substitute B_t for M_t in the cash-in-advance constraint until diminishing returns drive the marginal benefit, $i_t^*k'(\cdot)$, down to the marginal cost, $i_t^* - i_t$; (4b) gives the stopping point. In the standard model, where $k'(\cdot) = 0$, there is no transactions demand for government bonds, and $i_t^* = i_t$.

How good a substitute is B for M in reality? Answering this empirical question is not as straightforward as it might seem. U.S. Treasury bills clearly facilitate transactions in a number of ways: they serve as collateral in many financial markets, banks hold them to manage the liquidity of their portfolios, and individuals hold them in money-market accounts that offer checking services. If we interpret B as T- bills and B^* as (say) commercial paper, then interest rate spreads could be used as a measure of $i_t^*k'(\cdot)$; the spreads would presumably be small, implying that B is a poor substitute for M. But such a calculation would overlook the fact that commercial paper – and other money-market assets issued by the private sector – provide some of the same transactions services as T- bills; for example, they too are held in money-market accounts with checking privileges.

An alternative approach would be to note that $(1+i_t^*)(P_t/P_{t+1})$ is the CCAPM real interest rate; it could be constructed using price and consumption data in the Euler Equation. We may infer from the equity-premium literature that this approach would imply a very large spread between i_t^* and i_t – essentially, as large as the observed equity premium.⁵ We can not settle these empirical questions here.⁶ However, our arguments in the next three sections of the paper only require that government bonds have *some* liquidity value; both $k(\cdot)$ and $k'(\cdot)$ can be arbitrarily

small. The assertion that government bonds provide some small transactions services would not seem to be controversial.

In equilibrium, $c_t + G_t/P_t = y$, where G_t denotes government purchases. For algebraic simplicity, we will however let $G_t = 0$ and $y = 1$. We will also require that $i_t^* > 0$ in equilibrium, so that (2) is binding. In equilibrium, $B_t^* = 0$ and $c_t = 1$, and our model reduces to:

$$\frac{M_t}{P_t} + k\left(\frac{B_t}{P_t}\right) = 1, \quad (5a)$$

$$1 + i_t^* = (1/\beta)\Pi_t \quad (5b)$$

$$i_t^* - i_t = i_t^* k' \left(\frac{B_t}{P_t} \right), \quad (5c)$$

where $\Pi_t = P_{t+1}/P_t$, and $1/\beta$ is the CCAPM gross real rate of interest.

The government's flow budget constraint can be written as:

$$(1 + i_{t-1})B_{t-1} = T_t - G_t + B_t + (M_t - M_{t-1}). \quad (6)$$

Letting $L_t = M_t + B_t$, the budget constraint becomes:

$$L_t = L_{t-1} + D_t, \quad (7)$$

where $D_t = i_{t-1}B_{t-1} + G_t - T_t$ is the total deficit, inclusive of interest payments.

Both M and B provide transactions services in this model; so, both monetary and fiscal policy play a potential role in price determination. We will refer to $L (= M + B)$ as "total transactions balances", but we note that B's are generally less effective than M's in satisfying the cash-in-advance constraint. So, we will refer to $M + k(B/P)P$ as "effective transactions

balances”. The government budget constraint determines L_t , and open market operations determine the composition of L_t . So, fiscal policy determines total transactions balances, and then monetary policy converts total transactions balances into effective transactions balances. In the next two sections, we will show that fiscal policy can (but need not) provide a nominal anchor when monetary policy does not.

