

Equilibrium Search with Time-Varying Unemployment Benefits*

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Abstract

In this paper we show how time-varying unemployment benefits can generate equilibrium wage dispersion in an economy in which identical firms post wages and homogeneous risk-averse workers search for acceptable offers. We model a two-tier unemployment benefit system that is similar to real-world unemployment insurance programs. We assume that the unemployed initially receive benefits at rate b . Eventually, if a worker does not in the meantime find and accept a wage offer, the benefit falls to a lower level, s . The duration of high-benefit receipt is treated as an exponential random variable, so the model is stationary. We characterize the equilibrium, and we derive the comparative statics effects of changes in the unemployment compensation system (i.e., in the two benefit levels and in the expected duration of the high-benefit state) on the equilibrium wage distribution and the unemployment rate.

1 Introduction

In this paper, we analyze the implications of time-varying unemployment insurance in an equilibrium search model. We consider a two-tier system, i.e., a system with a high and a low benefit level. This is the form taken by

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most real-world UI programs. Unemployed workers initially receive unemployment compensation, but eventually, if a worker does not find a job in the meantime, this benefit is terminated. The worker then has access only to lower social assistance benefits or in some countries to no benefits at all.

Specifically, we consider an economy in which newly unemployed workers initially receive unemployment benefits at rate b . Eventually, if a worker does not find and accept a job in the meantime, the unemployment benefit falls to a lower level, s . We assume the event that triggers the fall from b to s occurs at Poisson rate λ .¹ This allows us to do our equilibrium analysis in a stationary framework while focusing on the most important aspect of time-varying unemployment compensation, namely, that after some point the benefit falls. This representation of time-varying unemployment compensation enables us to derive a two-point equilibrium wage distribution in a simple stationary setting. We derive this equilibrium distribution using a wage-posting model of sequential search. We allow for free entry and exit of jobs and for matching frictions in the sense that the rate at which unemployed workers and vacant jobs contact one another depends on overall labor market tightness. The use of a matching function to determine the job contact rate for workers (and the worker contact rate for jobs) is relatively unusual in wage-posting models, which typically assume a fixed contact rate. Our model can thus be viewed as a combination of the wage-posting and job-matching (e.g, Pissarides 2000) traditions.

Equilibrium wage dispersion can arise in our model because time-varying unemployment benefits lead to a distribution of worker reservation wages.² Even though workers are identical *ex ante*, a worker who is receiving b can afford to be choosier about the jobs that he or she will accept (has a higher

¹Our model can be interpreted as one in which the search activity of unemployed workers is imperfectly monitored by a government agency. Suppose unemployed workers are punished by a reduction in their benefits from b to s when found to be putting forth less search effort than required and that detection of insufficient search effort occurs at Poisson rate λ . If all workers choose to put forth less than the required effort – and this is a plausible assumption, given that workers are homogeneous –, then this “sanctions” model is equivalent to our model with time-varying benefits. That is, our model can be interpreted as a (simplified) equilibrium version of Abbring, van den Berg, and van Ours (2000).

²The insight that time-varying unemployment benefits generate a distribution of reservation wages is an old one. Mortensen (1977) and Burdett (1979) initially developed the idea that with a (deterministic) time limit on unemployment benefits, a worker’s reservation wage falls as elapsed duration gets closer to the time limit. This individual problem of nonstationary search has now been analyzed in considerable generality, in particular, by van den Berg (1990). One contribution of our paper is to incorporate the basic insight from this literature into an equilibrium wage-posting model.

reservation wage) than can a worker who is receiving s . Time-varying unemployment compensation thus provides a new way to overcome the Diamond (1971) paradox, which suggests that in a wage-posting model with homogeneous workers and firms, search costs, no matter how small, will lead to an equilibrium wage distribution that is degenerate at the monopsony wage. Other models have generated equilibrium wage dispersion from a distribution of reservation wages. Albrecht and Axell (1984) did this by simply assuming that workers were *ex ante* heterogeneous with respect to the value of leisure and hence had differing reservation wages. Burdett and Mortensen (1998) use on-the-job search to generate an endogenous distribution of reservation wages for *ex ante* identical workers – an employed job seeker’s reservation wage is his or her current wage. We also generate an endogenous distribution of reservation wages for *ex ante* identical workers. We use a real-world institution (time-varying unemployment benefits) to generate an endogenous distribution of worker types and hence reservation wages. Unlike Burdett and Mortensen (1998), our model has a distribution of reservation wages among unemployed workers. This can lead to job rejection by some unemployed workers and to search unemployment.³

In addition to providing a new foundation for equilibrium wage dispersion, the introduction of time-varying unemployment benefits into an equilibrium model leads to interesting comparative statics. The possibility of falling from the high-benefit to the low-benefit state creates an incentive for workers to accept jobs in order to be reentitled to high benefits. This reentitlement incentive causes some comparative static effects to differ considerably from those that would be found in a model in which unemployment benefits do not vary over time. For example, an increase in the high benefit leads to a reduction in the reservation wage for high-benefit recipients. Absent time-varying unemployment compensation, the increased value of being unemployed would increase the reservation wage, but with the chance of falling into the low-benefit state, the increase in the high benefit raises the worker’s incentive for reentitlement and leads to a lower reservation wage. This captures the real-world phenomenon of workers who are about to lose unemployment compensation taking jobs in order to requalify for unemployment benefits.

The next section presents our model. Section 3 presents the special case of worker risk neutrality. We present this case because with risk neutrality,

³Using a strategic bargaining approach, Coles and Masters (2003) also focus on the effect of time-varying unemployment benefits on reservation wages. However, their model is one with complete information, so no wages are rejected in equilibrium.

the model can be solved analytically and we can explicitly derive the comparative statics results. In Section 4, we present a numerical example with log utility. We show that the comparative static effects of the main policy variables, b , s , and λ are virtually the same as in the case of risk neutrality. Section 5 considers an extension of the model to allow workers to quit the low-wage job and receive the high unemployment benefit with some probability. (In the basic model, this probability is implicitly set to zero.) In Section 6, we discuss the implications of our model for the optimality of time-varying unemployment insurance. Section 7 contains conclusions.

2 The Model

We consider a continuous-time model in which *ex ante* homogeneous workers are infinitely-lived. The measure of workers is fixed and normalized to 1. The decision that workers make is whether or not to accept job offers. Jobs are likewise *ex ante* homogeneous. The decision that a firm (job owner) makes is whether the job should be in the market (entry/exit) and what wage to post when the job is vacant. The measure of jobs in the market (vacancies plus filled jobs) is endogenous. Both workers and firms discount the future at the rate r .

2.1 Workers

At any moment, a worker is either unemployed or employed. When unemployed, a worker receives the (income-equivalent) value of leisure or home production, h , plus unemployment compensation. When initially unemployed, a worker receives b and then moves to the lower level s at Poisson rate λ . Thus, when unemployed, the worker's income, y , can equal either $b+h$ or $s+h$. When employed, a worker's income is the wage that he or she is paid; that is, $y = w$. The worker's instantaneous utility function is $\xi(y)$, which is common across workers. We assume that $\xi'(y) > 0$ and $\xi''(y) \leq 0$.