But first, we want to demonstrate the Ricardian nature of the price indeterminacy problem. This demonstration runs off the government budget constraint, and its applicability extends well beyond our present model. Dividing both sides of (7) by P_t , and letting $l_t = L_t/P_t$ and $d_t = D_t/P_t$,

$$L_{t-1}/P_t + d_t = l_t = (M_t/P_t) + (B_t/P_t) \quad (8)$$

Since L_{t-1} is predetermined at time t , any model that fails to pin down P_t will also fail to determine l_t and/or d_t . Suppose for example that fiscal policy determines d_t , and that M_t/P_t is determined in the money market, then price level indeterminacy is synonymous with B_t/P_t indeterminacy. Standard models used to illustrate the price indeterminacy problem are typically Ricardian. Bonds do not “matter”, and the indeterminacy of B_t/P_t does not keep the model from determining the real variables of interest; the problem is sometimes labeled a “nominal indeterminacy”. But, a model (like ours) in which real bonds “matter” must either determine the price level or fail to determine the real variables that depend upon B_t/P_t . The problem is either solved or made much worse. In the next three sections, we will see that the problem is solved for a wide range of monetary and fiscal policies.

III. Price Determinacy When Fiscal Policy Sets a Nominal Aggregate

If government bonds provide liquidity services, then fiscal policy can provide a nominal

anchor when monetary policy does not. Here, we illustrate this principle using the classic example of an interest rate peg. Schmitt-Grohe and Uribe (2000) show how price indeterminacy arises in a standard model if monetary policy pegs the nominal interest rate and fiscal policy adopts a balanced budget rule. We base our discussion on their example. The central bank sets i_t equal to \bar{i} , while fiscal policy sets D_τ equal to zero; the government budget constraint (7) becomes:

$$L_\tau = L_{\tau-1} = \bar{L}, \quad (9)$$

where \bar{L} is a positive constant. In effect, fiscal policy sets total transactions balances each period.

Our model reduces to a standard cash-in-advance model if we let $k(\cdot) = k'(\cdot) = 0$. Then, (5a) implies that $M_t/P_t = 1$, (5c) implies that $i_t^* = \bar{i}$, and (5b) implies that $\Pi_t = (1+\bar{i})\beta$; however, the model does not pin down either P_t or B_t/P_t .⁷ Fiscal policy sets a nominal aggregate $-\bar{L} = M_t + B_t$ – but this does not provide a nominal anchor in the standard model. By contrast, Proposition 1 states that both P_t and B_t/P_t are pinned down if bonds do provide transactions services.

Proposition 1: If $k(\cdot)$ satisfies (A1), (A2) and (A3), there is a unique bounded equilibrium, as long as the central bank's choice of \bar{i} is consistent with a positive equilibrium value of B_t/P_t .

First, we conjecture that a steady-state value for real liabilities, l_t , exists. Since nominal liabilities are constant ($L_t = \bar{L}$), our conjecture implies that Π_t must be equal to 1 in a steady-state solution. Using this fact, (5b) and (5c) imply

$$\bar{i} = \left[1 - k' \left(\frac{B_t}{P_t} \right) \right] \left(\frac{1}{\beta} - 1 \right). \quad (10)$$

We restrict the central bank's choice of \bar{i} to make the equilibrium value of B_t/P_t positive:

$$[1 - k'(0^+)](\beta^{-1} - 1) < \bar{i} < (\beta^{-1} - 1). \quad (11)$$

(A2) and (A3) imply that $k'(\cdot)$ is invertible for positive values of B_t/P_t . So, for \bar{i} within the interval prescribed by (11), (5a) and (10) can be solved implicitly for M_t/P_t and B_t/P_t :

$$\frac{M_t}{P_t} = m(\bar{i}), \quad m'(\cdot) < 0, \quad (12a)$$

$$\frac{B_t}{P_t} = b(\bar{i}), \quad b'(\cdot) > 0. \quad (12b)$$

(12a) and (12b) confirm our conjecture that there is a steady-state solution for real liabilities, and dividing (9) by P_t , the price level is uniquely determined as the solution to:

$$m(\bar{i}) + b(\bar{i}) = \frac{\bar{L}}{P_t}. \quad (13)$$

In appendix I, we show that non-steady-state solutions to our model involve divergent paths (for the price level as well as real money and bond holdings).⁸ So, the steady-state solution is the only equilibrium in the space of bounded sequences. This completes the proof of Proposition 1.

Note that bonds do not need to be a good substitute for money for the price level to be determined. $k(\cdot)$ and $k'(\cdot)$ can be arbitrarily small; all that is required is that they appear in (5a) and (5c), and that they be invertible functions. Note also that both monetary and fiscal policy affect the price level. Our balanced budget rule means that monetary policy cannot affect the level of total transactions balances, \bar{L} ; and this implies that fiscal policy determines steady-state inflation. But, both \bar{L} and \bar{i} appear in (13), which determines the value of P . A higher value of \bar{L} results in a higher price level, and it is not difficult to show that a higher value of \bar{i} results in a lower price level.

IV. Local Determinacy When Fiscal Policy Does Not Set a Nominal Aggregate

In the last section, we assumed that fiscal policy set a nominal aggregate – total transactions balances – and we saw that this could provide a nominal anchor when monetary policy did not. In this section, we assume that fiscal policy is instead formulated in real terms. We will generalize the analysis of the last section in the sense that we let monetary and fiscal policy to be governed by rules that imply dynamic equilibria. But, we will restrict the analysis to looking for local determinacy: the price level will be uniquely determined if the number of eigenvalues outside the unit circle is equal to the number of forward looking variables.

In this section, we rewrite the government budget constraint (8) as:

$$l_t = l_{t-1}/\Pi_{t-1} + d_t, \quad (14)$$

and we assume that fiscal policy is governed by a rule for setting the real deficit:

$$d_t = z_t - \rho l_{t-1}, \quad (15)$$

where z_t captures other economic and political factors. (15) says the real deficit (or deficit-to-GDP ratio, since $y = 1$) responds to movements in total government liabilities, and the parameter ρ measures the strength of this response. Note that setting $\rho = 0$ implies that the real primary surplus must increase in response to an increase in real liabilities (to cover the interest payments).⁹

We assume that monetary policy is governed by a rule for setting the nominal interest rate:

$$1 + i_t = A\Pi_{t-1}^\theta, \quad (16)$$

where A is a constant related to the steady-state interest rate and θ is a parameter that describes how aggressively monetary policy reacts to a rise in observed inflation.

Summarizing, the model consists of (5a), (5b), (5c), the budget constraint (14), and policy rules (15) and (16). In appendix II, we linearize the model around its steady-state equilibrium, and we show that the homogeneous equations for the linearized system can be expressed as:

$$\begin{bmatrix} \hat{l}_t \\ \hat{\Pi}_t \end{bmatrix} = \begin{bmatrix} \frac{1}{\bar{\Pi}} - \rho & -\frac{1}{\bar{\Pi}} \\ \delta_2 \left(\frac{1}{\bar{\Pi}} - \rho \right) & \frac{\theta}{\delta_1} - \frac{\delta_2}{\bar{\Pi}} \end{bmatrix} \begin{bmatrix} \hat{l}_{t-1} \\ \hat{\Pi}_{t-1} \end{bmatrix}, \quad (17)$$

where hats denote percentage deviations from the steady-state,

$$\delta_1 = \frac{\bar{\Pi}[1-k'(\bar{b})]}{\bar{\Pi}[1-k'(\bar{b})] + \beta k(\bar{b})}, \quad \delta_2 = \frac{\bar{l}}{\bar{\Pi}} \cdot \frac{(\bar{\Pi} - \beta)k''(\bar{b})}{[1-k'(\bar{b})]^2},$$

and \bar{b} denotes real bond balances in the steady state. The eigenvalues of (17) are

$$\mu_1, \mu_2 = \frac{1}{2} \left\{ \delta_3 \pm \sqrt{\delta_3^2 - \left(\frac{4\theta}{\delta_1} \right) \left(\frac{1}{\bar{\Pi}} - \rho \right)} \right\}, \quad \text{where} \quad \delta_3 = \frac{1 - \delta_2}{\bar{\Pi}} - \rho + \frac{\theta}{\delta_1}.$$