Workers move from employment to unemployment (worker/job matches break up) at an exogenous Poisson rate δ . The transition rate from unemployment to employment is endogenous and depends on labor market tightness and on worker choice. Specifically, we assume a constant returns to scale contact function, $M(u, v) = m(\theta)u$, where u is the unemployment rate, v is the measure of vacant jobs, and $\theta = v/u$ represents labor market tightness. The Poisson rate at which an unemployed worker contacts a vacant job is thus $m(\theta)$, and the rate at which a vacancy meets an unemployed worker is $m(\theta)/\theta$. The contact function is increasing in its arguments and

satisfies $M(0, v) = M(u, 0) = 0$. These assumptions imply $m(0) = 0$ and $\lim_{\theta \rightarrow 0} m(\theta)/\theta = +\infty$, as well as $m'(\theta) > 0$ and $\frac{d[m(\theta)/\theta]}{d\theta} < 0$. Finally, the fact that the offer arrival rate for workers is increasing in θ while the applicant arrival rate for vacancies is decreasing in θ implies the standard elasticity condition, $0 < m'(\theta)\theta/m(\theta) < 1$.

Given any distribution of wage offers across vacancies, $F(w)$, there will be two reservation wages among the unemployed, one for those receiving b and one for those receiving s . Firms have no incentive to offer a wage that is not someone's reservation wage; thus, in equilibrium, at most two wages will be offered. We let w_b denote the reservation wage for workers with unemployment benefit b , w_s the reservation wage for workers with unemployment benefit s , and ϕ the fraction of offers at w_b . Since workers receiving b will reject offers of w_s , not all offers need be accepted in equilibrium.

The higher reservation wage is determined by equating the value of unemployment for those receiving b , $U(b)$, to the value of employment at w_b , $N(w_b)$. Similarly, the lower reservation wage is determined by equating the value of unemployment for those receiving s , $U(s)$, to the value of employment at w_s , $N(w_s)$. The unemployment values are defined by

$$rU(b) = \xi(b + h) + \phi m(\theta)[N(w_b) - U(b)] + \lambda[U(s) - U(b)] \quad (1)$$

$$rU(s) = \xi(s + h) + \phi m(\theta)[N(w_b) - U(s)] + (1 - \phi)m(\theta)[N(w_s) - U(s)]. \quad (2)$$

The value for an unemployed worker who is receiving b reflects the fact that only the higher wage offer, w_b , is acceptable. The value for an unemployed worker who is receiving s reflects the fact that such a worker will be less selective; that is, either wage offer will be accepted. Similarly, the employment values are defined by

$$rN(w_b) = \xi(w_b) + \delta[U(b) - N(w_b)] \quad (3)$$

$$rN(w_s) = \xi(w_s) + \delta[U(b) - N(w_s)]. \quad (4)$$

Using the reservation wage property, that is,

$$U(b) = N(w_b) \text{ and } U(s) = N(w_s),$$

and substituting in equation (3) yields

$$N(w_b) = \frac{\xi(w_b)}{r} = U(b).$$

Using equations (1) and (4) and the reservation wage property gives

$$\xi(w_s) = \xi(w_b) + \frac{(r + \delta)}{\lambda} [\xi(w_b) - \xi(b + h)]. \quad (5)$$

Since $w_b > w_s$, equation (5) implies $b + h > w_b$. The intuition for this is that for a worker employed at w_b , the only possible transition is into the high-benefit unemployment state. Such a transition entails no loss of value. On the other hand, a worker in the high-benefit state faces two possible transitions – into employment at w_b or into the low-benefit unemployment state. The first of these transitions entails no loss of value, but the second does. Thus, to maintain $N(w_b) = U(b)$, the flow utility in the high-benefit unemployment state has to exceed the flow utility when employed at w_b ; that is, $\xi(b + h) > \xi(w_b)$.

Using equations (1) and (2) and the reservation wage property gives

$$\xi(w_b) = \frac{[r + \phi m(\theta)]\xi(b + h) + \lambda \xi(s + h)}{r + \lambda + \phi m(\theta)}. \quad (6)$$

This equation shows that $\xi(w_b)$ is a weighted sum of the flow utilities of unemployment in the high- and low-benefit states. Finally, combining equations (5) and (6), we have

$$\xi(w_s) = \frac{[\phi m(\theta) - \delta]\xi(b + h) + [r + \delta + \lambda]\xi(s + h)}{r + \lambda + \phi m(\theta)}. \quad (7)$$

Equations (6) and (7) summarize the worker side of the model. It is worth noting that these equations remain valid even when $\phi = 0$ or $\phi = 1$. For example, if $\phi = 1$, equation (7) gives the reservation wage for unemployed workers receiving s . This reservation wage is well-defined even if, in equilibrium, no firm chooses to offer that wage.

2.2 Firms

Jobs are either filled or vacant. A job incurs a flow cost of c whether filled or vacant and produces an output valued at x when filled. Thus, the instantaneous profit for a job paying a wage of w is $-c$ when the job is vacant and $x - w - c$ when the job is filled. There is free entry and exit of vacancies.

Let $\pi(w_b)$ and $\pi(w_s)$ be the values of having vacancies offering w_b and w_s , respectively, and let $J(w_b)$ and $J(w_s)$ be the values of having filled jobs paying w_b and w_s , respectively. As noted above, the rate at which vacant jobs meet unemployed workers is $m(\theta)/\theta$. However, not all unemployed workers

will accept w_s . Letting γ denote the fraction of unemployed with reservation wage w_b , we have

$$\begin{aligned} r\pi(w_b) &= -c + \frac{m(\theta)}{\theta}[J(w_b) - \pi(w_b)] \\ r\pi(w_s) &= -c + \frac{m(\theta)}{\theta}(1 - \gamma)[J(w_s) - \pi(w_s)] \\ rJ(w_b) &= x - w_b - c + \delta[\pi(w_b) - J(w_b)] \\ rJ(w_s) &= x - w_s - c + \delta[\pi(w_s) - J(w_s)]. \end{aligned}$$

Eliminating $J(w_b)$ and $J(w_s)$ gives

$$\pi(w_b) = -c + \frac{m(\theta)}{\theta} \frac{x - w_b - c}{r + \delta} \quad (8)$$

$$\pi(w_s) = -c + \frac{m(\theta)}{\theta} (1 - \gamma) \frac{x - w_s - c}{r + \delta}. \quad (9)$$

If $0 < \phi < 1$, that is, if some firms post w_b while others post w_s , then free entry/exit requires that $\pi(w_b) = \pi(w_s) = 0$. If only w_s is offered, that is, if $\phi = 0$, then free entry/exit requires $\pi(w_s) = 0$ but only that $\pi(w_b) \leq 0$. Similarly, if only w_b is offered, i.e., if $\phi = 1$, then $\pi(w_b) = 0$ and $\pi(w_s) \leq 0$ must hold in equilibrium.

2.3 Steady-State Conditions

In steady state, the measures of workers in each possible state must be constant through time. We use two steady-state conditions to derive expressions for γ and u .

Workers can be classified into three categories – employed, unemployed and receiving b , and unemployed and receiving s . The measure of employed is $1 - u$, the measure of unemployed receiving b is γu , and the measure of unemployed receiving s is $(1 - \gamma)u$. Since the measure of workers is normalized to one, we need only equate inflows and outflows for two of these states. We work with the two unemployment states.

The condition that equates the flows into and out of the high-benefit unemployment state is

$$\delta(1 - u) = [\phi m(\theta) + \lambda]\gamma u.$$

Workers flow into this state from employment at rate δ ; workers flow out of this state either back into employment (at rate $\phi m(\theta)$) or into the low-benefit unemployment state (at rate λ). The comparable equation for the low-benefit unemployment state is

$$\lambda\gamma u = m(\theta)(1 - \gamma)u;$$

that is,

$$\lambda\gamma = m(\theta)(1 - \gamma).$$

These conditions imply that we can write γ and u in terms of the other endogenous variables of the model, namely,

$$\gamma = \frac{m(\theta)}{\lambda + m(\theta)} \quad (10)$$

$$u = \frac{\delta}{\delta + \gamma(\phi m(\theta) + \lambda)}. \quad (11)$$

2.4 Equilibrium

A steady-state equilibrium is a vector $\{w_b, w_s, \phi, \theta, \gamma, u\}$ such that

- (i) $U(b) = N(w_b)$ and $U(s) = N(w_s)$,
- (ii) No wage other than w_b or w_s is offered, and one of the following is satisfied:

- (a) $0 < \phi < 1$ and $\pi(w_b) = \pi(w_s) = 0$
- (b) $\phi = 0$ and $\pi(w_s) = 0$ but $\pi(w_b) \leq 0$
- (c) $\phi = 1$ and $\pi(w_b) = 0$ but $\pi(w_s) \leq 0$

- (iii) the steady-state conditions (10) and (11) hold.