Since there is one forward looking variable, Π_t , there is a unique, stable solution if one eigenvalue is inside the interval $(-1, 1)$ and the other is outside. If there is a unique stable solution, then the model determines $\Pi_{t-1} = P_t/P_{t-1}$, and since P_{t-1} is given, P_t is determined.¹⁰

Once again, it is instructive to consider the standard model where government bonds do not provide liquidity services. Letting $k(\cdot) = k'(\cdot) = k''(\cdot) = 0$, we have $\delta_1 = 1$ and $\delta_2 = 0$. So, in the standard model, the eigenvalues are $\mu_1 = \bar{\Pi}^{-1} - \rho$ and $\mu_2 = \theta$. Assuming fiscal policy makes μ_1 stable, we get the familiar requirement that (a positive) θ must be greater than one to deliver price determinacy.¹¹ And once again, we can give the price indeterminacy problem a Ricardian inter-pretation. When bonds do not provide liquidity services, the dynamic equation for inflation (coming from the Euler Equation) is decoupled from the dynamics of government debt. Since

we are assuming debt dynamics to be stable, monetary policy has to create an unstable eigenvalue for inflation dynamics if there is to be a unique stable solution to the model. The central bank does this by making the nominal interest rate respond vigorously to any increase in observed inflation. When government bonds do provide liquidity services, inflation dynamics are not decoupled from debt dynamics, and fiscal policy plays a role in creating the unstable eigenvalue.

We begin with a simple case, essentially the analogue of our balanced-budget example in Section III: we let $\rho = 0$, so that fiscal policy sets the real deficit each period (instead of the nominal deficit); we let $\theta = 0$, so that monetary policy continues to peg the interest rate (and from above, the price level would be indeterminate in the standard model); and we let $\bar{d} = 0$, so that $\bar{\Pi} = 1$ (via the budget constraint, (14)). The eigenvalues of (17) are $\mu_1 = 0$ and $\mu_2 = 1 - \delta_2$. Assumptions (A2) and (A3) imply $\delta_2 < 0$; so, $\mu_2 > 1$, and there is a unique stable solution. Here, neither monetary policy nor fiscal policy sets a nominal aggregate, but the price level is pinned down.

Once we allow for non-zero values of ρ and θ , our model implies more complicated inter-actions between monetary and fiscal policy: the values of ρ and θ directly affect the eigenvalues; in addition, the two policies interact indirectly because they both affect the steady-state values of inflation and real liabilities. To illustrate these interactions, we have to resort to a numerical example. We give $k(\cdot)$ a functional form that satisfies the restrictions (A1), (A2) and (A3); letting $b_t = B_t/P_t$, we assume that:

$$k(b_t) = \kappa[1 - \exp(-b_t)], \quad (18)$$

where $0 \leq \kappa < 1$. We want to set the parameter κ so that the implied values of the observable

variables are plausible. To this end, we modify (just for the remainder of this section) our cash-in-advance constraint to allow for a non-unitary velocity of effective transactions balances, V :¹²

$$\frac{M_t}{P_t} + k \left(\frac{B_t}{P_t} \right) = 1 / V, \quad (2a)$$

In effect, we follow Woodford (1991) in assuming that a fixed fraction of current income can be spent in the current period (without holding bonds).

In our benchmark case, we set $\bar{\Pi} = 1.03$, $\bar{d} = 0.02$, $\kappa = 0.2$, and $V = 5$. In the corresponding steady-state solution, $\bar{l} = 0.69$, $\bar{b} = 0.57$, $\bar{m} = 0.11$ and $\bar{i} = 0.0453$; given the simplicity of our model, these benchmark values seem surprisingly reasonable for the U.S. or for a typical EU country. From (18), we calculate $k(\bar{b}) = k'(\bar{b}) = 0.09$, which suggests a very limited role for government bonds in the transactions technology. We also find that the interest rate spread, $\bar{i}^* - \bar{i}$, is only 57 basis points. This too suggests that government bonds are a poor substitute for money in our benchmark case.

In what follows, we let $\theta \in [0, 10]$; these values of θ would seem to encompass the relevant range. We begin by setting $\rho = 0$. The first part of Table 1 (all but the last column) gives the critical values of θ required for a unique stationary solution for different values of \bar{d} and $\bar{\Pi}$; that is, the price level is determined for any value of θ between the reported figure and our arbitrary upper bound of $\theta = 10$. When $\bar{d} = 0.001$ and steady-state inflation is zero ($\bar{\Pi} = 1$), any positive value of θ will do; this is essentially the “simple example” we reported above. As we increase steady-state inflation and/or the steady-state deficit, the critical value of θ tends to rise. In our benchmark solution ($\bar{\Pi} = 1.03$, $\bar{d} = 0.02$), the critical value is 0.75. Indeed, for reasonable values of the parameters, the critical value of θ seems to be about 0.75; the critical

value of θ in the standard model was equal to one.

The strength of the fiscal response to government liabilities also matters. The last column of Table 1 reports critical values of θ for benchmark values of steady-state inflation and the steady-state deficit ($\bar{\Pi} = 1.03$, $\bar{d} = 0.02$) and different values of ρ . The first number in a cell is the value of ρ , and the second number is the critical value for θ . As ρ rises from 0 to 0.01, the critical value of θ increases from 0.75 to 0.79; as ρ falls to -0.01, the critical value of θ decreases to 0.68. In other words, as the fiscal response to liabilities is strengthened, monetary policy has to respond more vigorously to inflation to create the unstable eigenvalue that is required for price determinacy.

Our model is probably too simple for us to have much confidence in specific numerical values. However, we think that our analysis in this section does establish two basic facts: (1) In a model in which government bonds provide transactions services, the minimum value of θ required for price determinacy is less than 1; monetary policy can respond less aggressively to inflation and still achieve price determinacy. (2) In a model in which government bonds provide transactions services, the minimum value of θ depends upon the dynamics of government debt; if fiscal policy responds more aggressively to changes in government debt, then monetary policy has to respond more aggressively to inflation to achieve price determinacy.

V. Global Determinacy in an “Inflation Targeting” Framework

In the last two sections, fiscal policy played a prominent role in determining steady-state inflation. This fact is not due to our assumption that bonds provide transactions services. The government budget constraint, (14), implies that $(\bar{\Pi}-1)/\bar{\Pi} = \bar{d}/\bar{l}$ in any steady state equilibrium. In this section, we consider the global determinacy of equilibrium without restricting our

discussion to equilibrium paths that converge to a steady state. In non-stationary equilibria, monetary policy can play a strong role in determining inflation.

We will illustrate this principle in an inflation targeting framework that would exhibit the price indeterminacy problem in a standard model. In particular, we will show that monetary policy can select the interest rate as its instrument of monetary policy and “target” a wide range of inflation rates, provided that fiscal policy is “disciplined”. In standard models (where government bonds do not provide transactions services), the price level would not be determined if the central bank set an interest rate and announced an inflation target (or a growth rate for the money supply). In our model, the price level will be pinned down.

We do have to put some restrictions on the choice of monetary and fiscal policy. Letting the symbols d , i , i^* , P , Π , B , and L (without subscripts) denote sequences that start at date t , we require:

$$d \equiv \{d_\tau\}_{\tau=t}^\infty \in \ell_\infty \quad \text{and} \quad \|d\|_\infty < 1 \quad (19)$$

$$\pi \equiv \{\pi_\tau\}_{\tau=t}^\infty \in \ell_\infty \quad \text{and} \quad \pi_\tau > \beta. \quad (20)$$

Recalling that output has been normalized to one, (19) says that the deficit to GDP ratio is bounded, and always less than one. This assumption rules out Ponzi games and assures us that fiscal policy balances the government present value budget constraint for any choice of monetary policy. (20) says that the central bank’s inflation target is bounded, and (in light of (5b)) high enough that the CCAPM interest rate is always positive. With these restrictions, we are ready to state our result.