Condition (i) states that workers search optimally given the wage offer distribution, $\{w_b, w_s, \phi\}$.⁴ Condition (ii) states that firms optimize with respect to their wage offers in the sense that no wage is offered that is not some worker's reservation wage and with respect to their entry/exit decisions.

⁴Since all offers at w_b are accepted, whereas only a fraction $1 - \gamma$ of offers at w_s are accepted, the equilibrium distributions of wages offered and of wages paid are not the same. The relationship between the two distributions can be derived from the condition that the flows of workers into and out of high-wage employment must be the same. (Equivalently, one can use the condition that the flows into and out of low-wage employment are the same.) Let η denote the fraction of employed workers who are paid w_b . The steady-state condition is then $\phi m(\theta)u = \delta\eta(1 - u)$. Using equations (10) and (11) to eliminate u , this implies $\eta = \phi \left(\frac{\lambda + m(\theta)}{\lambda + \phi m(\theta)} \right)$.

There are three types of equilibria to consider – equilibria in which only the low wage is offered ($\phi = 0$), equilibria with wage dispersion ($0 < \phi < 1$), and equilibria in which only the high wage is offered ($\phi = 1$).

We want to know which parameter configurations are consistent with the existence of a steady-state equilibrium and whether equilibrium is unique. Further, we want to know which parameter configurations imply $\phi = 0$, which lead to wage dispersion, and which imply $\phi = 1$. We can address these issues by considering the effects of varying a single parameter, holding all others constant. The most intuitive parameter to vary is x , the flow output from filled jobs. Given any collection of fixed values for the other parameters of the model (subject only to trivial restrictions such as $b > s$, $r > 0$, etc.), it is clear that for x sufficiently close to zero, no steady-state equilibrium (with $\theta > 0$) exists. For x sufficiently small, it is not worth posting a vacancy at the lower wage, even if that vacancy could be filled arbitrarily quickly.

For somewhat larger values of x , there is a unique equilibrium in which $\phi = 0$. To see this, suppose provisionally that $\phi = 0$. With $\phi = 0$, equations (6) and (7) show that neither w_b nor w_s vary with θ and that $w_b > w_s$. There are thus values of x such that $x - w_s - c > 0 > x - w_b - c$. At such values, it is profitable for at least a small number of firms to create low-wage vacancies while at the same time it is unprofitable to create high-wage vacancies. That is, for sufficiently small values of x (but not so small that $x - w_s - c < 0$), there is a unique equilibrium in which only the low wage is posted.

As x increases further, eventually $x - w_b - c > 0$. It becomes worthwhile at some point for a first (individually negligible) firm to post the higher wage. At this value of x , there is a corresponding value of θ such that $\pi(w_b) = \pi(w_s) = 0$. The assertion that there is an (x, θ) such that

$$-c + \frac{m(\theta)}{\theta} \frac{x - w_b - c}{r + \delta} = -c + \frac{m(\theta)}{\theta} \frac{\lambda}{\lambda + m(\theta)} \frac{x - w_s - c}{r + \delta} = 0$$

is easy to verify since we are still at the point (because the posting of the first high-wage vacancy does not measurably increase ϕ above zero) at which w_b and w_s can be treated as constants. As x increases even further, more firms have an incentive to post the higher wage. The situation becomes more complicated because w_b and w_s are no longer constants.

To show the existence of a unique equilibrium when x is such that $0 < \phi < 1$, we reduce the system of equations defining equilibrium to a single equation in θ , show that equation has a unique solution, and then check that given θ , the other endogenous variables of the model are uniquely

determined. First, equation (5) gives w_s as a function of w_b and parameters; in particular, w_s is an increasing function of w_b . Next, from $\pi(w_b) = 0$ (cf. equation (8)),

$$w_b = x - c - \frac{c(r + \delta)\theta}{m(\theta)}.$$

Thus, w_b is a decreasing function of θ , and so too is w_s . Finally, $\pi(w_s) = 0$ (equation (9)) gives our equation for θ , namely,

$$c = \left(\frac{m(\theta)}{\theta}\right)\left(\frac{\lambda}{\lambda + m(\theta)}\right)\left(\frac{x - c - w_s}{r + \delta}\right).$$

As $\theta \rightarrow 0$, the right-hand side of this equation goes to infinity; as $\theta \rightarrow \infty$, the right-hand side goes to zero; and the right-hand side is decreasing in θ so long as $m(\theta) \geq r + \delta$. Thus, if the equilibrium θ is not too small, the equation has a unique solution in θ .

The last step is to check that the other endogenous variables are uniquely determined. We have already shown that the two wages are uniquely determined by θ . Equation (6) can be rearranged to give:

$$\phi = \frac{(r + \lambda)\xi(w_b) - r\xi(b + h) - \lambda\xi(s + h)}{m(\theta)[\xi(b + h) - \xi(w_b)]},$$

so ϕ is also uniquely determined by labor market tightness. Finally, equation (10) gives γ as a function of θ , and equation (11), after substituting for γ and ϕ , gives u as a function of γ .

Finally, for x sufficiently large, $\phi = 1$, as it is no longer worthwhile to incur the “delay cost” implied by posting w_s . Again, it is easy to see that for any x such that $\phi = 1$, equilibrium is unique. To see this, note that with $\phi = 1$, equation (6) gives

$$\xi(w_b) = \frac{[r + m(\theta)]\xi(b + h) + \lambda\xi(s + h)}{r + \lambda + m(\theta)}.$$

At the same time, $\pi(w_b) = 0$ (equation (8)) implies

$$w_b = x - c - \frac{c(r + \delta)\theta}{m(\theta)};$$

thus in any equilibrium with $\phi = 1$,

$$\xi\left(x - c - \frac{c(r + \delta)\theta}{m(\theta)}\right) = \frac{[r + m(\theta)]\xi(b + h) + \lambda\xi(s + h)}{r + \lambda + m(\theta)}.$$

The right-hand side of this equation is monotonically increasing in θ , ranging from $\frac{r\xi(b+h) + \lambda\xi(s+h)}{r+\lambda}$ to $\xi(b+h)$. The left-hand side is monotonically decreasing in θ , starting from $\xi(x-c)$ at $\theta = 0$. So long as $x > c + b + h$, the above equation has a unique solution; that is, equilibrium with $\phi = 1$ is unique.

We have thus shown that for any fixed values of the other parameters of the model, a range of equilibrium possibilities can be traced out as we vary x .