Proposition 2: Assume $k(\cdot)$ satisfies (A1) to (A3). Let d be any fiscal policy sequence satisfying (19), let L_{t-1} be any positive initial debt level, let Π be any inflation sequence satisfying

(20), and let i_t be almost any initial interest rate satisfying:

$$(A4) \quad [1 - \kappa'(0^+)](\beta^{-1}\Pi_\tau - 1) < i_\tau < \beta^{-1}\Pi_\tau - 1.$$

Given the central bank's choice of Π and i_t , equations (5a), (5b), (5c), (7) or (8) uniquely determine the initial price level P_t , the interest rate sequences i and i^* , the money sequence M , and the bond sequence B ; moreover, $P_\tau > 0$, $i_\tau > 0$, $i_\tau^* > 0$ and $M_\tau > 0$ for all $\tau \geq t$.

We prove the proposition by construction. In *Step 1* of the proof, we find the initial (period t) values of the equilibrium sequences. The central bank's choice of Π_t uniquely determines i_t^* via (5b), and (20) implies that i_t^* is positive. The restriction on i_t in (A4) implies that (5c) can be inverted to give $\frac{B_t}{P_t} = k'^{-1}\left(1 - \frac{i_t}{i_t^*}\right) > 0$. Given real bond balances, (5a) determines a unique $\frac{M_t}{P_t} > 0$. Given real money and bond balances, (8) determines an initial real debt level, $\frac{L_{t-1}}{P_t} = \frac{M_t}{P_t} + \frac{B_t}{P_t} - d_t$. Since $k(0) = 0$ and $k'(\cdot) < 1$, $k\left(\frac{B_t}{P_t}\right) < \frac{B_t}{P_t}$. So, (5a) implies $\frac{M_t}{P_t} + \frac{B_t}{P_t} > 1$. This fact, together with the requirement that $d_t < 1$, implies that $\frac{L_{t-1}}{P_t} > 0$, and since $L_{t-1} > 0$, the initial price level is uniquely determined and strictly positive. Finally, (7) determines L_t . All of the initial (period t) variables are uniquely determined and strictly positive.

In *Step 2* of the proof, we find the $t+1$ values of the equilibrium sequences. The central bank's choice of Π_t determines P_{t+1} , and its choice of Π_{t+1} determines i_{t+1}^* (via an updated (5b)); restriction (20) implies that $i_{t+1}^* > 0$. Then, updated versions of (7) and (8) give L_{t+1} . Updating (5a), M_{t+1} must satisfy $\frac{M_{t+1}}{P_{t+1}} + k\left(\frac{L_{t+1} - M_{t+1}}{P_{t+1}}\right) = 1$. Under our assumptions about $k(\cdot)$, the LHS of this equation is strictly monotone increasing in M_{t+1} ; moreover, the LHS tends to $\pm \infty$ with M_{t+1} . So, the updated version of (5a) determines a unique M_{t+1} ; since $0 \leq k(\cdot) < 1$, $M_{t+1} > 0$. The bond supply is determined, since $B_{t+1} = L_{t+1} - M_{t+1}$. If $B_{t+1} \neq 0$ (so that $k'(\cdot)$ is well defined), then an updated (5c) uniquely determines i_{t+1} . Since $0 \leq k'(\cdot) < 1$, $i_{t+1} > 0$. So, all of the period $t+1$

variables are uniquely determined, and bear the appropriate signs, as long as $B_{t+1} \neq 0$. We avoid the technical complications associated with $B_{t+1} = 0$ by making sure that $B_{t+1} \neq 0$ in equilibrium.¹³ In appendix III, we show that this can be done by ruling out a specific value of i_t , the initial interest rate.