3 Risk Neutrality

We now consider the special case of risk neutrality. Risk neutrality allows us to solve the model analytically and to derive qualitative comparative statics results. In this case, the equations defining the equilibrium with wage dispersion are

$$\gamma = \frac{m(\theta)}{\lambda + m(\theta)} \quad (10)$$

$$u = \frac{\delta}{\delta + \gamma(\phi m(\theta) + \lambda)} \quad (11)$$

$$w_s = w_b + \frac{(r + \delta)}{\lambda} [w_b - b - h] \quad (5')$$

$$w_b = b + h - c\theta \quad (12)$$

$$\phi = \frac{\lambda(b-s) - c\theta(r+\lambda)}{c\theta m(\theta)} \quad (13)$$

$$c\theta m(\theta) - (r + \delta)c\theta + (x - c - b - h)m(\theta) = 0. \quad (14)$$

Equations (10) and (11) are repeated from the last section. Equation (5') is equation (5) in the case of risk neutrality. The derivation of equations (12)-(14) is given in Appendix 1. Equation (14) has a unique solution for θ so long as $x - c - b - h \leq 0$.⁵ Using equations (10)-(14) and (5') then gives

⁵To see this, rewrite equation (14) as

$$cm(\theta) - (r + \delta)c = -(x - c - b - h)m(\theta)/\theta.$$

The left-hand side, which is monotonically increasing in θ , equals $-(r + \delta)c$ when $\theta = 0$ and tends to infinity as $\theta \rightarrow \infty$. The right-hand side is monotonically decreasing in θ (if and only if $x - c - b - h < 0$), tends to infinity as $\theta \rightarrow 0$ and tends to zero as $\theta \rightarrow \infty$. Thus, when $x - c - b - h < 0$, this equation has a unique solution. When $x - c - b - h = 0$, the unique solution is the θ that solves $m(\theta) = r + \delta$.

a unique solution for the other endogenous variables.

3.1 Comparative Statics

We now address the basic comparative statics associated with time-varying unemployment compensation for the case in which workers are risk neutral. As noted above, in this case, we can solve analytically for the comparative statics. Specifically, we derive the qualitative effects of changes in b , s , and λ in equilibria with wage dispersion. Table 1 gives our results. The derivations are in Appendix 2. With the exception of $\partial u/\partial b$, all of these effects are unambiguous.

Table 1
Basic Comparative Statics

	θ	u	ϕ	γ	w_b	w_s
b	+	?	-	+	-	-
s	0	+	-	0	0	0
λ	0	-	+	-	0	+

Many of these results differ from what one would find in a standard equilibrium search model. For example, all else equal, the standard model predicts that an increase in b would increase the reservation wage of the unemployed who are receiving b . Our prediction differs because time-varying unemployment benefits create a reentitlement incentive for workers that dominates the standard effect. Because the short-term unemployed face a limited expected duration of receiving the high benefit, they are interested in avoiding the low-benefit state. This creates an incentive to take a job in order to be reentitled to the high benefit level and exerts downward pressure on the reservation wage.

To be more specific, consider the effect of an increase in b . Workers prefer to cycle between employment at the high wage and unemployment at the high benefit level, that is, to avoid unemployment at the low benefit level. An increase in b makes unemployed workers in the high-benefit state willing to accept a lower value of w_b . A similar intuition holds for the negative effect of b on w_s . Unemployed workers receiving s are more eager than they were before the increase in b to get back to the high-benefit unemployment state. This makes them more willing to accept low-wage employment and so reduces their reservation wage. In fact, w_s must fall by more than w_b does. The reason is that the increase in b has countervailing effects on w_b .

On the one hand, the direct effect of the increase in b is to make high-benefit unemployment more attractive. If this were the only effect, as in the standard model, w_b would increase. On the other hand, as indicated above, the increase in b makes workers in the high-benefit unemployment state more eager to avoid the low-benefit unemployment state, so w_b falls. The second effect dominates. The effect of an increase in b on w_s is, however, unambiguously negative.

With the fall in both w_b and w_s , firms have an incentive to open more vacancies. Because w_s falls by more than w_b , entry at the low wage is particularly attractive; thus, ϕ decreases. With a smaller fraction of vacancies at the high wage, there is an increase in the fraction of unemployed who are receiving b ; that is, γ increases. The zero-value condition for high-wage jobs implies that the matching rate, $m(\theta)/\theta$, must fall (and, accordingly, θ must increase) to offset the decrease in w_b . Finally, the ambiguous effect on unemployment is a result of two offsetting factors. Job offers arrive faster than they did before the increase in b (θ increases), but relatively fewer of these offers are acceptable (ϕ decreases).

Next, consider the effects of an increase in s . An unemployed worker receiving b now has less incentive to avoid falling into the low-benefit state; all else equal, this places upward pressure on w_b . For an unemployed worker receiving s , matters are more complicated. On the one hand, an increase in s makes low-benefit unemployment more attractive, so there is a tendency for w_s to rise. On the other hand, the high-benefit state has become more attractive, so there is downward pressure on the reservation wage of workers receiving s . In short, there is stronger pressure on w_b to increase than there is on w_s . However, this pressure is counterbalanced by a change in vacancy composition – it is now relatively less attractive for firms to open high-wage vacancies; that is, ϕ falls. This means that the low-benefit unemployed are less likely to find high-wage jobs and this causes the value of unemployment at s to fall. These two effects on $U(s)$ balance, so $U(s)$ and w_s remain unchanged. The reservation wage of the high-benefit unemployed, w_b , also does not change. This is because the only effect of s on $U(b)$ is through changes in $U(s)$. From the zero-value condition for the high-wage jobs, the fact that w_b is unchanged requires that the matching rate for these firms remains the same, i.e., θ is unchanged. Since neither w_s nor θ are affected by a change in s , zero value for low-wage vacancies implies that changes in s do not affect γ . Finally, the fall in ϕ , all else equal, implies relatively fewer acceptable offers for unemployed workers at the high benefit level; that is, the average duration of unemployment rises and with it, u increases.

The final comparative statics effects are those for λ . Consider unemployed workers receiving s . When λ increases, the reentitlement incentive for these workers is reduced, so their reservation wage rises. The reentitlement incentive for unemployed workers receiving b is also reduced, putting upward pressure on w_b , but this is counterbalanced by the first-order effect of the reduction in the value of unemployment at b . Since the upward pressure on w_s is greater than on w_b , ϕ rises. This further increases the upward pressure on w_s and w_b . In equilibrium, this leaves w_s higher and just offsets the first-order negative effect on w_b , leaving it unchanged. As noted above, there is a direct link between w_b and θ via the zero-value condition for high-wage vacancies. If w_b is unaffected by the increase in λ , then θ is unchanged. An increase in λ , by reducing the expected duration of high-benefit unemployment, reduces γ , the fraction of unemployed who are in the high-benefit state. In addition, since w_s increases with λ while θ is unaffected, γ must fall in order to maintain zero value for low-wage vacancies. Finally, the increase in ϕ in conjunction with the increase in λ makes the unemployed accept job offers more quickly, and that is why the unemployment rate falls with λ .

The next section provides a numerical example of the comparative statics for a case in which workers are risk averse. For comparison, we also present the example for the case of risk neutrality. The effects of changes in b and s are virtually the same in both cases, while the effect of changes in λ are somewhat different. While this indicates that the comparative statics are similar when we allow for risk aversion, this is only a numerical example, which need not hold for all forms of risk aversion.

4 Numerical Example

In this section, we present a numerical example to illustrate the properties of the model in the case of risk aversion. We also present the numerical example with the same parameters for the case of risk-neutral workers. We do this to illustrate the effect of risk aversion. The example uses the contact function, $m(\theta) = 8\sqrt{\theta}$, and in the baseline case, we assume that $b = 1$, $s = 0$, $h = 1$, $\lambda = 2$, $x = 2$, $\delta = .2$, $c = .5$, and $r = .05$. The baseline parameter values were chosen with two criteria in mind. First, the parameter values themselves should be reasonable. Second, the values of the endogenous variables that follow from these parameter values should also be reasonable. For the case of risk aversion, we use the instantaneous utility function $\xi(y) = \ln y$.

Table 2 presents the solution for our baseline case with worker risk aversion (in row 1) and the comparative statics of changes in b .