Repeating step 2 for dates $t+2$ and beyond, we derive unique equilibrium sequences for all of the endogenous variables. In each period, we must make sure that bonds are not equal to zero, and this requires us to rule out another specific value of i_t ; so, there is a countable set of initial interest rates that the central bank can not choose. This accounts for the “almost any” restriction in the statement of the proposition, and concludes our proof.

VI. Conclusion

In this paper, we emphasized the Ricardian nature of the price indeterminacy problem that has been associated with interest rate policy rules. Cushing (1999) showed that making consumers “finite lived” would not resolve the problem; he argued that introducing Non-Ricardian elements into the consumer’s decision problem was not sufficient to resolve the problem. However, we showed that price indeterminacy implies the indeterminacy of real bond balances in a large class of macro-economic models. This implies that any Non-Ricardian model – that is, any model in which the real value of bonds “matters” – must either determine the price level or else fail to determine all the real variables that depend on real bond balances. Making bonds “matter” either solves the problem or makes it much worse.

We made bonds matter by assuming that bonds were imperfect substitutes for money in a cash-in-advance constraint; we showed that this modification of an otherwise standard model can pin down the price level for a wide class of monetary and fiscal policies, including interest rate

pegs. In particular, we showed that if government bonds provide transactions services, (1) fiscal policy can (but need not) provide a nominal anchor when monetary policy does not, (2) interest rate rules do not have to respond as aggressively to movements in observed inflation to achieve price determinacy, but (3) the required interest rate response increases with the strength of the fiscal response to movements in government liabilities.

Bonds do not have to be a good substitute for money for our resolution of the price indeterminacy problem to work. If, on the other hand, bonds are a reasonably good substitute for money, our model has a number of other implications for monetary theory and policy. In particular, Canzoneri, Cumby and Diba (2002) have shown that the CCAPM interest rate is negatively correlated with the federal funds rate. This is a challenge for standard monetary policy models (which equate the two interest rates). However, our model can in principle face this challenge; it implies that a monetary contraction will decrease the spread between the CCAPM rate and the instrument of monetary policy. The practical relevance of this point, however, hinges on how good a substitute bonds actually are for money, and this remains an open empirical issue.

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Table 1: Critical values for θ

$\bar{\Pi}$ \backslash \bar{d}	0.001	0.01	0.02	0.03	ρ
1.00	0.00	0.00	0.00	0.00	-0.01, 0.68
1.01	0.69	0.67	0.81	0.90	0.00, 0.75
1.03	0.74	0.73	0.75	0.79	0.01, 0.79
1.05	0.75	0.74	0.75	0.77	0.05, 0.84
1.10	0.76	0.76	0.76	0.77	0.10, 0.86

Endnotes:

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1. This a very old problem in monetary theory, with roots that trace back to Wicksell ([1898], 1965). The “rational expectations” incarnation of the problem is due to Sargent and Wallace (1975), which spawned a large literature. Papers that have proposed resolutions of the multiplicity problem include Canzoneri, Henderson and Rogoff (1983) and McCallum (1981). More recent discussions include: Leeper (1991) Woodford (1995), Kerr and King (1996), Cushing (1999), Schmitt-Grohe and Uribe (2000), and Alstadtheim and Henderson (2003).

Benhabib Schmitt-Grohe and Uribe (2001), among others, emphasize the theoretical distinction between nominal indeterminacy (of the price level) and real indeterminacy (of the inflation rate and other real variables). Carlstrom and Fuerst (2001) suggest that “good” monetary policy should deliver real determinacy, while discounting the problem of nominal indeterminacy on the grounds that it has no welfare effect.