Table 2: $\xi(y) = \ln y$: Risk Aversion
Comparative Statics for b
Solution with $m(\theta) = 8\theta^{\frac{1}{2}}$
 $x = 2, h = 1, \delta = .2, c = .5, r = .05$
 $s = 0$ and $\lambda = 2$

b	θ	$m(\theta)$	u	γ	ϕ	w_b	w_s
1	.8731	7.4752	.0521	.7889	.3492	1.4854	1.4312
1.5	1.5099	9.8304	.0652	.8309	.1474	1.4808	1.3870
2	2.0005	11.3150	.0716	.8498	.0931	1.4779	1.3527

Table 2a: $\xi(y) = y$: Risk Neutrality
Comparative Statics for b
Solution with $m(\theta) = 8\theta^{\frac{1}{2}}$
 $x = 2, h = 1, \delta = .2, c = .5, r = .05$
 $s = 0$ and $\lambda = 2$

b	θ	$m(\theta)$	u	γ	ϕ	w_b	w_s
1	1.0317	8.1258	.0611	.8025	.2249	1.4842	1.4197
1.5	2.0447	11.4390	.0754	.8507	.0773	1.4777	1.3499
2	3.0546	13.9820	.0817	.8749	.0407	1.4727	1.2818

With risk aversion (Table 2), the baseline case generates an unemployment rate of about 5%. The equilibrium value of $\theta = .8731$ implies a steady-state measure of vacancies of $v = \theta u = (.8731)(.0521) = .0455$. The average duration of unemployment is slightly more than one and one half months ($12 \times (1/7.4752) = 1.6053$), while the average duration of a vacancy is similar ($12 \times (.8731/7.4752) = 1.4016$). The comparative statics of this example are consistent with those derived in our analytic solution of the risk neutral case. As we increase b , labor market tightness increases, i.e., θ rises as does the unemployment rate, u . The fraction of vacancies offering the higher wage (ϕ) falls and the fraction of short-term unemployed (γ) rises. Wages for all workers fall, and the low wage falls by more than the high wage. With risk neutrality (Table 2a), the results are quite similar. For all values of b , the values of θ , u , and γ are higher, while the values of ϕ , and both wages are lower.

Tables 3 and 3a present the comparative statics results for changes in s . We set $b = 1$ so that the results can be compared to those in row 1 of Tables 2 and 2a.

Table 3: $\xi(y) = \ln y$: Risk Aversion
Comparative Statics for s
Solution with $m(\theta) = 8\theta^{\frac{1}{2}}$
 $x = 2, h = 1, \delta = .2, c = .5, r = .05$
 $b = 1$ and $\lambda = 2$

s	θ	$m(\theta)$	u	γ	ϕ	w_b	w_s
.1	.8731	7.4752	.0600	.7889	.2635	1.4854	1.4312
.2	.8731	7.4752	.0697	.7889	.1852	1.4854	1.4312
.3	.8731	7.4752	.0818	.7889	.1132	1.4854	1.4312
.4	.8731	7.4752	.0974	.7889	.0466	1.4854	1.4312

Table 3a: $\xi(y) = y$: Risk Neutrality
Comparative Statics for s
Solution with $m(\theta) = 8\theta^{\frac{1}{2}}$
 $x = 2, h = 1, \delta = .2, c = .5, r = .05$
 $b = 1$ and $\lambda = 2$

s	θ	$m(\theta)$	u	γ	ϕ	w_b	w_s
.1	1.0317	8.1258	.0676	.8025	.1771	1.4842	1.4197
.2	1.0317	8.1258	.0755	.8025	.1294	1.4842	1.4197
.3	1.0317	8.1258	.0856	.8025	.0817	1.4842	1.4197
.4	1.0317	8.1258	.0968	.8025	.0340	1.4842	1.4197

As predicted by the analytic comparative statics in the risk neutrality case, changing s has no effect on θ , γ , w_b , or w_s . As in Tables 2 and 2a, the values of θ and γ are higher with risk neutrality, while the values of w_b and w_s are lower. In both Tables 3 and 3a, as s increases, the fraction of high-wage vacancies, ϕ , falls and the unemployment rate rises. Comparing the effect of an increase in s from .1 to .2 with the increase in b from 1 to 1.5, one can see that in this example the effect of increasing the low unemployment benefit on unemployment is greater than a comparable increase in the high benefit.

Finally, Tables 4 and 4a present the comparative statics results for changes in λ . The third row corresponds to the baseline case. Higher levels of λ correspond to steady states with shorter average durations of high unemployment benefit receipt. As λ increases, the fraction of unemployed receiving b falls (i.e., γ decreases), unemployment falls, and the fraction of vacancies offering the higher wage rises. As we argued above, this is because of the positive effect of an increase in λ on the lower wage, which is apparent in both Tables 4 and 4a. Unlike the case of changes in b and s , there

is a difference between the cases of risk aversion and risk neutrality when λ changes. With risk neutrality, neither θ nor w_b changes when λ changes. With risk aversion, initially as λ rises, w_b falls and θ rises. After $\lambda = 2$, both w_b and θ are constant. This effect seems slight and may reflect an initially larger direct effect of λ on w_b , i.e., the increase in λ making unemployment at b relatively less attractive. The decrease in w_b leads to the increase in vacancies and consequently in θ .

Table 4: $\xi(y) = \ln y$: Risk Aversion
Comparative Statics for λ
Solution with $m(\theta) = 8\theta^{\frac{1}{2}}$
 $x = 2, h = 1, \delta = .2, c = .5, r = .05$
 $b = 1$ and $s = 0$

λ	θ	$m(\theta)$	u	γ	ϕ	w_b	w_s
1	.8494	7.3728	.0905	.8806	.1738	1.4856	1.3792
1.5	.8612	7.4240	.0672	.8319	.2621	1.4855	1.4137
2	.8731	7.4752	.0521	.7889	.3492	1.4854	1.4312
2.5	.8731	7.4752	.0442	.7494	.4382	1.4854	1.4419
3	.8731	7.4752	.0388	.7136	.5272	1.4854	1.4490

Table 4a: $\xi(y) = y$: Risk Neutrality
Comparative Statics for λ
Solution with $m(\theta) = 8\theta^{\frac{1}{2}}$
 $x = 2, h = 1, \delta = .2, c = .5, r = .05$
 $b = 1$ and $s = 0$

λ	θ	$m(\theta)$	u	γ	ϕ	w_b	w_s
1	1.0317	8.1258	.1063	.8904	.1094	1.4842	1.3553
1.5	1.0317	8.1258	.0766	.8442	.1671	1.4842	1.3982
2	1.0317	8.1258	.0611	.8025	.2249	1.4842	1.4197
2.5	1.0317	8.1258	.0517	.7647	.2826	1.4842	1.4326
3	1.0317	8.1258	.0453	.7304	.3404	1.4842	1.4412

The example illustrates that time-varying unemployment benefits can generate equilibrium wage dispersion for a range of parameter values with both risk aversion and risk neutrality and shows that the comparative statics are similar in the two cases. The equilibrium in either case is one in which unemployment arises not only because of matching frictions but also from the rational rejection of wage offers by job seekers in favor of further search.

5 Quits into Unemployment

We have assumed up to now that a worker employed at the low wage cannot quit. There is, however, a clear incentive to do so since quitting a low-wage job would imply an expected lifetime utility gain of $U(b) - N(w_s)$. In the real world, workers who quit their jobs are typically not eligible for UI, but as there is often ambiguity about whether a separation is a quit or a layoff, we now consider a variation on our model in which a worker who quits receives the high level of unemployment compensation, b , with exogenous probability q and the low benefit, s , with probability $1 - q$. In this case, we interpret s as the level of minimum social benefits.⁶ The model we have considered up to this point corresponds to the special case of $q = 0$.