2. Woodford (1995) defines a *Ricardian* fiscal regime as one in which, for any path of the price level might take, the fiscal authority adjusts its primary surpluses to ensure fiscal solvency. In his *Non-Ricardian* regime, primary surpluses do not necessarily adjust to ensure fiscal solvency for every possible path of prices; in this regime, the equilibrium path of prices may be determined by the requirement of fiscal solvency.

3. For example, Patinkin (1965) presents a model in which both money and bonds appear in the household utility function. More recently, Bansal and Coleman (1996) used the approach to study the equity premium puzzle and related issues. See also Calvo and Vegh (1995).

4. According to Calvo and Vegh, in the 1980's Argentine commercial banks issued short-term demand deposits and lent the proceeds to the government, mainly to meet compulsory reserve requirements. To capture this structure in a simple way, Calvo and Vegh modeled a government that issued both currency and an interest bearing asset that provided transactions services. They let the government control the interest rate on this asset and the total quantity of the two assets, and they showed that their model pinned down the price level. Interestingly, Calvo and Vegh did not view their arguments as a resolution of the price indeterminacy problem we discuss here; they say, “it should be emphasized at the outset that our results are not directly comparable to those of the interest rate targeting literature because we are in fact undertaking a different exercise (that is, the interest rate controlled by the policymakers is not the same). Hence, our model should be viewed as providing a framework within which to think about the policy issues described above, and not as having a direct bearing on the standard results of the interest rate targeting literature.”

5. Drawing on earlier work by Weil (1989), Giovannini and Labadie (1991) argue that the risk-free rate implied by the CCAPM is close to the return on equity observed in US data. Bensal and Coleman (1996) address the equity premium puzzle and related issues using a model that ascribes transactions services to bonds.

6. Canzoneri, Cumby and Diba (2002) follow the approach suggested above, calculating time series for the CCAPM interest rate in models with and without “habit”. They show that the CCAPM rate is negatively correlated with the federal funds rate, and that the spread between them is systematically related to the stance of monetary policy: a monetary tightening decreases the spread. This empirical finding is a challenge for standard models (which tend to equate the

interest rate in the Euler Equation with the interest rate in the monetary policy rule). However, the model outlined here has precisely this implication: a contractionary open market operation raises B in equation (4b), decreasing $k'(\cdot)$, and decreasing the spread $i_t^* - i_t$. Thus, Canzoneri, Cumby and Diba provide some indirect empirical support for our model.

7. Establishing price-level indeterminacy entails one more step to show that the government's present-value budget constraint is satisfied for any price level. Schmitt-Grohe and Uribe (2000) show that this is the case under the balanced budget rule.

8. The three appendices referred to in this paper will be posted (for a period of time) on Matthew Canzoneri's web page.

9. (15) implies that the real primary surplus inclusive of central-bank transfers follows the rule $s_t = [\rho + (i_{t-1}/\Pi_{t-1})]l_{t-1} - z_t$. Canzoneri, Cumby and Diba (2001) shows that this rule is "Ricardian" in Woodford's (1995) sense if $\rho + (i_{t-1}/\Pi_{t-1})$ is positive infinitely often. We will assume that this is the case; so, prices will not be pinned down in the manner described by the "fiscal theory of the price level". We will consider negative values of ρ , for which fiscal policy is active in Leeper's (1991) sense, as well as positive values, which correspond to Leeper's passive fiscal policies.

10. Another way to see this is to recall equation (8) in Section II: if real liabilities and the real deficit are determined, the budget constraint implies that the price level is also determined.

11. This is the analogue in our setup of what Leeper (1991) calls a passive fiscal policy and an active monetary policy. The analogy is a bit loose because we are assuming a constant response, ρ , of the total deficit to total liabilities, while Leeper assumed a constant response of the primary surplus to bond holdings.

12. This modification does not affect our analysis of the eigenvalues; it just adds a constant onto

the log-linearized dynamic equations.

13. $k'(0)$ is not well defined because of assumptions made about $k(\cdot)$; this raises minor technical questions about the equilibrium conditions when debt is equal to zero.