Let Q be the value of quitting. Then

$$Q = qU(b) + (1 - q)U(s).$$

Workers are homogeneous, so if w_s is to be offered in equilibrium, they must choose not to quit low-wage jobs. This implies a no-quit constraint of

$$N(w_s) \geq Q.$$

The no-quit constraint has an efficiency wage interpretation – w_s has to be high enough so that low-wage jobs can keep their workers. As $Q > U(s)$ for $q > 0$, the reservation wage condition, $N(w_s) \geq U(s)$, does not bind for low-wage jobs, and w_s is determined by the efficiency wage constraint.

Suppose the parameters of the model are such that both wages are offered. Substituting $N(w_b) = U(b)$ and $N(w_s) = Q$ into equations (1) and (2) and using the definition of Q gives

$$\begin{aligned} rU(b) &= \xi(b + h) + \lambda[U(s) - U(b)] \\ rU(s) &= \xi(s + h) + \phi m(\theta)[U(b) - U(s)] + (1 - \phi)m(\theta)[q(U(b) - U(s))]. \end{aligned}$$

Solving simultaneously,

$$\begin{aligned} rU(b) &= \frac{[r + m(\theta)(\phi + q(1 - \phi))]\xi(b + h) + \lambda\xi(s + h)}{r + \lambda + m(\theta)(\phi + q(1 - \phi))} \\ rU(s) &= \frac{m(\theta)(\phi + q(1 - \phi))\xi(b + h) + (r + \lambda)\xi(s + h)}{r + \lambda + m(\theta)(\phi + q(1 - \phi))}. \end{aligned}$$

⁶It is straightforward to generalize to the case in which quitters who are detected receive flow benefit $z < s$.

Using $N(w_b) = U(b) = \xi(w_b)/r$ from equation (3) gives

$$\xi(w_b) = \frac{[r + m(\theta)(\phi + q(1 - \phi))] \xi(b + h) + \lambda \xi(s + h)}{r + \lambda + m(\theta)(\phi + q(1 - \phi))}. \quad (15)$$

Similarly, using $N(w_s) = \frac{\xi(w_s) + \delta U(b)}{r + \delta}$ from equation (4) and $N(w_s) = Q = qU(b) + (1 - q)U(s)$ gives

$$\xi(w_s) = \frac{[m(\theta)(\phi + q(1 - \phi)) + (r + \delta)q - \delta] \xi(b + h) [r + \delta + \lambda - (r + \delta)q] \xi(s + h)}{r + \lambda + m(\theta)(\phi + q(1 - \phi))}. \quad (16)$$

Equations (15) and (16) are the generalizations of equations (6) and (7) to allow for quits.

The other equations of the model are as before. It is still the case that $\pi(w_b) \leq 0$ ($= 0$ if some high-wage vacancies are posted) and similarly for low-wage vacancies. Likewise, the steady-state conditions still need to be satisfied in equilibrium.

Intuitively, the first-order effect of introducing the possibility of quitting is to increase w_s . As q increases, all else equal, a higher wage is required to keep low-wage workers from quitting their jobs. Of course, as q changes, the other endogenous variables of the model are also affected. To get a feel for the comparative static effects of increasing q , we have solved the model numerically using log utility and our baseline-case parameters. The results are shown in Table 5.

Table 5: $\xi(y) = \ln y$
 Comparative Statics for q
 Solution with $m(\theta) = 8\theta^{\frac{1}{2}}$
 $x = 2, h = 1, \delta = .2, c = .5, r = .05$
 $b = 1$ and $s = 0$

q	θ	$m(\theta)$	u	γ	ϕ	w_b	w_s
0	.8674	7.4508	.0522	.7884	.3504	1.4854	1.4312
.1	.7812	7.0709	.0585	.7795	.3007	1.4862	1.4374
.2	.6948	6.6686	.0671	.7693	.2425	1.4870	1.4436
.3	.6083	6.2395	.0787	.7573	.1749	1.4878	1.4498
.4	.5216	5.7779	.0953	.7429	.0964	1.4887	1.4561
.5	.4348	5.2752	.1193	.7251	.0073	1.4897	1.4625

As suggested above, an increase in q leads to a corresponding increase in w_s . Note that as w_s increases so too must w_b . This in turn leads to a

fall in vacancy creation so that θ falls, while u increases. The increase in unemployment makes posting a low-wage vacancy relatively more attractive even though the low wage rises more than the high wage.

6 Implications for Optimal Unemployment Insurance

Although the main purpose of our paper is to analyze the positive effects of time-varying unemployment benefits, it is also interesting to speculate on the implications of our model for optimal unemployment insurance. The basic issue in the optimal UI literature is how best to manage the tradeoff between the insurance benefit of unemployment compensation – presuming, of course, that workers are risk averse and unable to self-insure – and the corresponding moral hazard cost, that is, the less-than-optimal job search intensity, job acceptance decisions, and/or job retention effort that UI may induce. This insurance/moral hazard tradeoff is one that can be understood at the level of the individual worker without worrying about equilibrium complications, and it is at this level that Shavell and Weiss (1979) make the argument for the optimality of an unemployment benefit that declines with elapsed duration of unemployment. In their model, the moral hazard problem is one of insufficient search intensity among the unemployed. If UI were time invariant, it would be possible to reduce the moral hazard cost of UI by shifting some benefits forward in time, that is, to move in the direction of a declining-benefit system. The reason is that unemployed job seekers would have an incentive to increase their search intensities early on in order to avoid low benefits later. This shift in benefits can be engineered in such a way as to leave the insurance benefit of UI unchanged. If the distribution of unemployment durations were the same as in the time-invariant baseline, time-varying compensation would reduce the insurance benefit of UI. However, since a declining-benefit system gives workers an incentive to increase their search intensities early on, unemployment durations are shorter on average.

Fredriksson and Holmlund (2001) show that the Shavell-Weiss result holds in an equilibrium setting. Their approach to modeling time-varying UI is the same as ours; namely, benefits switch from a high level to a low level (zero in their model) at an exogenous Poisson rate. They use a standard Pissarides (2000) framework to analyze the equilibrium effects of time-varying UI. A key feature of their model is that all workers are paid the same wage. This results from their use of Nash bargaining and their strong

assumption that all workers have the same threat point. They justify the common threat point assumption by arguing that if employers were to try to exploit the weak bargaining position of the unemployed who are receiving the low benefit by offering a correspondingly low wage, then these workers could threaten to respond by immediately quitting. This threat to quit immediately has the effect of giving low-benefit job applicants the same bargaining position as their higher-benefit counterparts. When the discount rate goes to zero, Fredriksson and Holmlund (2001) show analytically that a time-varying UI system is preferable to a constant-rate system; numerical calibrations indicate that a time-varying system is also preferable in an economy with a positive discount rate.

Our model suggests that these results should be viewed with caution. In a wage-posting model – and in our view most of the workers who make use of real-world unemployment insurance take jobs for which wages are posted rather than bargained over – time-varying unemployment benefits can lead to inefficient job rejection. This occurs either if the parameters of the model are such that there is wage dispersion or if only w_s is offered in equilibrium. In either case, high-benefit recipients reject low-wage jobs. Such job rejection is inefficient. There is a positive surplus created by a match between any vacancy and any worker. Incomplete information, i.e., the fact that the potential employer “guessed wrong” about the job applicant’s benefit status, is what keeps this surplus from being realized. In short, while time-varying unemployment benefits may lessen the moral hazard problem associated with search effort, they may at the same time worsen the moral hazard problem associated with the job acceptance decision.⁷

7 Concluding Remarks

We could extend our model in a number of ways, e.g., by introducing search intensity or on-the-job search. While such extensions would be interesting, we feel that the basic model is sufficient for our purposes. That is, it allows us to explore how time-varying unemployment compensation can generate equilibrium wage dispersion, even though both workers and firms are *ex ante* homogeneous. We have thus added to the equilibrium search literature by demonstrating the implications of a new approach to overcoming the

⁷The third potential moral hazard problem, insufficient job retention effort, is analyzed in Wang and Williamson (1996). Considering this problem together with that of insufficient search intensity, they find that the optimal UI profile initially increases but later decreases.

Diamond (1971) paradox.

In addition to showing that time-varying unemployment benefits can lead to equilibrium wage dispersion, we find that the time-varying benefit structure leads to interesting comparative statics by introducing a role for benefit reentitlement effects. For example, an increase in the high benefit level leads to a reduction in the reservation wages of high-benefit recipients. With constant unemployment benefits, the increase in the value of being unemployed would increase workers' reservation wages, but with time-varying benefits, the increase in the high benefit also raises the incentive for reentitlement and on net leads to lower reservation wages. This reflects the phenomenon of workers taking jobs primarily to requalify for unemployment benefits.

We present simulations of our model to show that the comparative statics results that we derive in the risk-neutral case can hold when workers are risk averse. These simulations suggest that changes in the level of the low benefit and in the duration of the high benefit can have larger effects than changes in the high benefit level on the equilibrium unemployment rate. We also allow for the possibility that workers can quit in order to collect unemployment benefits. So long as quits are detected with a high enough probability, our basic results continue to hold. Finally, we discuss the implications of our model for the optimality of time-varying unemployment insurance. While the literature emphasizes that declining unemployment benefits can have positive effects on search effort, our model indicates that time-varying benefits can lead to wage dispersion and thus to inefficient job rejection.

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Appendix 1: Derivation of Equations (12) to (14)

We start by deriving equation (12). From equation (9), when $\pi(w_s) = 0$,

$$w_s = \frac{m(\theta)(1-\gamma)(x-c) - c\theta(r+\delta)}{m(\theta)(1-\gamma)}.$$

Equating this to the expression for w_s given by (5) for the case of risk neutrality gives,

$$\frac{m(\theta)(1-\gamma)(x-c) - c\theta(r+\delta)}{m(\theta)(1-\gamma)} = w_b + \frac{(r+\delta)(w_b - b - h)}{\lambda}.$$

Using equation (8) with $\pi(w_b) = 0$,

$$m(\theta)(x-c) = m(\theta)w_b + c\theta(r+\delta).$$

Substitution then gives

$$\frac{-\gamma c\theta(r+\delta)}{m(\theta)(1-\gamma)} = \frac{(r+\delta)(w_b - b - h)}{\lambda}.$$

Thus,

$$w_b - b - h = \frac{-\gamma \lambda c\theta}{m(\theta)(1-\gamma)} = \frac{-\left(\frac{m(\theta)}{\lambda+m(\theta)}\right)\lambda c\theta}{m(\theta)\left(\frac{\lambda}{\lambda+m(\theta)}\right)} = -c\theta,$$

which verifies (12).

Next, set the expression for w_b from (12) equal to the one given by (6) with risk neutrality; that is,

$$b + h - c\theta = \frac{(r + \phi m(\theta))(b + h) + \lambda(s + h)}{r + \lambda + \phi m(\theta)}.$$

Solving for ϕ verifies (13).

Finally, from (8)

$$c\theta(r+\delta) = m(\theta)(x - w_b - c).$$

That is,

$$c\theta(r+\delta) = m(\theta)(x - b - h + c\theta - c),$$

which, after rearrangement, verifies (14).

Appendix 2: Comparative Statics Derivations

a. Comparative statics for θ : Using (14),

$$\frac{\partial \theta}{\partial b} = \frac{m(\theta)}{c[m(\theta) - (r + \delta)] + m'(\theta)[c\theta + (x - c - b - h)]}.$$

The denominator of this expression is positive since, from (14), we have

$$c[m(\theta) - (r + \delta)] = -[m(\theta)/\theta](x - c - b - h) > 0$$

and

$$c\theta + (x - c - b - h) = c\theta(r + \delta)/m(\theta) > 0.$$

Thus, $\partial\theta/\partial b > 0$. Since neither s nor λ enters into (14), we have $\partial\theta/\partial s = \partial\theta/\partial\lambda = 0$.

b. Comparative statics for w_b : Using (12), $\partial w_b/\partial b = 1 - c(\partial\theta/\partial b)$. That is,

$$\frac{\partial w_b}{\partial b} = \frac{-c(r + \delta) + m'(\theta)[c\theta + (x - c - b - h)]}{c[m(\theta) - (r + \delta)] + m'(\theta)[c\theta + (x - c - b - h)]}.$$

From (14), $c(r + \delta) = \frac{m(\theta)}{\theta}[c\theta + (x - c - b - h)]$, so by substitution

$$\frac{\partial w_b}{\partial b} = \frac{[-\frac{m(\theta)}{\theta} + m'(\theta)][c\theta + (x - c - b - h)]}{c[m(\theta) - (r + \delta)] + m'(\theta)[c\theta + (x - c - b - h)]}.$$

As $m'(\theta)\theta < m(\theta)$ and $c\theta + (x - c - b - h) > 0$ we have $\partial w_b/\partial b < 0$. Since s and λ appear in neither (12) nor (14) it follows that $\partial w_b/\partial s = \partial w_b/\partial\lambda = 0$.

c. Comparative statics for w_s : From (5'),

$$\frac{\partial w_s}{\partial b} = \left(\frac{r + \delta + \lambda}{\lambda}\right) \frac{\partial w_b}{\partial b} - \frac{r + \delta}{\lambda} < 0 \text{ and}$$

$$\frac{\partial w_s}{\partial \lambda} = \frac{(b + h - w_b)(r + \delta)}{\lambda^2} = \frac{c\theta(r + \delta)}{\lambda^2} > 0.$$

Finally, $\partial w_s/\partial s = 0$.

d. Comparative statics for ϕ : From (13),

$$\frac{\partial \phi}{\partial b} = \frac{\lambda}{c\theta m(\theta)} + \frac{\partial \phi}{\partial \theta} \frac{\partial \theta}{\partial b},$$

where

$$\frac{\partial \phi}{\partial \theta} = \frac{c\theta m(\theta)[-c(r + \lambda)] - [\lambda(b - s) - c\theta(r + \lambda)][cm(\theta) + c\theta m'(\theta)]}{[c\theta m(\theta)]^2}$$

and

$$\frac{\partial \theta}{\partial b} = \frac{m(\theta)}{c[m(\theta) - (r + \delta)] + m'(\theta)[c\theta + (x - c - b - h)]}.$$

We thus have

$$\frac{\partial \phi}{\partial b} = \frac{\lambda}{c\theta m(\theta)} - \frac{m(\theta)\lambda(b - s) + \theta m'(\theta)[\lambda(b - s) - c\theta(r + \lambda)]}{c\theta^2 m(\theta)(c[m(\theta) - (r + \delta)] + m'(\theta)[c\theta + (x - c - b - h)])}.$$

Multiplying both sides by $c\theta m(\theta)$, the sign of $\partial \phi / \partial b$ is the same as that of

$$\lambda - \frac{m(\theta)\lambda(b - s) + \theta m'(\theta)[\lambda(b - s) - c\theta(r + \lambda)]}{c\theta[m(\theta) - (r + \delta)] + m'(\theta)\theta[c\theta + (x - c - b - h)]}.$$

Since the denominator of the fraction is positive, the sign of $\partial \phi / \partial b$ is the same as that of

$$\begin{aligned} & \lambda c\theta[m(\theta) - (r + \delta)] + \lambda m'(\theta)\theta[c\theta + (x - c - b - h)] \\ & - m(\theta)\lambda(b - s) - \theta m'(\theta)[\lambda(b - s) - c\theta(r + \lambda)]. \end{aligned}$$

From (14), $x - c - b - h = [c\theta(r + \delta)/m(\theta)] - c\theta$; thus, the sign of $\partial \phi / \partial b$ is the same as that of

$$\lambda m(\theta)[c\theta - (b - s)] - \lambda c\theta(r + \delta)\left[1 - \frac{m'(\theta)\theta}{m(\theta)}\right] - \theta m'(\theta)[\lambda(b - s) - c\theta(r + \lambda)].$$

The first and third of these three terms are negative by equation (13); specifically, by the condition $\phi > 0$. The second term is negative by $m'(\theta)\theta < m(\theta)$. We thus have $\partial \phi / \partial b < 0$.

Next,

$$\frac{\partial \phi}{\partial \lambda} = \frac{b - s - c\theta}{c\theta m(\theta)} > 0$$

since, again from (13), $b - s - c\theta > 0$.

Finally,

$$\frac{\partial \phi}{\partial s} = -\frac{\lambda}{c\theta m(\theta)} < 0.$$

e. Comparative statics for γ : From (10),

$$\frac{\partial \gamma}{\partial \theta} = \frac{\lambda m'(\theta)}{(\lambda + m(\theta))^2} > 0.$$

Then $\partial \gamma / \partial \lambda = -m(\theta) / [\lambda + m(\theta)]^2 < 0$, and the rest of the derivatives of γ have the same signs as the partials of θ with respect to the various parameters. Specifically, $\partial \gamma / \partial b > 0$ and $\partial \gamma / \partial s = 0$.

f. Comparative statics for u : From (11),

$$\frac{\partial u}{\partial b} = \frac{\partial u}{\partial \phi m(\theta)} \frac{\partial(\phi m(\theta))}{\partial b} + \frac{\partial u}{\partial \gamma} \frac{\partial \gamma}{\partial b},$$

where

$$\frac{\partial u}{\partial \phi m(\theta)} = \frac{-\delta \gamma}{(\delta + \gamma[\phi m(\theta) + \lambda])^2} < 0$$

and

$$\frac{\partial u}{\partial \gamma} = \frac{-\delta(\phi m(\theta) + \lambda)}{(\delta + \gamma[\phi m(\theta) + \lambda])^2} < 0.$$

Let $\Psi = \frac{\delta}{(\delta + \gamma[\phi m(\theta) + \lambda])^2} > 0$. Then, we have

$$\frac{\partial u}{\partial b} = -\Psi \left(\gamma \frac{\partial \phi m(\theta)}{\partial b} + (\phi m(\theta) + \lambda) \frac{\partial \gamma}{\partial b} \right).$$

Next from equation (13) we have

$$\phi m(\theta) = \frac{\lambda(b-s)}{c\theta} - (r + \lambda)$$

so

$$\frac{\partial \phi m(\theta)}{\partial b} = \frac{\lambda}{c\theta} - \frac{\lambda(b-s)}{c\theta^2} \frac{\partial \theta}{\partial b} = \frac{\lambda}{c\theta} \left(1 - \frac{(b-s)}{\theta} \frac{\partial \theta}{\partial b} \right).$$

From above,

$$\frac{\partial \gamma}{\partial b} = \frac{\lambda m'(\theta)}{(\lambda + m(\theta))^2} \frac{\partial \theta}{\partial b} > 0.$$

Thus,

$$\frac{\partial u}{\partial b} = -\lambda \Psi \left(\frac{\gamma}{c\theta} + \frac{\partial \theta}{\partial b} \left[\frac{m'(\theta)(\phi m(\theta) + \lambda)}{(\lambda + m(\theta))^2} - \frac{\gamma(b-s)}{c\theta^2} \right] \right).$$

Since

$$\frac{m'(\theta)(\phi m(\theta) + \lambda)}{(\lambda + m(\theta))^2} = \frac{\gamma m'(\theta)(\phi m(\theta) + \lambda)}{m(\theta)(\lambda + m(\theta))},$$

we have

$$\frac{\partial u}{\partial b} = -\frac{\lambda\gamma}{c\theta}\Psi\left(1 + \frac{\partial\theta}{\partial b}\left[\frac{m'(\theta)(\phi m(\theta) + \lambda)c\theta}{m(\theta)(\lambda + m(\theta))} - \frac{(b-s)}{\theta}\right]\right).$$

Then,

$$\text{sign}\left[\frac{\partial u}{\partial b}\right] = -\text{sign}\left[1 + \frac{\partial\theta}{\partial b}\left(\frac{m'(\theta)(\phi m(\theta) + \lambda)c\theta}{m(\theta)(\lambda + m(\theta))} - \frac{(b-s)}{\theta}\right)\right]$$

Without imposing more restrictions on $m(\theta)$, this latter sign is indeterminate.

Next, since $\frac{\partial\gamma}{\partial s} = \frac{\partial\theta}{\partial s} = 0$, we have

$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial\phi}\frac{\partial\phi}{\partial s}.$$

Since

$$\frac{\partial u}{\partial\phi} = \frac{-\delta\gamma m(\theta)}{(\delta + \gamma[\phi m(\theta) + \lambda])^2} < 0$$

and, as shown above, $\frac{\partial\phi}{\partial s} < 0$, we have $\frac{\partial u}{\partial s} > 0$.

Finally, using $\frac{\partial\theta}{\partial\lambda} = 0$, we have

$$\frac{\partial u}{\partial\lambda} = \frac{\partial u}{\partial\phi}\frac{\partial\phi}{\partial\lambda} + \frac{\partial u}{\partial\gamma}\frac{\partial\gamma}{\partial\lambda} - \frac{\delta\gamma}{(\delta + \gamma[\phi m(\theta) + \lambda])^2}.$$

The final term is the direct effect of λ on u . Substitution gives,

$$\begin{aligned} \frac{\partial u}{\partial\lambda} = & -\left(\frac{\delta\gamma m(\theta)}{(\delta + \gamma[\phi m(\theta) + \lambda])^2}\right)\left(\frac{b-s-c\theta}{c\theta m(\theta)}\right) + \\ & \left(\frac{\delta(\phi m(\theta) + \lambda)}{(\delta + \gamma[\phi m(\theta) + \lambda])^2}\right)\left(\frac{m(\theta)}{(\lambda + m(\theta))^2}\right) - \frac{\delta\gamma}{(\delta + \gamma[\phi m(\theta) + \lambda])^2}. \end{aligned}$$

Or,

$$\frac{\partial u}{\partial\lambda} = -\Psi\left(\frac{\gamma(b-s-c\theta)}{c\theta} - \frac{m(\theta)(\phi m(\theta) + \lambda)}{(\lambda + m(\theta))^2} + \gamma\right).$$

The sign of $\frac{\partial u}{\partial\lambda}$ is the same as that of

$$-\frac{\gamma(b-s)}{c\theta} + \frac{m(\theta)(\phi m(\theta) + \lambda)}{(\lambda + m(\theta))^2}.$$

Using equations (10) and (13) for γ and ϕ , the sign of $\frac{\partial u}{\partial \lambda}$ is the same as that of

$$\frac{-\frac{m(\theta)}{\lambda + m(\theta)}(b - s)}{c\theta} + \frac{m(\theta)\left[\frac{\lambda(b - s) - c\theta(r + \lambda)}{c\theta} + \lambda\right]}{(\lambda + m(\theta))^2} = \frac{-m(\theta)[(b - s)m(\theta) + c\theta r]}{c\theta(\lambda + m(\theta))^2} < 0.$$

Thus, $\frac{\partial u}{\partial \lambda} < 0$.